BRIGHTNESS OF THE SCATTERED LIGHT FROM A PROJECTOR IN A CLOUDY MEDIUM

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A formula for calculating the brightness distribution of scattered light within the image of a directed light source, which is observed through a cloudy medium, is proposed. It takes into account the viewing geometry, the salient features of the light scattering properties of the medium, and their stratification along the observation path. The range of applicability of the proposed formula is determined by comparing it with the data obtained by the Monte Carlo method. The results of comparison show that this formula can be employed to describe the brightness of the image of the projector with the width of the angular distribution of light intensity $\theta_0^2 > 0.4\cdot\tau\cdot\beta^2$, where τ is the optical thickness and β is the effective width of the light scattering phase function in a cloudy medium. In this case the distributions of brightness of the scattered light are adequately described within the viewing angles $\varphi < \theta_0$ when the angles of deflection of the axis of the angular distribution of light intensity from the direction toward the observer do not exceed $2\theta_0$.

In order to estimate the visibility of spatially bounded directed light sources (projectors) in a turbid atmosphere, it is necessary to know the brightness of the scattered light produced by them. Some investigations of the brightness distributions within the image of the projector in a cloudy medium based on numerical solution of the radiation transfer equation by the Monte Carlo method were performed in Refs. 1 and 2. When a light beam propagates through the cloudy medium, in which the light scattering phase function is strongly elongated, the angular distribution of brightness everywhere retains considerable anisotropy. For this case, the formulas for calculating some integral parameters of the spatial and angular brightness distributions produced by the directed light source in the scattering medium were derived in Refs. 3–5. There, the shape of both angular and spatial distributions was assumed to be Gaussian. At the same time, the real light fields, especially near the sources with narrow angular distribution of light intensity, can substantially differ from the Gaussian fields. Based on the results of Refs. 1–5, we propose the formula, which adequately describes the brightness distribution of the scattered light within the image of the narrow–beam projectors observed through the scattering medium.

Let the projector be observed through the scattering medium from the point located at the distance L from the source. The light scattering parameters of the medium, namely, the scattering coefficient σ, the extinction coefficient ε, and the scattering phase function $\chi(\theta)$ can vary along the observation path. The angular distribution of light intensity of the projector is assumed to be Gaussian with the following parameters: θ_0 is the half–width of the distribution and I_0 is the maximum light intensity. We assume that the image of the source is formed by angular scanning of the light field with the receiver with narrow field–of–view angle located at the observation point. Let us specify the viewing direction of the receiver by the vector $\boldsymbol{\varphi}\,(\varphi_x, \varphi_y)$, whose components prescribe the angular coordinates of the point in the image plane counted off from the direction toward the projector. If the axis of the angular distribution of light intensity of the projector makes an angle θ with the direction toward the receiver, the brightness distribution of scattered light within the image of the projector observed through a scattering medium can be calculated based on the following formula:

$$B(\varphi, \theta, L) = \frac{I_0}{L^2} f(\varphi, L) \exp\left[-\frac{(\theta - \varphi\cdot L\cdot A/D_r)^2}{\theta_0^2 + D_\theta - A^2/D_r}\right], \qquad (1)$$

where

$$D_\theta = D_\theta(L) = \int_0^L \sigma(z)\cdot\beta^2(z)\,dz\,; \qquad (2)$$

$$A = A(L) = \int_0^L z\cdot\sigma(z)\cdot\beta^2(z)\cdot dz\,; \qquad (3)$$

$$D_r = D_r(L) = \int_0^L z^2\,\sigma(z)\cdot\beta^2(z)\cdot dz\,, \qquad (4)$$

z is the current coordinate along the observation path, β is the effective width of the light scattering phase function in the medium, and the vector $\boldsymbol{\theta}\,(\theta_x, \theta_y)$ specifies the angle of deflection of the axis of the angular distribution of light intensity of the projector from the direction toward the observation point. The function $f(\varphi, L)$ describes the brightness distribution of the scattered light within the image of a point diffusely glowing source of light of unit power observed from the distance L. The methods for calculating this function have been developed in detail in Refs. 5 and 6. The formulas for calculating the parameters A, D_r, and D_θ were taken from Ref. 5.

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Formula (1) is analogous to the solution of the radiation transfer equation in a small–angle diffuse approximation, which has been obtained in Refs. 3–5. In this approximation, the solution for the function $f(\varphi, L)$ is sought after in the form of a Gaussian distribution. But, as it has already been mentioned above, the real distributions of the brightness of the point light sources can substantially differ from Gaussian even for the wide–beam sources. In addition, as has been noted in Refs. 1–2, the brightness of the scattered light near the directed source, which is observed through a cloudy medium, virtually coincides with that from a diffusely glowing source with the intensity of light equal to the intensity of a directed source in the direction toward the observation point. It has been also shown there that the effect of finite width of the angular distribution of light intensity of the source is manifested in narrowing the brightness distributions when the width of the angular distribution decreases. In this connection, it seems possible to employ formula (1) in order to calculate the brightness distributions within the image of the projector observed through the cloudy medium and fog. The finite width of the angular distribution of light intensity of the projector is taken into account in Eq. (1) by means of the exponent, in which the parameters A, D_r, and D_θ as well as the function $f(\varphi, L)$ depend on the stratification of the scattering properties in the medium. Formula (1) permits one to estimate the behavior of the image of the projector depending on the light–scattering parameters of the medium and on their stratification along the observation path as well as on the viewing geometry. As is shown below, formula (1) ensures sufficiently high accuracy of calculations of the brightness of the images of signalling projectors used in practice under the real conditions of the turbid atmosphere.

In order to calculate the images of diffusely glowing objects in the scattering medium, the small–angle approximation developed for media with the strongly elongated scattering phase function is efficient.[5,6] In this approximation the brightness distribution of the scattered light formed by a point receiver with the half–width of the field–of–view angle φ_0 can be calculated according to the following relation:

$$f(\varphi, L) = \frac{1}{\pi\varphi_0}\int\limits_0^\infty \left[F(\nu, \tau) - \exp(-\tau)\right] J_0(\nu\varphi)\cdot J_1(\nu\varphi_0)\,d\nu\,, \quad (5)$$

where $F(\nu, \tau)$ is the optical transfer function (OTF) of the layer of the scattering medium,[5,6] $\tau = \int\limits_0^L \varepsilon(z)\,dz$ is the optical thickness of the medium, and $J_0(x)$ and $J_1(x)$ are the Bessel functions. The OTF characterizes the degree of distortions introduced in the spatial frequency spectrum of the image of the object observed through the medium by this scattering medium. In general, the OTF depends on the stratification of the light scattering properties in the medium[7]

$$F(\nu, \tau) = \exp\left[-\tau + \int\limits_0^1 \sigma(z)\frac{Q(\nu z, z)}{Q(0, z)}\,dz\right], \quad (6)$$

where

$$Q(x, z) = \frac{1}{2\pi}\int\limits_0^\infty \chi(\theta, z)\cdot\theta\cdot J_0(\theta x)\,d\theta.$$

In calculating the OTF, the shape of the scattering phase function should be taken into account over entire range of the angles of scattering in the forward direction.[8] As the analysis shows, in polydisperse media such as clouds and fogs the angular dependence of the scattering phase function can be described by the following formula:

$$\chi(\theta) = \frac{2\alpha}{\beta^2}\exp\left(-\frac{\theta^2}{\beta^2}\right) + \frac{(1-\alpha)}{\gamma^2}\exp\left(-\frac{\theta}{\gamma}\right), \quad (7)$$

where, adjusting the parameters α, β, and γ, it is possible to ensure sufficiently high accuracy for a wide range of the scattering angles. The first term in Eq. (7) describes the spike of the scattering phase function strongly elongated in the forward direction as a result of the diffraction of light on the particles of the scattering medium. The second term takes into account the refraction and reflection of light by the particles of the medium in the geometric optics approximation. In order to describe the size distributions of the droplets of fog and clouds, the gamma–distribution with the parameters μ and r_M, which specify the distribution width and modal radius of particles, is often employed.[9] Using the method of optimal parameterization based on the results of calculation of the scattering phase functions from Mie's formulas we found that for the visible range of wavelengths within a wide range of variation of the parameters of gamma distribution ($5 < \mu < 15$ and $1\ \mu m < r_M < 10\ \mu m$) the parameters α and γ, which enter into Eq. (7) describing the phase functions vary within relatively narrow limits, namely, $\alpha = 0.4$–0.44 and $\gamma = 14$–$16°$. The formula from Ref. 10 $\beta = 2/\kappa\rho_M$, where $\kappa = \sqrt{(1 + 5/\mu)(1 + 6/\mu)}$, $\rho_M = 2\pi r_M/\lambda$, and λ is the radiation wavelength, adequately describes the parameter β as a function of the microstructure of the scattering medium. Thus, for example, for Deirmendjian's model of water–droplet C1 cloud[9] ($\mu = 6$ and $r_M = 4\ \mu m$) with the parameters $\alpha = 0.43$, $\beta = 1.76°$, and $\gamma = 14.6°$ the light scattering phase function at the wavelength $\lambda = 0.7\ \mu m$ is described by Eq. (7) with an error exceeding 5% within the scattering angles $0° \le \theta \lesssim 75°$ with the exception of the region $5° < \theta < 7°$, in which the error does not exceed 25%.

The use of Eq. (7) permits one to obtain an analytic formula for the OTF and considerably simplify the calculations of the function $f(\varphi, L)$ in integrating numerically Eq. (5). In so doing, the calculational error due to approximate description of the scattering phase function by formula (7) is insignificant. Taking into account the fact that a considerable anisotropy of scattering in the cloudy medium is primary due to the diffraction spike of the scattering phase function, in calculating the image of the projector according to formulas (1)–(4), it is necessary to use the width of the diffraction part of the scattering phase function (7) as the characteristic widths of the phase function β. In calculations, we must use the "effective" scattering coefficient $\sigma_e = (1 - \eta)\cdot\sigma$, where σ is the real characteristic of the cloudy medium and η is the fraction of light scattered by unit volume into the back hemisphere at the angles $\theta > 90°$ (see Refs. 3 and 5). For the Deirmendjian's C1 cloud model $\eta = 0.045$.

The limits of applicability of formula (1) were determined by comparing the calculated results with the data obtained by the Monte Carlo method.[1,2,11] The statistical error in the data used for comparison was about 10–20%. In what follows, we will consider the data for the images of the projector formed by the receiver with the half–width of the field–of–view angle $\varphi_0 = 1'$, i.e., for the images analogous to those formed by a human eye. Figure 1 shows the calculated profiles of the brightness distribution of scattered light within the image of the projector with the half–width of the angular distribution of light intensity $\theta_0 = 3°$ observed through the homogeneous cloudy medium. We employ Deirmendjian's C1 cloud model as the scattering medium. We present the data for the cases, in which the observation was carried out from the point located on the optical axis of the light beam ($\theta = 0°$) and at the angle $\theta = 6°$ with respect to the optical axis. In the latter case, the profiles of the brightness distributions are shown in the plane, which contains the optical axis of the beam and the viewing line directed toward the beam. The viewing angles φ for the plots in Fig. 1 are counted off from the direction toward the light source and the viewing directions toward the deflection of the beam axis correspond to the positive values of the angle φ.

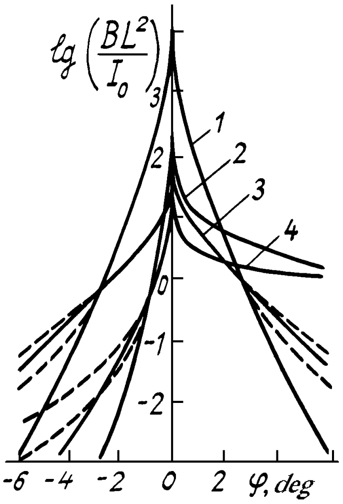

FIG. 1. The profiles of the brightness distribution of the image of the projector with $\theta_0 = 3°$ for the homogeneous path. The solid curves indicate the results of calculations according to formula (1) and the dashed curves denote the results of calculations by the Monte Carlo method: 1) $\tau = 1$ and $\theta = 0°$, 2) $\tau = 1$ and $\theta = 6°$, 3) $\tau = 7$ and $\theta = 0°$, and 4) $\tau = 7$ and $\theta = 6°$.

As can be seen from Fig. 1, the results calculated according to formula (1) and obtained by the Monte Carlo method virtually coincide when the viewing angles are close to the direction toward the projector. Such a coincidence is observed at the optical thickness of the cloudy medium up to $\tau \approx 9$, i.e., for the optical thicknesses, where the small–angle approximation is applicable.[11] When the viewing angles are large, formula (1) gives underestimated results. Note that here the brightness of the scattered light is less than the brightness in the maximum of the distributions by 3 orders of magnitude or more and it can be neglected in most practical cases. The analysis of the calculated data shows that with broadening of the angular distribution of light intensity of the projector the range of applicability of formula (1) extends. Thus, within the viewing angles $\varphi \lesssim \theta_0$, the error of calculations based on formula (1) does not exceed 30%. Here the width of the angular distribution of light intensity of the projector, whose image can be described by formula (1), must satisfy the relation $\theta_0^2 > 0.4 \cdot \tau \cdot \beta^2$ in order for formula (1) to be applicable to describe the image of the projector.

When observing the projector at an angle with respect to its optical axis, the brightness distributions of scattered light become asymmetric. In this case formula (1) describes the brightness distributions with sufficiently high accuracy (not worse than the accuracy of calculations by the Monte Carlo method) when the angles of deflection of the axis of the angular distribution of light intensity of the projector from the direction toward the observer $\theta \lesssim 2\theta_0$.

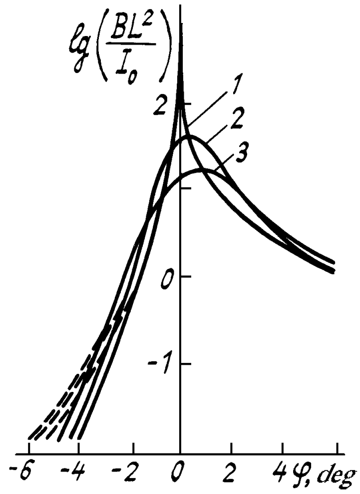

FIG. 2. The profiles of the angular distribution of brightness of the image of the projector with $\theta_0 = 5°$ observed at an angle $\theta = 6°$ for $\tau = 3$ on the path with the gap m: 1) $m = 0$, 2) $m = 0.35$, and 3) $m = 0.9$.

As shown in Refs. 1 and 11, the formula for the brightness distributions of the scattered light depends strongly on the stratification of the scattering properties of the medium along the observation path. Thus, in particular, when observing through a cloudy layer located at a certain distance from the projector the distributions of the scattered light are bell–shaped. The calculated results of the brightness distributions of scattered light when the projector is observed at an angle with respect to the optical axis in the case, in which the cloudy medium occupies only a part of the observation path and there is a gap between the projector and the cloudy layer, in which the light is not scattered, are shown in Fig. 2. We give the calculated results for identical optical thicknesses of the layer of a cloudy medium and for different relative length of the gap m normalized to the path length. As can be seen from the plots of Fig. 2, for the considered viewing geometry, the maximum in the distributions shifts toward the direction of deflection of the optical axis of the projector, while the value of the shift of the maximum depends on the relative length of the gap. The comparison of the results calculated according to formula (1) with the data obtained by the Monte Carlo method shows that the use of Eq. (1) for this viewing geometry also gives quite good results and the range of its applicability is the same as in the above–considered case of the homogeneous path.

In conclusion, let us note that until now the numerical Monte Carlo method is virtually the unique method, which allows one to take into account in general the viewing geometry, the salient features of the light scattering properties of the medium, and their stratification along the observation path. For this reason, the use of formula (1) for estimating the brightness of the image of the projector under conditions of the turbid atmosphere seems to be useful. In spite of some limitations on the range of applicability of this formula, it permits one to take into account all main features peculiar to the image of the directed light sources observed through the scattering medium and can be widely used for solving the applied problems connected with the utilization of signaling projectors in the turbid atmosphere, for example, when calculating the visibility of the light–signaling pattern of an airfield observed by a pilot, who lands his aircraft under conditions of clouds and fog.[12]

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