Motion of an aerosol particle in a sound field

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The oscillating motion and drift of an aerosol particle in a gas under the action of a sound field are studied at the arbitrary Knudsen numbers. The relaxation time, the flow coefficient, the phase shift of gas and particle oscillations and the drift velocity are calculated as functions of Knudsen number and the sound wave frequency. It is shown that the drift velocity of aerosol particles can change its direction at the intermediate Knudsen numbers.

Introduction

One of the possible methods of solving the problem of aerosol precipitation is to affect an aerodisperse system by high-intensity sound waves. A sufficiently complete overview of the fundamental experimental and theoretical investigations in the field of acoustic aerosol precipitation is given in Ref. 1. It should be noted that until so far the results of these investigations not only remain of current interest but also are the main source of data on the aerosol motion under the effect of sound. In recent years, this problem has received little attention for the reason that the theory agrees well with the experimental data[2] obtained at the atmospheric pressure for large particles of radius from 1 to 4 μm.

In a gaseous medium exposed to sound the particles take part in the vibrational gas motion and simultaneously move progressively, i.e., drift under the action of some effects of the second order (radiation pressure, asymmetry of the shape of a sound wave, and so on).

At present, only the hydrodynamic theory of aerosol motion in the gas exposed to sound has been developed quite adequately. It is known that the hydrodynamic description of aerosol motion is based on the solution of the Navier–Stokes equation with the boundary condition of "sticking" of gas to the particle surface. It is evident that such a theory cannot describe the motion of submicron aerosol particles, for which the Knudsen number (Kn is the ratio of the mean free path of gas molecules l to the particle radius r) is not small. Therefore, it is of prime interest to develop a model applicable to aerosol particles of any size, at any gas pressure, and sound wavelengths.

The goal of work was to study theoretically the vibrational motion and drift of an aerosol particle in a sound field as functions of Kn number and the acoustic wave frequency.

Statement of the problem

Now we consider a spherical particle of the radius r, suspended in a gas, whose state is disturbed by a running or standing sound waves. Let ω be the cyclic frequency, $\lambda = 2\pi c/\omega$ be the wavelength, and c be the sound speed in the gas.

We have restricted ourselves to the case when the mean free path of gas molecules l is much less than the acoustic wavelength λ. This implies that in relation to the sound propagation the gas is a continuous medium. On the other hand, we consider that the mean free path of gas molecules can be arbitrary with respect to the radius of an aerosol particle (the arbitrary Knudsen number $\mathrm{Kn} = l/r$). This implies that for an aerosol particle gas is a rarefied medium. Consequently, the hydrodynamic theory of particle motion in a gas is inapplicable. Let the particle radius be much less than the sound wavelength. This is a realistic condition. Really, for particles of micron size this condition is fulfilled in a wide wavelength range including the ultrasonic range.

Thus, the formulation of the problem is based on the following relations of the particle radius, mean free path of gas molecules, and acoustic wavelength:

$$l \ll \lambda, \; r \; \forall \; l, \; r \ll \lambda.$$

Table 1 illustrates the field of application of the theory under standard conditions ($p = 10^5$ Pa, $l = 0.1$ μm).

Table 1. Dimensions of particles and wavelength range corresponding to the formulation of the problem under standard conditions ($p = 10^5$ Pa, $l = 0.1$ μm)

Kn	r, μm	λ, μm
0.01	10	≥ 100
0.1	1	≥ 10
1	0.1	≥ 1
10	0.01	≥ 1

Hence it follows that the accepted approximation in the case of standard conditions covers practically the entire range of degree of dispersion of atmospheric aerosol and the entire range of wavelengths except for a narrow band of the upper boundary of the ultrasonic range (minimum wavelength equals to 0.3 μm). In the last line of Table 1 at Kn = 10 the wavelength range $\lambda \geq 1$ μm is given because the smaller values of λ break the accepted inequality $\lambda \gg l$.

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Vibrational motion

The rate of gas flow vibrating harmonically along the axis x varies according to the law

$$u_g = U_g \sin \omega t, \quad (1)$$

where U_g is the amplitude of the rate variation. In this case, the particle experiences the action of "static" and "kinematic" forces.[3] Static forces are connected with the gradient of pressure in acoustic wave, are proportional to the mass of a displaced gas and therefore are small. Kinematic forces are mainly caused by the resistance when flowing around the particle.[1,3]

In Ref. 4, based on the molecular-kinetic theory, the calculations have been made of the force of the resistance when flowing around a spherical particle by a flow of rarefied gas at arbitrary Kn numbers. In this paper the analytical expression is derived approximating the numerical calculation for a force of resistance

$$F = 6\pi\eta r f (u_g - u_p),$$
$$f = \frac{0.619}{\mathrm{Kn} + 0.619}\left(1 + \frac{0.310\mathrm{Kn}}{\mathrm{Kn}^2 + 1.152\mathrm{Kn} + 0.785}\right), \quad (2)$$

where η is the coefficient of gas viscosity; u_p is the particle motion rate.

Then the equation of motion for the particle rate of mass m_p is written in the form

$$m_p \frac{du_p}{dt} = 6\pi\eta r f (u_g - u_p). \quad (3)$$

This equation differs from that used previously[1] by the factor f depending on the Kn number. Therefore the steady periodic solution of Eq. (3) is of the same form as in Ref. 1:

$$u_p = \mu_p U_g \sin(\omega t - \varphi), \ \mu_p = \frac{1}{\sqrt{1 + \omega^2 \tau^2}}, \quad (4)$$

where μ_p is the coefficient of particle entraining in the vibrational motion, which shows the number of times of difference of amplitudes of vibrations of a particle and gas; τ is the time of particle relaxation determined by the expression:

$$\tau = \frac{\tau_0}{f}, \ \tau_0 = \frac{2}{9}\frac{\rho_p}{\eta} r^2. \quad (5)$$

where τ_0 is the time of relaxation in the hydrodynamic limit $\mathrm{Kn} \to 0$ (Ref. 1); ρ_p is the particle density. The phase shift angle φ in Eq. (4) for the particle motion rate is determined by the equation

$$\tan\varphi = \omega\tau. \quad (6)$$

The calculation indicates that with increasing the Kn number the value of relaxation time can both decrease and increase. If the gas pressure is fixed the larger Kn number and lesser relaxation time correspond to the particles of smaller size. In this case the relaxation time is proportional to the square of the particle radius ($\tau \sim r^2$) in hydrodynamic regime ($\mathrm{Kn} \to 0$) and linearly depends on the radius ($\tau \sim r$) in the free molecular regime ($\mathrm{Kn} \to \infty$). If the particle size is fixed then when decreasing gas pressure the Kn number also increases but in this case the relaxation time increases. In the free molecular regime of particle flow the time of relaxation is determined by the following expression:

$$\tau_k = \frac{\pi}{\pi + 8}\frac{\rho_p}{p}\bar{v} r, \ \bar{v} = \left(\frac{8 k_B T_0}{\pi m}\right)^{1/2}, \quad (7)$$

where p is the gas pressure; k_B is the Boltzmann constant; m is the mass of gas molecule; T_0 is the mean gas temperature; $\bar{v}$ is the mean rate of thermal molecular motion.

As the relaxation time changes, the particle flow coefficient and phase shift of particle oscillation also change according to formulae (4) and (6). Table 2 shows the values of relaxation time, particle flow coefficient and phase shift for particles of different size, which are in the air under normal conditions ($\rho_p = 1$ g/cm^3).

Table 2. Values of the relaxation time, the particle flow, and the phase shift coefficient for particles in the air ($\rho_p = 1$ g/cm^3, $p = 10^5$ Pa, $\eta = 1.85 \cdot 10^{-5}$ Pa·s) at different cyclic frequencies

r, μm	Kn	τ_0, s	τ, s	ω, 10^3 s^{-1}			
				6.28	62.8	188.4	314
				μ_p; φ, deg.			
10	0.01	$1.21 \cdot 10^{-3}$	$1.22 \cdot 10^{-3}$	0.129 82.6	0.013 89.3	0.0044 89.8	0.0026 89.9
5	0.02	$3.01 \cdot 10^{-4}$	$3.08 \cdot 10^{-4}$	0.459 62.7	0.052 87.0	0.017 89.0	0.010 89.4
2	0.05	$4.80 \cdot 10^{-5}$	$5.10 \cdot 10^{-5}$	0.952 17.8	0.298 72.7	0.104 84.1	0.062 86.4
1	0.1	$1.20 \cdot 10^{-5}$	$1.35 \cdot 10^{-5}$	0.996 4.85	0.763 40.3	0.366 68.5	0.230 76.7
0.5	0.2	$3.00 \cdot 10^{-6}$	$3.75 \cdot 10^{-6}$	1.00 1.35	0.973 13.3	0.817 35.2	0.647 49.7
0.2	0.5	$4.81 \cdot 10^{-7}$	$7.93 \cdot 10^{-7}$	1.00 0.285	0.999 2.85	0.989 8.50	0.970 14.0
0.1	1	$1.20 \cdot 10^{-7}$	$2.84 \cdot 10^{-7}$	1.00 0.102	1.00 1.02	0.999 3.06	0.996 5.10
0.05	2	$3.01 \cdot 10^{-8}$	$1.17 \cdot 10^{-7}$	1.00 0.042	1.00 0.421	1.00 1.26	0.999 2.10
0.02	5	$4.80 \cdot 10^{-9}$	$4.14 \cdot 10^{-8}$	1.00 0.015	1.00 0.149	1.00 0.447	1.00 0.745
0.01	10	$1.20 \cdot 10^{-9}$	$2.00 \cdot 10^{-8}$	1.00 0.0072	1.00 0.072	1.00 0.216	1.00 0.360

Table 2 shows that for small particles ($r \le 0.1$ μm) the hydrodynamic relaxation time of a particle τ_0 is comparable with the mean time of a free molecular path in a gas that is incorrect. The smaller is the particle, the greater is the degree of gas flow in the vibrational motion and the smaller is the phase shift. For particles of the fixed size the coefficient of gas flow decreases with the increasing gas vibrational frequency and the phase shift grows. The calculation shows that at low frequencies the gas flow coefficient

weakly depends on the Kn number, and in the infrasonic range this dependence is lacking. The greater is the frequency of sound vibrations, the stronger is the dependence of the gas flow coefficient on the Kn number.

Particle drift

It is known[1] that the directed translational motion of aerosol particles (drift) in a gas exposed to sound is determined by a number of factors: the radiation pressure of sound on particles, periodic variation of viscosity of fluctuating gas, distortion of acoustic wave shape, asymmetry of vibrational motion in a standing acoustic wave.

The radiation pressure is the mean over time value of the acoustic wave pressure in the close vicinity of an obstacle, calculated with the account of the interaction between the acoustic wave and unperturbed medium. In a running wave, the force of radiation pressure is directed toward the wave propagation, and in a standing wave – from the node to the antinode of vibrations where the sound pressure tends to zero. The expression for this force was derived in Ref. 5 and corrected with regard to the vibrational motion of particles in Ref. 1 for the case of running and standing acoustic waves. Because we are interested in the translational motion of a particle, it is assumed that the force of radiation pressure at each moment in time is equalized by the force of gas resistance determined by Eq. (2). Then the rate of radiation drift of a particle in the field of running and standing acoustic waves is determined by the following expressions:

$$V_R = \frac{11}{54\eta f}\left(\frac{\omega}{c}\right)^4 r^5 \mu_g^2 \bar{E}, \quad V_R = \frac{4}{9\eta f}\left(\frac{\omega}{c}\right) r^2 \mu_g^2 \bar{E} \sin 2kx,$$

$$\mu_g = \sqrt{1-\mu_p^2} = \frac{\omega\tau}{\sqrt{1+\omega^2\tau^2}}; \quad \bar{E} = \frac{I}{c}, \qquad (8)$$

where $\bar{E}$ is the energy density in an incident wave; μ_g is the coefficient of flowing around a particle; k is the wave number; x is the coordinate of the aerosol particle center; I is the intensity of the acoustic wave. Equations (8) can be used either at low gas pressures or for sufficiently large particles when the energy dissipation of acoustic wave in the atmospheric boundary layer is insignificant. As the numerical estimations indicate, under standard conditions in the case of angular frequency $62.8 \cdot 10^3$ s^{-1} Eqs. (8) are valid for particles of radius $r > 700$ μm in a running wave and $r > 25$ μm in a standing wave. The motion of such particles is well described by the hydrodynamic theory. In our case, the sound pressure can be neglected as compared with the mechanisms given below.

Adiabatic gas compressions and rarefactions in the sound field produce periodic temperature variations and the appropriate variations of viscosity. As a result, the force appears affecting a particle in the direction toward the sound source in the running wave or toward a node of oscillations in the standing wave. The calculation of the above force is given in Ref. 6.

From the condition of equality of this force and the resistance force (2) we obtain the following expressions for the drift rate in the case of the running and standing waves:

$$V_\eta = \frac{\gamma-3}{2\rho_0 c_0 f}\mu_g^2 \bar{E}, \quad V_\eta = \frac{\gamma-3}{2\rho_0 c_0 f}\mu_g^2 \bar{E} \sin 2kx, \qquad (9)$$

where γ is the adiabatic exponent; ρ_0 and c_0 are the gas density and the speed of sound at average temperature T_0. In Ref. 6 it was assumed that the oscillations of temperatures of a particle and gas occur simultaneously. Therefore, Eqs. (9) are valid for particles with high thermal conductivity.

In case of unharmonic sound oscillations of a gas the suspended particles undergo nonsinusoidal oscillation motions. In this case a certain force occurs, which shifts the equilibrium configuration, relative to which the suspended particle oscillates, that is, the shift of this particle takes place. A mechanism of occurrence of the shift force for a sawtooth form of acoustic wave is discussed in detail in Ref. 1. In Ref. 6 acoustic wave is presented as a superposition of harmonic oscillations, and in the approximation of the second harmonic the force is calculated affecting the particle. Equating this force to the Stokes resistance force (2), we obtain the following expression for the shift velocity:

$$V_h = -\frac{h_2 \sin\psi}{\pi\eta f} r \mu_g^2 \bar{E}, \qquad (10)$$

where h_2 is the ratio of the amplitude of the second harmonic of the acoustic wave and the amplitude of the fundamental harmonic; ψ is the phase shift of the second harmonic. It is evident that the direction of this force coincides with the direction of acoustic wave propagation if the phase shift ψ is negative. If $\psi > 0$, then the force is directed toward the acoustic wave source.

In the standing acoustic wave the shift amplitudes and gas velocities increase sinusoidally with distance from the node of oscillations. This results in the accelerated gas motion, when gas moves in the direction of antinode, and in slowing down when moving back. The suspended particle, having certain inertia, lags behind the gas motion at direct shift and is ahead at back shift. As a result, the asymmetry of vibrational gas motion results in the appearance of a force affecting an aerosol particle. Using the expression for this force, obtained in Ref. 7, for the particle drift velocity we have

$$V_a = \frac{k\mu_p r^2}{18\eta f}\left[\frac{9}{2}\left(b^2+b\right)\mu_g - \left(3+\frac{9}{2}b\right)\mu_p\right]\bar{E}\sin 2kx,$$

$$b = \frac{1}{r}\sqrt{\frac{2\nu}{\omega}}, \qquad (11)$$

where ν is the kinematic gas viscosity.

It should be noted that Eqs. (8)–(11) differ from the expressions for velocities given in Ref. 1 not only by the function f, but also by that the coefficients of entrainment μ_p and flow around μ_g depend on the Kn number.

The resulting velocity of the particle drift is

$$V = V_\eta + V_h + V_a . \qquad (12)$$

Table 3 shows the calculated results on the drift velocity of particles of different size in the air: $\eta = 1.85 \cdot 10^{-5}$ Pa·s, $\rho = 1.29$ kg/m^3 at the atmospheric pressure $p = 10^5$ Pa and the angular frequency $\omega = 62.8 \cdot 10^3$ s^{-1}.

Table 3. Values of the drift velocity of particles of different size in the atmosphere exposed to sound ($\eta = 1.85 \cdot 10^{-5}$ Pa·s, $\rho = 1.29$ kg/m^3, $p = 10^5$ Pa)

r, μm	Kn	$\omega = 62.8 \cdot 10^3$ s^{-1}; $I = 0.1$ W/cm^2; $h_2 = 0.5$; $\psi = \pi/2$			
		$-V_\eta \cdot 10^3$, cm/s	$-V_h \cdot 10$, cm/s	$V_a \cdot 10^2$, cm/s	V, cm/s
10	0,01	559	260	6,63	−26,5
5	0.02	565	131	22.4	−13.4
2	0.05	535	50	111	−4.43
1	0.1	260	12	190	0.44
0.5	0.2	36.2	0.842	88.9	0.769
0.2	0.5	2.28	$2.12 \cdot 10^{-2}$	23.4	0.230
0.1	1	0.423	$1.97 \cdot 10^{-3}$	10.9	0.108
0.05	2	0.116	$2.70 \cdot 10^{-4}$	6.75	0.0674
0.02	5	$3.22 \cdot 10^{-2}$	$2.99 \cdot 10^{-5}$	4.96	0.0496
0.01	10	$1.46 \cdot 10^{-2}$	$6.80 \cdot 10^{-6}$	4.56	0.0456

We notice that the components of the drift velocity connected with oscillations of the gas viscosity and the acoustic wave distortion, decrease fast with reducing the particle size. The drift velocity, due to asymmetry of vibrational motion, at a preset gas pressure varies slowly with the increasing Kn number. The resulting drift velocity at the intermediate numbers of Kn changes its sign. This means that in the sound field an aerodisperse system should be divided into large and small fractions. It is self-evident that this conclusion calls for further experimental check.

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