SCALAR MODEL OF RADIATION TRANSFER THROUGH THE ATMOSPHERE ABOVE AN INHOMOGENEOUS NON–LAMBERTIAN REFLECTING SURFACE

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We present a general solution of the scalar integro–differential equation of transfer of natural radiation through the atmosphere above an inhomogeneous non–Lambertian surface in the visible and near infrared ranges. The atmosphere is treated as plane–parallel, with standard vertical profiles of the volume coefficients of scattering and extinction. Within the physically acceptable limits the dependence of the reflection coefficient of the surface on horizontal coordinates and angles of incidence and reflection may remain arbitrary. A model is proposed of the field of brightness, based on this general solution. Such a model provides a methodologically high computational accuracy of algorithms of the theory of radiation transfer.

In Refs. 1–7 the authors developed some models to describe the radiation transfer through the atmosphere above an inhomogeneous non–Lambertian Earth's surface. Most general solution of the boundary–value problem for a stationary equation describing the radiation transfer through an atmospheric layer over a surface with an arbitrary reflection coefficient is presented in Refs. 4 and 5. Certain simplifying assumptions on the character of multiple reflections of radiation from the surface are usually made in such a case.

When constructing such a model, the authors of Refs. 4 and 5 referred to an assumption,[2] according to which the first order reflection from a surface agrees with the exact coefficient of reflection, while multiple reflections are described by a hemispherical albedo. Such an assumption makes it possible to present the field of brightness in a compact form and at the same time significantly reduce the computer time without any extra loss of accuracy. The authors of Refs. 4 and 5 presented the coefficient of reflection as a sum of weighted coefficients of reflection of the basic surfaces, factorized over the angular and spatial variables. Such a presentation of the reflection coefficient is a necessary condition for the technique of optical spatial frequency characteristics to be used.

In this study we use the general solution, which is transformed following Ref. 5 and the above simplifying assumption concerning the mode of interaction of radiation with the surface. However, in contrast to Ref. 5, the reflection coefficient is assumed to be an arbitrary function of both the angular and spatial variables, within the physically acceptable limits. The absence of any additional limitations on the reflection coefficient makes it possible to construct a model which is more general and compact than that form Ref. 5. When constructing the model presented here we used the technique of multiple reflections developed in Ref. 8 for the Lambertian underlying surface.

Earlier models of brightness distribution resulting from a preset values of the reflection coefficients and from the model proposed in Refs. 3–5 and 8 are only particular cases of the present model.

Consider the transfer of radiation through a plane–parallel atmosphere over a flat surface with inhomogeneous non–Lambertian reflection. Let z be the vertical coordinate; $\mathbf{r} = \{x, y\}$ be the vector of horizontal coordinates; $z = 0$ be the top of the atmosphere; $z = h$ be the level of the Earth's surface; πS_λ be the spectral solar constant; $\mathbf{s} = \{\mu, \mathbf{s}_\perp\}$ be the unit vector of propagating radiation, $\mathbf{s} \in \Omega$, Ω be the unit sphere; $\mathbf{s}_\perp = \sqrt{1-\mu^2}\,\{\cos\varphi, \sin\varphi\}$; $\mu = \cos\Theta$; Θ, φ be the zenith and the azimuth angles; $\mathbf{s}_0 = \{\zeta, \sqrt{1-\zeta^2}, 0\}$ be the unit vector of incidence of the solar radiation; $\zeta = \cos\Theta_0$; Θ_0 be the zenith angle of the Sun; $\Gamma_0 = \{z = 0, \mathbf{s} \in \Omega_+\}$, $\Gamma_h = \{z = h, \mathbf{s} \in \Omega_-\}$ be the inner boundaries of the atmospheric layer; Ω_+ and Ω_- be the lower and the upper hemispheres; $\alpha(z)$, $\sigma(z)$ be the volume coefficients of extinction and scattering; $f(z, \mathbf{s}, \mathbf{s}_0)$ be the single scattering phase function; $\rho(\mathbf{r}, \mathbf{s}, \mathbf{s}_0)$ be the coefficient of reflection; $I(z, \mathbf{r}, \mathbf{s}, \mathbf{s}_0)$ be the spectral brightness of radiation.

Spectral brightness of radiation satisfies the boundary–values problem

$$LI = SI, \qquad I\Big|_{\Gamma_0} = \pi S_\lambda\, \delta(\mathbf{s} - \mathbf{s}_0), \qquad I\Big|_{\Gamma_h} = RI; \tag{1}$$

where $L = (\nabla, \mathbf{s}) + \alpha(z)$ is the transfer operator; S is the operator of scattering

$$SI = \frac{\sigma(z)}{4\pi} \int\limits_{\Omega} f(z, \mathbf{s}, \mathbf{s}')\, I(z, \mathbf{r}, \mathbf{s}', \mathbf{s}_0)\, d\mathbf{s}';$$

R_ρ is the operator of reflection

$$R_\rho\, I = \frac{1}{\pi} \int\limits_{\Omega_+} \rho(r, \mathbf{s}, \mathbf{s}')\, I(h, \mathbf{r}, \mathbf{s}', \mathbf{s}_0)\, d\mathbf{s}'.$$

Solution of the boundary–value problem (1) is presented in the following form:[4,5]

$$I(z, \mathbf{r}, \mathbf{s}, \mathbf{s}_0) = D(z, \mathbf{s}, \mathbf{s}_0) + Z(\mathbf{r} - \tilde{\mathbf{r}}, \mathbf{s}, \mathbf{s}_0)\, T(\mu) +$$

$$+ \int\limits_{\Omega_-} \int\limits_{-\infty}^{\infty} \tilde{O}(z, \mathbf{r} - \tilde{\mathbf{r}} - \mathbf{r}', \mathbf{s}, \mathbf{s}')\, Z(\mathbf{r}', \mathbf{s}', \mathbf{s}_0)\, d\mathbf{r}'\, d\mathbf{s}'\,; \tag{2}$$

where

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$$Z(\mathbf{r}, \mathbf{s}, \mathbf{s}_0)=E_\rho(\mathbf{r}, \mathbf{s}, \mathbf{s}_0)+\sum_{n=1}^{\infty} \int_{\Omega} \int_{-\infty}^{\infty} \dots \int_{\Omega} \int_{-\infty}^{\infty} Q(\mathbf{r}, \mathbf{r}-\mathbf{r}_n, \mathbf{s}, \mathbf{s}_n) \times$$

$$\times\, Q(\mathbf{r}_n, \mathbf{r}_n - \mathbf{r}_{n-1}, \mathbf{s}_n, \mathbf{s}_{n-1}) \dots Q(\mathbf{r}_2, \mathbf{r}_2 - \mathbf{r}_1, \mathbf{s}_2, \mathbf{s}_1)\times$$

$$\times\, E_\rho(\mathbf{r}_1, \mathbf{s}_1, \mathbf{s}_0)\, d\mathbf{r}_1\, d\mathbf{s}_1 \dots d\mathbf{r}_n\, d\mathbf{s}_n; \tag{3}$$

$D(z, \mathbf{s}, \mathbf{s}_0)$ is the brightness of an atmospheric haze;

$\tilde{O}_\delta(z, \mathbf{r} - \tilde{\mathbf{r}}, \mathbf{s}, \mathbf{s}')=O_\delta(z, \mathbf{r} - \tilde{\mathbf{r}}, \mathbf{s}, \mathbf{s}')-T(\mu')\, \delta(\mathbf{r} - \tilde{\mathbf{r}})\, \delta(\mathbf{s} - \mathbf{s}')$;

$O_\delta (z, \mathbf{r}, \mathbf{s}, \mathbf{s}')$ is the pulse transient function of the system of transfer of directional radiation through the atmospheric layer;

$O_h(\mathbf{r}, \mathbf{s}', \mathbf{s}_1) = O_\delta (h, \mathbf{r}, \mathbf{s}', \mathbf{s}_1)$; $T(\mu) = \exp \{- (\tau_0 - \tau) / \eta\}$;

$\eta = |\mu|$; $\tau = \int_0^z \alpha(z')\, dz'$ is the optical vertical coordinate;

$\tau_0 = \int_0^h \alpha(z')\, dz'$ is the optical thickness of the atmosphere;

$\tilde{r} = s_\perp(h - z) / \eta$ is the displacement vector;

$$E_\rho(\mathbf{r}, \mathbf{s}, \mathbf{s}_0) = 2 \int_0^1 \rho^0(\mathbf{r}, \mu, \mu')\, D^0 (h, \mu', \zeta)\, \mu'\, d\mu' +$$

$$+\, \zeta\, \rho(\mathbf{r}, \mathbf{s}, \mathbf{s}_0)\, S_k\, e^{-\tau_0/\xi};$$

$$Q(\mathbf{r}, \mathbf{r}-\mathbf{r}_1, \mathbf{s}, \mathbf{s}_1) = \frac{1}{\pi} \int_{\Omega_+} \rho(\mathbf{r}, \mathbf{s}, \mathbf{s}')\, O_h(\mathbf{r}-\mathbf{r}_1, \mathbf{s}', \mathbf{s}_1)\, \mu'\, d\mathbf{s}';$$

$$D^0(h, \mu, \zeta) = \frac{1}{2\pi} \int_0^{2\pi} D(h, \mathbf{s}, \mathbf{s}_0)\, d\varphi,\quad \rho^0(\mathbf{r}, \mu, \mu') =$$

$$= \frac{1}{2\pi} \int_0^{2\pi} \rho(\mathbf{r}, \mathbf{s}, \mathbf{s}')\, d\varphi.$$

Functions D and O_δ satisfy the basic boundary–value problems

$$\bar{L}D = SD + SI_{\text{dir}}, \qquad D\,\big|_{\Gamma_0} = 0, \qquad D\,\big|_{\Gamma_h} = 0;$$

$$L\, O_\delta = S\, O_\delta, \qquad O_\delta\big|_{\Gamma_0} = 0, \qquad O_\delta\big|_{\Gamma_h} = \delta(\mathbf{r})\, \delta(\mathbf{s} - \mathbf{s}');$$

where $\bar{L} = \mu\, d/d\,z + \alpha(z)$, and $I_{\text{dir}} = \pi\, S_\lambda\, \delta(\mathbf{s} - \mathbf{s}_0)\, e^{-\tau/\xi}$ is the brightness of the direct nonscattered radiation.

Presentation (2)–(3) has the highest level of generalization among the solutions of the initial boundary–value problem (1). In order to transform this presentation into a form suitable for making a computational procedure, certain simplifying model assumptions should be made. Assume that the first order reflection occurs from the surface with a preset reflection coefficient $\rho\, (\mathbf{r}, \mathbf{s}, \mathbf{s}_0)$, while reflections of every other order happen from an effective Lambertian surface with a mean albedo $\bar{q}$. The value of $\bar{q}$ is an average component of the hemispherical albedo

$$q(\mathbf{r}) = \frac{1}{\pi^2} \int_{\Omega_+} \int_{\Omega_-} \rho(\mathbf{r}, \mathbf{s}, \mathbf{s}')\, \mu\, \mu'\, d\mathbf{s}'\, d\mathbf{s} \tag{4}$$

within some range of $\mathbf{r} = \{x, y\}$ variation. Estimates from Refs. 2 and 9 indicate that the accepted simplification of the model of multiple reflections from the surface results only in insignificant additional error in computed values of I. In accordance with the simplification assumed above the sought approximate solution of the boundary–value problem (1), as it follows from (2) and (3) has the form

$$I_\rho(z, \mathbf{r}, \mathbf{s}, \mathbf{s}_0) = D(z, \mathbf{s}, \mathbf{s}_0) + \bar{\Psi}_{\delta,0}(z, \mathbf{s}, \mathbf{s}_0) +$$

$$+\frac{\bar{q}\, \bar{C}_{\delta,0}(z)\, \Psi_0(z, \mu)}{1 - \bar{q}\, C_0} + \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \left\{ \tilde{\Psi}_\delta(z, \mathbf{p}, \mathbf{s}, \mathbf{s}_0) + \frac{\Psi(z, \mathbf{p}, \mathbf{s})}{1 - \bar{q}\, C(\mathbf{p})} \times \right.$$

$$\left. \times \left[\bar{q}\, \tilde{C}_\delta\, (\mathbf{p}, \mathbf{s}_0) + \frac{\hat{\tilde{q}}(\mathbf{p})\, \bar{C}_{\delta,0}(\xi)}{1 - \bar{q}\, C_0} \right] \right\} e^{-i(\mathbf{p},\, \mathbf{r})}\, d\mathbf{p}; \tag{5}$$

where

$$\Psi_{\delta,0}(z, \mathbf{s}, \mathbf{s}_0) = \int_{\Omega_-} \Psi_{\delta,0}(z, \mathbf{s}, \mathbf{s}')\, E_{\bar{\rho}}(\mathbf{s}', \mathbf{s}_0)\, d\mathbf{s}', \tag{6}$$

$$\bar{C}_{\delta,0}(\zeta) = \frac{1}{\pi} \int_{\Omega_+} \bar{\Psi}_{\delta,0}(h, \mathbf{s}, \mathbf{s}_0)\, \mu\, d\mathbf{s}, \tag{7}$$

$$\Psi_0(z, \mu) = \int_{\Omega_-} \Psi_{\delta,0}(z, \mathbf{s}, \mathbf{s}')\, d\mathbf{s}', \tag{8}$$

$$C_0 = 2 \int_0^1 \Psi_0(h, \mu)\, \mu\, d\mu, \tag{9}$$

$$\tilde{\Psi}_\delta(z, \mathbf{p}, \mathbf{s}, \mathbf{s}_0) = \int_{\Omega_-} \Psi_\delta(z, \mathbf{p}, \mathbf{s}, \mathbf{s}')\, \hat{E}_{\tilde{\rho}}(\mathbf{p}, \mathbf{s}', \mathbf{s}_0)\, d\mathbf{s}', \tag{10}$$

$$\Psi(z, \mathbf{p}, \mathbf{s}) = \int_{\Omega_-} \Psi_\delta(z, \mathbf{p}, \mathbf{s}, \mathbf{s}')\, d\mathbf{s}', \tag{11}$$

$$\tilde{C}_\delta(\mathbf{p}, \mathbf{s}_0) = \frac{1}{\pi} \int_{\Omega_-} \tilde{\Psi}_\delta(h, \mathbf{p}, \mathbf{s}, \mathbf{s}_0)\, \mu\, d\mathbf{s}, \tag{12}$$

$$C(\mathbf{p}) = \frac{1}{\pi} \int_{\Omega_-} \Psi(h, \mathbf{p}, \mathbf{s})\, \mu\, d\mathbf{s}, \tag{13}$$

$$E_{\bar\rho}(\mathbf{s}, \mathbf{s}_0)=2\int_0^1 \bar\rho^0(\mu, \mu')D^0(h, \mu', \zeta)\,\mu'\,d\mu'+\zeta\, S_\lambda\, \bar\rho(\mathbf{s}, \mathbf{s}_0)e^{-\tau_0/\xi}, \tag{14}$$

$$\hat E_{\tilde\rho}(\mathbf{p}, \mathbf{s}, \mathbf{s}_0) = 2\int_0^1 \hat{\tilde\rho}^0(\mathbf{p}, \mu, \mu')\, D^0(h, \mu', \zeta)\, \mu'\, d\mu' +$$

$$+ \zeta\, S_\lambda\, \hat{\tilde\rho}(\mathbf{p}, \mathbf{s}, \mathbf{s}_0)\, e^{-\tau_0/\xi}; \tag{15}$$

$\mathbf{p} = \{p_x, p_y\}$ is the vector of spatial frequencies; ^ is the symbol of Fourier transform over the coordinates $\mathbf{r} = \{x, y\}$; $\bar\rho(\mathbf{s}, \mathbf{s}_0)$ is the average component of the reflection coefficient;

$\tilde\rho(\mathbf{r}, \mathbf{s}, \mathbf{s}_0) = \rho(r, s, s_0) - \bar\rho(s, s_0)$ is the variation of the reflection coefficient; $\tilde q(\mathbf{r}) = q(\mathbf{r}) - \bar q$ is the variation of the hemispherical albedo;

$$\bar\rho^0(\mu, \mu') = \frac{1}{2\pi}\int_0^{2\pi} \bar\rho(\mathbf{s}, \mathbf{s}')\, d\varphi;$$

$$\hat{\tilde\rho}^0(p, \mu, \mu') = \frac{1}{2\pi}\int_0^{2\pi} \hat{\tilde\rho}(\mathbf{p}, \mathbf{s}, \mathbf{s}')\, d\varphi;$$

$$\hat{\tilde q}(\mathbf{p}) = \int_{-\infty}^{\infty} \tilde q(\mathbf{r})\, e^{i(\mathbf{p},\, \mathbf{r})}\, d\mathbf{r};$$

$\Psi_\delta\,(z, \mathbf{p}, \mathbf{s}, \mathbf{s}')$ is the optical spatial frequency characteristic of an atmospheric layer in the case of a source of directed radiation placed at the bottom of the layer at the point $|\,\mathbf{r}\,| = 0$. The function $\Psi_\delta(z, \mathbf{p}, \mathbf{s}, \mathbf{s}')$ satisfies the boundary–value problem

$$\hat L\, \Psi_\delta = S\, \Psi_\delta; \quad \Psi_\delta\Big|_{\Gamma_+} = 0; \quad \Psi_\delta\Big|_{\Gamma_-} = \delta\,(\mathbf{s} - \mathbf{s}'), \tag{16}$$

where

$$\hat L = \mu\, \partial/\partial\, z + \alpha(z) - i(\mathbf{p}, \mathbf{s}_\perp).$$

The numerical technique for solving the problem (16) was developed in Ref. 10. Functions (6)–(15) are calculated by means of quadratures.

Expression (5) is the exact solution of our problem for the accepted simplified model of multiple reflections from the surface. Besides, it yields a compact expression for radiation brightness. Thus, such an expression combines compactness and high accuracy. Note that no additional limitations are imposed on the reflection coefficient. Thus obtained solution generalizes the result from Ref. 5, the latter giving the solution of the same problem under the same assumptions on the nature of multiple reflections from the surface but with the reflection coefficient of the form

$$\rho(\mathbf{r}, \mathbf{s}, \mathbf{s}_0) = \sum_{n=1}^{N} h_n(\mathbf{r})\, \bar\rho_n(\mathbf{s}, \mathbf{s}_n), \tag{17}$$

where $\bar\rho_n(\mathbf{s}, \mathbf{s}_0)$ are the coefficients of reflection of uniform basic surfaces; $h_n(\mathbf{r})$ are weighting functions. In the case when expression (17) is valid for the reflection coefficient, expression (5) is reduced into the expression similar to that from Ref. 5. Thus statement may be tested directly. In the case of Lambertian reflection we have $\rho(\mathbf{r}, \mathbf{s}, \mathbf{s}_0) \to q(\mathbf{r})$, and expression (5) is reduced to a well–known solution.[8]

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