Numerical analysis of observation data of regional pollution of an area source

V.F. Raputa,[1] S.E. Ol'kin,[2] and I.K. Reznikova[2]

[1]*Institute of Computational Mathematics and Mathematical Geophysics, Siberian Branch of the Russian Academy of Sciences, Novosibirsk*
[2]*State Research Center of Virology and Biotechnology "Vector," Koltsovo, Novosibirsk Region*

Received January 28, 2008

Models for long-term regional pollution estimation were worked out. Small-parameter representations of emission concentration fields of area sources were obtained by asymptotic expansion methods. Field observation results of snow cover pollution in the vicinity of Novosibirsk were numerically analyzed.

Introduction

Numerical modeling of area-source pollution propagation in atmosphere is one of the most complicated problems. Pollution propagation is closely connected with the current dynamical, thermal, and humidity air conditions, as well as the type of underlying surface. Especial difficulties arise in description of spatial distribution and temporal dynamics of pollutants emitted by an area source.[1–5]

When using direct modeling methods of pollution propagation in the atmosphere, accurate accounting for all the above factors results in quite bulky mathematical models, including many parameters needed a further refinement, which is not always possible due to present-day technical and economical possibilities. Therefore, a more accurate joint analysis of experimental data and theoretical description of pollution propagation in the ground and boundary atmospheric layers is required, as well as accounting for *a priori* data on source parameters and spatiotemporal scales of the processes under study. In particular, data on the wind speed and direction can be used instead of current meteorological data when modeling long-term ground pollution by stationary sources.[6] There is a possibility of comparatively simple mathematical description of pollution propagation to large distances from the source.[7–9] The use of the similarity and dimensionality methods, as well as asymptotic methods in a number of cases allows a decrease of the number of unknown parameters. Observation planning on the base of theoretical model of pollution propagation results in an increase of experimental data informativity.[10]

1. Models for regional pollution estimation

A description of the fields of pollutant concentrations in atmosphere at significant distances from sources allows essential simplifications. Experimental and theoretical investigations show that the distribution of pollutant concentrations equalizes throughout the height beginning from distances of about 7–10 km from the source located in the boundary atmospheric layer.[9] At such distances, the influence of some parameters, such as the source height, aerosol particle deposition rate, coefficient of vertical turbulent exchange, etc., becomes insignificant.

A point source. The field of long-term averaged concentration of a point-source pollution is described as[7,8]

$$\bar{q}(r,\varphi) = \frac{Mg(\varphi)}{2\pi u h r}, \tag{1}$$

where r and φ are the polar coordinates of the initial point centered at the source position; $g(\varphi)$ is the probability of the opposite wind direction φ at heights of the boundary atmospheric layer; M is the source intensity; u and h are the mean wind speed and the mixing layer thickness.

Under supposition that the density of aerosol precipitation is proportional to the air pollution concentration, obtain

$$\Phi(r,\varphi) = \frac{\theta g(\varphi)}{r}. \tag{2}$$

Here $\theta = \lambda M/(2\pi u h)$; λ is the factor of the pollutant–underlying surface interaction.

Note 1. Equation (2) can be represented otherwise, taking into account that u and h in this case have the sense of some average characteristics of the boundary atmospheric layer. Actually, if to define the function $B(u', h')$, describing the joint probability density of u', h' distribution in the preset direction φ from the source for the period under study, then the total density of aerosol precipitation can be represented as

$$\Phi(r,\varphi) = \frac{\lambda M g(\varphi)}{2\pi r}\iint_{\Omega}\frac{B(u',h')}{u'h'}\mathrm{d}\Omega = \frac{\theta' g(\varphi)}{r},$$

where

$$\theta' = \frac{\lambda M}{2\pi}\iint_\Omega \frac{B(u',h')}{u'h'}\mathrm{d}\Omega.$$

Then, in view of the generalized integral mean-value theorem for the single-connected domain Ω, obtaine[11]:

$$\iint_\Omega \frac{B(u',h')}{u'h'}\mathrm{d}\Omega = \frac{1}{uh}\iint_\Omega B(u',h')\,\mathrm{d}\Omega = \frac{1}{uh}.$$

An area source. Taking into account Eq. (2), the density of aerosol precipitation in the case of the area source S has the form

$$Q(x,y) = \frac{\lambda}{2\pi uh}\iint_S \frac{m(\xi,\eta)g(\varphi)}{d}\mathrm{d}\xi\mathrm{d}\eta, \quad (3)$$

where (ξ,η) are the current source coordinates; $(\xi,\eta)\in S$; $m(\xi,\eta)$ is the pollutant emission from this point;

$$\varphi(\xi,\eta,x,y) = \arctan\left(\frac{y-\eta}{x-\xi}\right);$$

$$d = |\mathbf{M}_1\mathbf{M}| = \sqrt{(x-\xi)^2+(y-\eta)^2},$$

the point (x,y) is also supposed as sufficiently distanced from S.

In practice, the intensity of area source emission $m(\xi,\eta)$ is usually unknown or can be set only approximately. Hence, the observation data interpretation with Eq. (3) is quite difficult. In this case, it is reasonable to estimate Eq. (3) with the use of its asymptotic representation and concentrations measured at different distances from S.

For

$$d = \sqrt{r^2+r_1^2-2rr_1\mu},\quad \frac{1}{d} = \frac{1}{r\sqrt{1+\alpha^2-2\alpha\mu}}, \quad (4)$$

where $r = |\mathbf{OM}|$, $r_1 = |\mathbf{OM}_1|$; $\alpha = r_1/r$; $\mu = \cos\theta$, the mutual positions of M, M_1, and the area source S are shown in Fig. 1.

Using the Eq. (4) series expansion in terms of Legendre polynomials, obtain[12]:

$$\frac{1}{d} = \frac{1}{r}\sum_{n=0}^{\infty}\alpha^n P_n(\mu), \quad (5)$$

where $P_n(\mu)$ is the nth order Legendre polynomial.

Substituting Eq. (5) in Eq. (3), we have

$$Q(x,y) = \frac{c}{r}\sum_{n=0}^{\infty}\iint_S \alpha^n P_n(\mu)m(\xi,\eta)g(\xi,\eta,x,y)\mathrm{d}\xi\mathrm{d}\eta =$$

$$= Q_1+Q_2+Q_3+\ldots = \frac{c}{r}\iint_S mgP_0(\mu)\mathrm{d}\xi\mathrm{d}\eta +$$

$$+\frac{c}{r^2}\iint_S mgr_1P_1(\mu)\mathrm{d}\xi\mathrm{d}\eta + \frac{c}{r^3}\iint_S mgr_1^2P_2(\mu)\mathrm{d}\xi\mathrm{d}\eta + \ldots. \quad (6)$$

Here $c = \lambda/(2\pi uh)$.

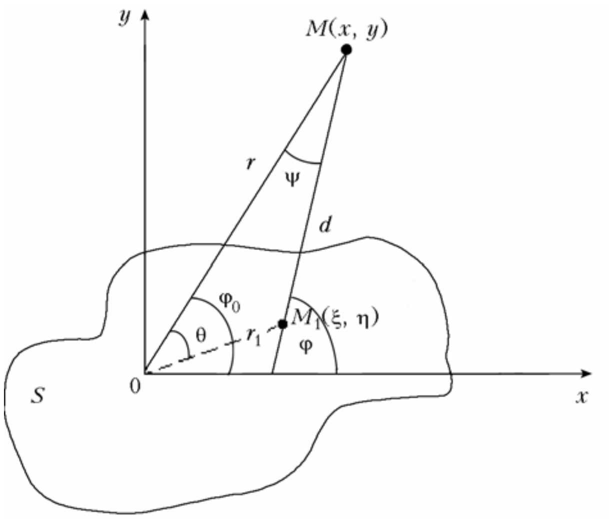

Fig. 1. Mutual pposition of M, M_1, and the area source S.

Consider the first item in Eq. (6), giving an initial approximation for calculation of the pollutant concentration field at large r. Approximate the function g:

$$g(\varphi) \cong g(\varphi_0) + g'(\varphi_0)\psi, \quad (7)$$

where

$$\varphi_0 = \arctan\frac{y}{x};\quad \psi = \varphi - \varphi_0.$$

Note, that

$$\cos\psi = \frac{x(x-\xi)+y(y-\eta)}{rd} = \frac{r^2-x\xi-y\eta}{rd}. \quad (8)$$

Then, taking the identity $\arcsin\psi + \arccos\psi = \frac{\pi}{2}$ and Eq. (5) into account, obtain

$$\psi = \arccos\frac{r^2-x\xi-y\eta}{rd} \approx \frac{\pi}{2} - \frac{r^2-x\xi-y\eta}{rd} =$$

$$= \frac{\pi}{2} - \frac{r^2-x\xi-y\eta}{r^2}\sum_{n=0}^{\infty}\alpha^n P_n(\mu). \quad (9)$$

Let $d \approx r$ at small angular sizes of the domain S. In this case, come to the following approximation:

$$\psi \approx \frac{\pi}{2} - \frac{r^2-x\xi-y\eta}{r^2} = \frac{\pi}{2} - 1 - \frac{x}{r^2}\xi - \frac{y}{r^2}\eta. \quad (10)$$

Taking into account Eqs. (7) and (10), obtain

$$g(\varphi) \cong g(\varphi_0) + g'(\varphi_0)\left[\frac{\pi}{2} - 1 - \frac{x}{r^2}\xi - \frac{y}{r^2}\eta\right]. \quad (11)$$

Substitution of Eq. (11) into the first term of expansion (6), yields

$$Q_1(x,y) = \frac{c}{r}\iint_S m(\xi,\eta)\times$$

$$\times\left\{g(\varphi_0)+\left(\frac{\pi}{2}-1\right)g'(\varphi_0)-g'(\varphi_0)\left(\frac{x}{r^2}\xi+\frac{y}{r^2}\eta\right)\right\}\mathrm{d}\xi\mathrm{d}\eta =$$

$$=\theta_1\frac{g(\varphi_0)+\left(\frac{\pi}{2}-1\right)g'(\varphi_0)}{r}+\theta_2\frac{g'(\varphi_0)x}{r^3}+\theta_3\frac{g'(\varphi_0)y}{r^3}, \quad (12)$$

$$\theta_1 = c\iint_S m(\xi,\eta)\mathrm{d}\xi\mathrm{d}\eta,\quad \theta_2 = -c\iint_S \xi m(\xi,\eta)\mathrm{d}\xi\mathrm{d}\eta,$$

$$\theta_3 = -c\iint_S \eta m(\xi,\eta)\mathrm{d}\xi\mathrm{d}\eta.$$

Note 2. If the wind rose is uniform, $g'(\varphi) = 0$. In this case, only θ_1 is to be estimated by Eq. (12). This property can be also used in estimating the fields of aerosol pollutant precipitation in the sectors, where $g(\varphi)$ varies insignificantly.

2. Experimental study of snow cover pollution in the vicinity of Novosibirsk

Study of snow and soil cover is a handy and economical way for obtaining data on pollution income from the atmosphere to the underlying surface. These studies are of special interest when investigating long-term pollution processes. The intensity and configuration of the concentration field are defined by the emission magnitude, the length of accumulation period, the source position, the wind direction repetition, etc.[13–15]

The field investigations of snow cover pollution by dust, heavy metals, and polyaromatic hydrocarbons were performed at the end of winter seasons of 2005/06 and 2006/07 in the vicinity of Novosibirsk (the scheme of snow sampling is shown in Fig. 2).

Sampling itineraries were chosen with accounting for the prevailing urban pollution propagation and the route system. The Tomsk route, located north-east of Novosibirsk, most satisfies these requirements. Winter repetition of southwest and west winds is about 60% at heights of the boundary atmospheric layer.[16]

Sampling was carried out throughout the snow cover depth. The chemical tests of the snow samples for heavy metals and polyaromatic hydrocarbons (PAH) were performed in laboratories of Institutes of SB RAS and the State Research Center of Virology and Biotechnology "Vector."

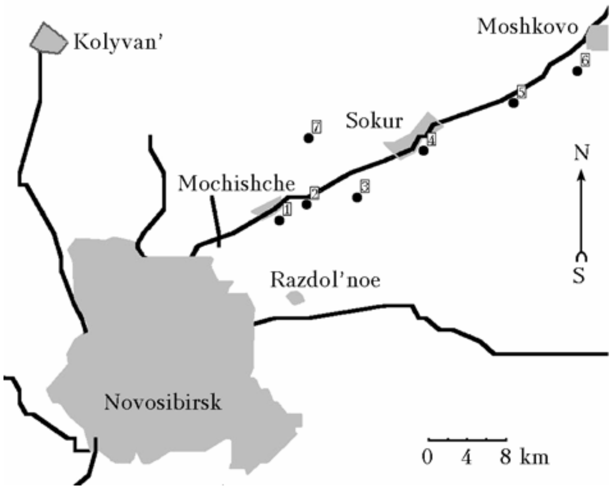

Fig. 2. Scheme of snow sampling in the vicinity of Novosibirsk. The points indicate the sites of sampling.

3. Numerical analysis of field observation data

The quantitative interpretation of the data of field and laboratory-analytical investigations was performed on the base of regression Eq. (12) as applied to the northeast sector of the regional pollutant propagation from urban sources. The grid origin was in the center of Novosibirsk (the Opera Theatre). Taking into account *Note 2*, the only unknown parameter θ_1 is to be estimated in this case. To do this, point No. 1, located before the Mochishche village and 18-km distanced from the city center, was taken as the reference one. The results of quantitative interpretation of the experimental data for the considered winter seasons are shown in Figs. 3 and 4.

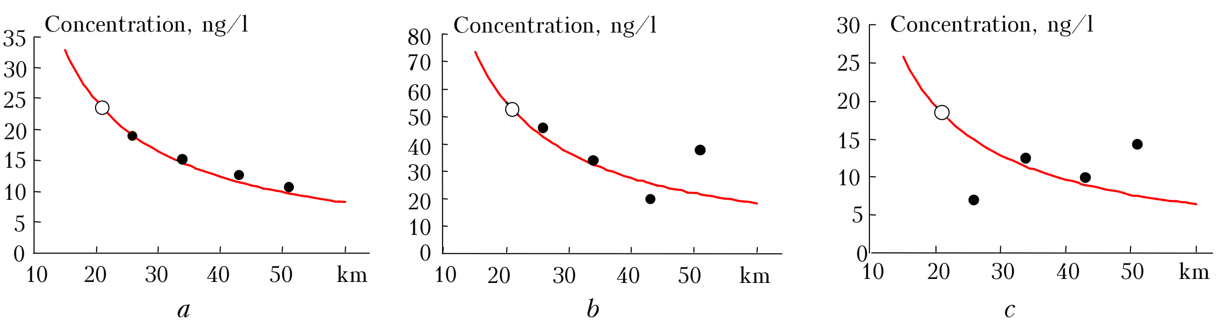

Fig. 3. Measured and retrieved concentrations of fluorene (*a*), phenanthrene (*b*), and benz(a)pirene (*c*) northeastward from Novosibirsk in the winter of 2005/06: ○ are the reference points, ● are the check points; curves show the concentration retrieved by model (12).

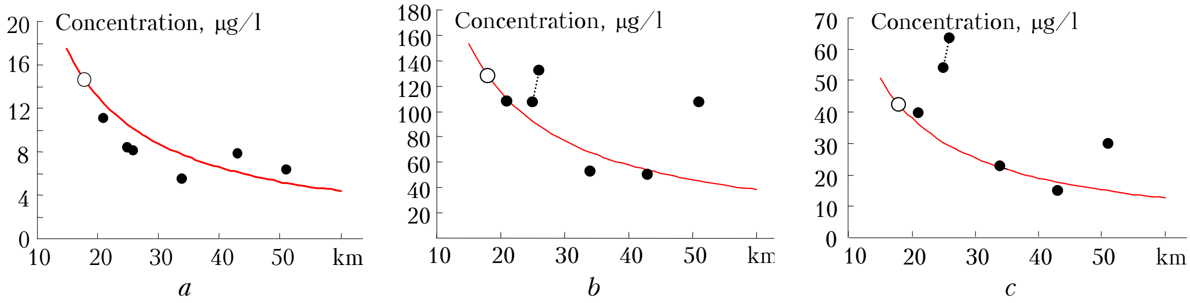

Fig. 4. Measured and retrieved precipitation levels of zinc (*a*), iron (*b*), and dust (*c*) northeastward from Novosibirsk in the winter of 2006/07.

Analysis shows a satisfactorily agreement between calculated and observational results in the check points. Despite a significant spread of heavy metals and PAH concentrations in some points, on the whole, the contribution of urban sources prevails in the snow cover pollution. Precipitation of dust and PAH components, in particular, benzapirene, are quite high and essentially affect the sickness rate of the city population. The real possibility of the use of dust precipitation as a tracer should be underlined, which can essentially decrease the number of relatively expensive chemical tests, having limited them to determination of concentrations of considered chemical elements and compounds in a number of reference points.

Conclusion

The results of experimental study of snow cover pollution at the end of winter seasons of 2005–2007 and numerical analysis of the obtained observation data have allowed us to find numerical laws of regional transfer of dust, heavy metals, and PAH from the Novosibirsk area. The retrieved density fields of pollutant precipitation onto the snow cover in the vicinity of the city serve as an integral characteristic of long-term environmental effect of the area source.

The obtained regularities allow the development of the economical monitoring system, estimation on its base of the long-term air pollution in the city, and determination of emission of characteristic pollutants from its area.

Acknowledgements

This work was financially supported by the Program of Fundamental Research of the Presidium of RAS (Project No. 16.6).

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