Methodological aspects of reconstructing optical characteristics of the atmosphere from data of laser radar measurements

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Based on the analytical expressions derived for errors in determination of both total and aerosol extinction coefficients of the atmosphere from laser sensing data, the features common for their reconstruction are explained having in mind complex influence of all factors that burden solving of the lidar equation. Some recommendations are given concerning the choice of calculation algorithms and the use of the *a priori* information about optical parameters in different atmospheric situations (clean, weakly turbid, and optically dense atmosphere) that may help to improve accuracy of the extinction coefficient measurements. The results obtained by numerical simulation, which confirm analytical conclusions, are presented. The possibility of using the obtained analytical expressions for the errors in the extinction coefficients in estimating measurement results in particular situations is shown.

Introduction

The methodological aspects of reconstructing optical characteristics of the atmosphere from backscattering signals have most completely been considered in Ref. 1. The main factors affecting the accuracy of determination of both local, the extinction coefficient $\varepsilon(r)$, and integral, the transmission of the sounding path's part $[r_i, r_j]$, $V^2(r_i, r_j)$, optical characteristics are revealed based on the proposed general vector model of different schemes for processing lidar signals. Analysis of noise immunity of different schemes for processing lidar returns is carried out using general model, the effect of different noises on the accuracy of interpretation is estimated, as well as the requirements to the accuracy of measurements are studied. Partial analytical formulas, which explain the effect of some factors (reference values of the characteristics to be determined, variability of the backscattering phase function g_π, and accuracy of recording the signals) have been obtained in Refs. 2 to 6.

Obviously, solving of such a problem as estimation of the complex effect of all interfering factors on the accuracy of interpretation of lidar measurements is quite urgent. Besides, no analytical expressions have been obtained so far, which would enable one to estimate the accuracy limit in measuring $\varepsilon(r)$ of a medium having *a priori* known the atmospheric situation and specifications of the measurement instrumentation that would make it possible to propose accuracy-optimal algorithms for processing the backscattering signals taking into account all the noise factors. The derived analytical formulas for the errors in solving the lidar equation relative to $\varepsilon(r)$ at complex effect of all aforementioned factors, as well as in the presence of the background and noise in the backscattering signal $P(r)$ are presented below.

1. Analytical formulas for the error in determining the extinction coefficient from data of laser sensing of inhomogeneous two-component media

In the most general case, one can write the lidar equation as follows[7]:

$$S(r) = [P(r) - B]r^2Y(r) = A\varepsilon(r)\exp\left\{-2\int_{r_0}^{r}\varepsilon(x)\mathrm{d}x\right\}, \quad (1)$$

where r_0 is the minimum distance, at which full overlap of the fields of view of the receiver and the transmitted pulse is reached; A is the instrumentation constant of the system;

$$\varepsilon(r) = \varepsilon_\mathrm{a}(r) + \frac{3}{8\pi g_{\pi,\mathrm{a}}(r)}\varepsilon_\mathrm{m}(r)$$

is the absorption coefficient of the medium, $\varepsilon_\mathrm{a}(r)$ and $\varepsilon_\mathrm{m}(r)$, respectively, are the aerosol and molecular extinction coefficients; $g_{\pi,\mathrm{a}}(r)$ is the aerosol backscattering phase function;

$$Y(r) = \frac{1}{g_{\pi,\mathrm{a}}(r)}\exp\left\{-2\int_{r_0}^{r}\left(\frac{3}{8\pi g_{\pi,\mathrm{a}}(x)} - 1\right)\varepsilon_\mathrm{m}(x)\mathrm{d}x\right\}$$

is the *a priori* set function; B is the background component of the signal.

General solution of Eq. (1) has the form[1]:

$$\varepsilon(r) = 0.5S(r)\left[I_k - \int_{r_k}^{r}S(x)\mathrm{d}x\right]^{-1}, \quad (2)$$

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where I_k is the calibration constant that may be determined using *a priori* data on the optical parameters of the medium. In using the local, $\varepsilon(r_k)$, and integral

$$V^2(r_0,r_k) = \exp\left\{-2\int_{r_0}^{r_k} \varepsilon(r)\mathrm{d}r\right\}$$

reference points, the solutions to Eq. (1) are as follows[1,2,7]:

$$\varepsilon(r) = S(r)\left[\frac{S(r_k)}{\varepsilon(r_k)} - 2\int_{r_k}^{r} S(x)\mathrm{d}x\right]^{-1}, \quad (3)$$

$$\varepsilon(r) = S(r)\left\{\frac{2}{1 - V^2(r_0,r_k)}\int_{r_0}^{r_k} S(x)\mathrm{d}x - 2\int_{r_0}^{r} S(x)\mathrm{d}x\right\}^{-1}. \quad (4)$$

Using expressions (3) and (4), one can easily show that the $\varepsilon(r)$ profile reconstructed using the integral reference point $V^2(r_0, r_k)$ is analogous to the profile reconstructed using the local reference point

$$\varepsilon(r_k) = \frac{S(r_k)[V^{-2}(r_0,r_k) - 1]}{2\int_{r_0}^{r_k} S(r)\mathrm{d}r}. \quad (5)$$

As follows from expressions (3) and (4), to solve Eq. (1) one has to know $g_{\pi,\mathrm{a}}(r)$, $\varepsilon_\mathrm{m}(r)$, and reference values of the optical characteristics to be determined.

By introducing the notation $I(r_k,r) = \int_{r_k}^{r} S(x)\mathrm{d}x$ and using the method of finite increments, it is easy to obtain, from Eq. (2), a formula for the error in reconstruction of $\varepsilon(r)$, which has the following form:

$$\delta\varepsilon(r) = \frac{\tilde{\varepsilon}(r) - \varepsilon(r)}{\varepsilon(r)} = \frac{\delta S(r) - \delta I(r)}{1 + \delta I(r)}, \quad (6)$$

where

$$\delta I(r) = \frac{1}{Q(r)V^2(r_k,r)}\Bigg\{Q(r_k)\delta I_k + \left(Q(r_k) - Q(r)V^2(r_k,r) - 2\int_{r_k}^{r} Q(x)\varepsilon(x)V^2(r_k,x)\mathrm{d}x\right) - 2\left(\int_{r_k}^{r} Q(x)\varepsilon(x)V^2(r_k,x)\mathrm{d}x\right)\delta I(r_k,r)\Bigg\}; \quad (7)$$

$$Q(r)=[g_{\pi,\mathrm{a}}(r)\varepsilon_\mathrm{a}(r)+3\varepsilon_\mathrm{m}(r)/8\pi]/[\tilde{g}_{\pi,\mathrm{a}}(r)\varepsilon_\mathrm{a}(r)+3\varepsilon_\mathrm{m}(r)/8\pi]$$

is the distortion factor determined by the difference between the backscattering phase function being preset, $\tilde{g}_{\pi,\mathrm{a}}(r)$, and its true value, $g_{\pi,\mathrm{a}}(r)$;

$$\delta I_k = \left[\delta S(r_k) - \delta\varepsilon(r_k)\right]/\left[1 + \delta\varepsilon(r_k)\right]$$

is the relative error in calculation of the calibration constant determined by the discrepancy between the reference $\varepsilon(r_k)$ value and the signal measured at the reference point r_k;

$$\delta I(r_k,r) = \int_{r_k}^{r} S(x)\delta S(x)\mathrm{d}x \Big/ \int_{r_k}^{r} S(x)\mathrm{d}x$$

is the relative error caused by the presence of the measurement errors in the backscattering signal.

Equation (7) was obtained for the preset local reference point $\varepsilon(r_k)$, however, it is also valid in using the integral calibration. In this case, based on Eq. (5), the value $\delta\varepsilon(r_k)$ in the formula for δI_k is determined by the following relationship

$$\delta\varepsilon(r_k) = \frac{Q(r_k)\left(1 + \delta S(r_k)\right)\left[1 - V^2(r_0,r_k)\right]}{2\int_{r_0}^{r_k} Q(x)\varepsilon(x)V^2(r_0,x)(1 + \delta S(x))\mathrm{d}x} - 1. \quad (8)$$

There are three components of the error in the formula for $\delta I(r)$, the one due to $\delta S(r)$, another one due to inaccuracy of calibration, and the third being the difference between $\tilde{g}_{\pi,\mathrm{a}}(r)$ and $g_{\pi,\mathrm{a}}(r)$. Let us consider the contribution of each of them separately.

2. The effect of accuracy of setting and arrangement of the reference points

To obtain the formula for the error in the reconstructed $\varepsilon(r)$ profile caused by inaccurate setting of the reference value $\varepsilon(r_k)$, let us assume in Eqs. (6) and (7) that $\tilde{g}_{\pi,\mathrm{a}} = g_{\pi,\mathrm{a}}$ and $\delta S(r) = 0$. Then the relationships following from Eqs. (6) and (7) coincide with that obtained earlier.[2–4] Therefore let us present here only main corollaries of these formulas.

1. The error $\delta\varepsilon(r)$ is a function of $\delta\varepsilon(r_k)$ and the optical thickness

$$\tau_\varepsilon(r_k,r) = \int_{r_k}^{r} \varepsilon(x)\mathrm{d}x.$$

In moving to the left from r_k, the error in reconstruction asymptotically tends to zero, while in moving to the right from this point, the error monotonically increases, tending to infinity at

$$\tau_\varepsilon(r_k,r) = -0.5\ln\{\delta\varepsilon(r_k)/[1 + \delta\varepsilon(r_k)]\}$$

that corresponds to the case, when the denominator in Eq. (3) becomes equal to zero at some point. If $\tilde{g}_{\pi,a}(r) = g_{\pi,a}(r)$, then the error in reconstruction $\delta\varepsilon(r)$ will have the same sign as the error in the reference value, more quick convergence of the obtained solution to the exact one is observed at positive deviations of the reference values, then at the negative of the same absolute value.

2. In principle, the position of the reference point is not important at small optical thickness $\tau_\varepsilon(r_k,r)$ and relatively small errors in setting the reference value $\delta\varepsilon(r_k)$. This is correct for lidar systems operating in a relatively clean atmosphere in the visible and infrared ranges, where the contribution of the molecular component is negligible.

3. The relative error in reconstruction becomes more sensitive to the position of the reference point at increasing contribution of the molecular component to the backscatter, and its position deep into the path becomes preferable. The behavior of the error at decreasing of the aerosol backscattering phase function will be similar.

3. Sensitivity of the solution to variability of the aerosol backscattering phase function

One can better demonstrate the effect of inaccurate setting $g_{\pi,a}(r)$ on reconstruction of the $\varepsilon(r)$ profile if using the formula for the error $\delta\varepsilon(r)$ derived from Eqs. (6) and (7) assuming exact local calibration at $\delta S(r) = 0$:

$$\delta\varepsilon(r) = Q(r)V^2(r_k,r)\times$$
$$\times\left[Q(r_k) - 2\int_{r_k}^{r} Q(x)\varepsilon(x)V^2(r_k,x)\mathrm{d}x\right]^{-1} - 1. \quad (9)$$

One can conclude based on Eq. (9) that if the value $Q(r) = \text{const}$ within the path segment $[r_k, r]$, then $\delta\varepsilon(r)$ equals to zero there even if $\tilde{g}_{\pi,a}(r) \neq g_{\pi,a}(r)$, because the integral in the denominator of Eq. (9) can be represented in the form

$$\int_{r_k}^{r} Q(x)\varepsilon(x)V^2(r_k,x)\mathrm{d}x = 0.5Q[1 - V^2(r_k,r)]\,.$$

However, in the general case $Q(r) \neq \text{const}$, that can lead to $\delta\varepsilon_a(r)$ increase along both directions from the point r_k, at which the boundary condition has been set. In using the integral calibration, as follows from Eq. (8), the equality $\delta\varepsilon(r_k) = 0$ is reached at any selected value $\tilde{g}_{\pi,a}$ only if the medium is homogeneous on the entire path's segment $[r_0, r_k]$. However, it is important to note that these corollaries are valid if the reference values, $\varepsilon_a(r_k)$ or $V^2(r_0,r_k)$, are the exact ones.

The profiles $\varepsilon_a(r_k) = \varepsilon(r) - 3\varepsilon_m(r)/8\pi g_{\pi,a}$ reconstructed using local and integral calibration are shown in Fig. 1. It was assumed, for simplicity, that $g_{\pi,a}(r) = \text{const}$, $\tilde{g}_{\pi,a} = g_{\pi,a}$; the profile $\varepsilon_m(r)$ was assumed to be known.

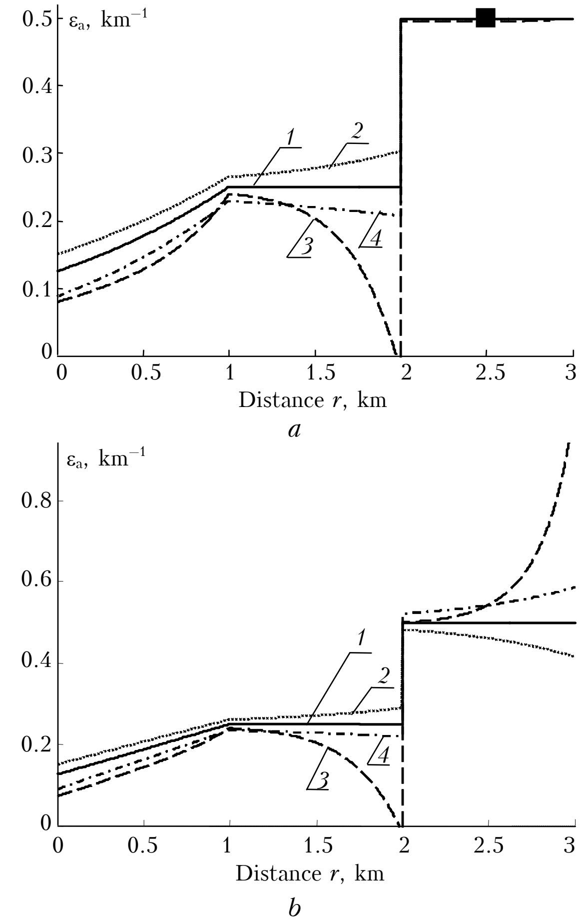

Fig. 1. The profile $\varepsilon_a(r)$ reconstructed using the local (*a*) and integral (*b*) calibration ($g_{\pi,a} = 0.03\ \text{sr}^{-1}$): (*1*) model profile $\varepsilon_a(r)$; (*2*, *3*) reconstructed profiles $\varepsilon_a(r)$ at $\varepsilon_m = 0.145\ \text{km}^{-1}$ and $\tilde{g}_{\pi,a} = 0.05$ and $0.01\ \text{sr}^{-1}$, respectively; (*4*) reconstructed profile $\varepsilon_a(r)$ at $\varepsilon_m = 0.0116\ \text{km}^{-1}$ and $\tilde{g}_{\pi,a} = 0.01\ \text{sr}^{-1}$.

As Figure 1*a* shows, $\delta\varepsilon_a(r) = 0$ independent of the value $\tilde{g}_{\pi,a}$ in a homogeneous medium along the path's segment from 2 to 3 km, where the exact reference value $\varepsilon_a(r_k)$ is set. In the case of integral calibration (Fig. 1*b*) the error in this part of the sounding path is not equal to zero because of variability of $\varepsilon_a(r)$ on the lower path's part 0–2 km that confirms all the above-said.

The profile reconstructed for the distance interval from 1 to 2 km converges to the exact value ε_a in a homogeneous medium if moving to the left from the reference point, the solution changes more quickly in this part, if values $\tilde{g}_{\pi,a}$ are underestimated because of the higher rate of $V^2(r_k, r)$ variation, which depends on $\tilde{g}_{\pi,a}$.

The reconstructed profile for inhomogeneous parts (Fig. 1) deviates from the exact values $\varepsilon_a(r)$ even when approaching to the beginning of the path. In the case of $Q(r) \neq$ const, i.e., although one of the components of $Q(r)$ has changed, divergence of the solution can exist at any position of the reference point. Hence, inhomogeneity of a two-component medium is the dominating factor in the errors of reconstruction of the $\varepsilon_a(r)$ profile because of inaccurate setting of the scattering phase function.

It is also seen from Fig. 1 that, at decreasing the contribution of the molecular component by increasing the wavelength of sounding radiation, the reconstructed profile $\varepsilon_a(r)$ becomes less sensitive to setting the scattering phase function.

To explain the difference between $\varepsilon_a(r)$ values reconstructed using overestimated and underestimated $\tilde{g}_{\pi,a}$ values, we obtained (based on Eq. (7)) the formula for the rate of the $Q(r)$ change as a function of $\varepsilon_a(r)$:

$$\frac{dQ(r)}{d\varepsilon_a(r)} = \frac{\beta_{\pi,m}(r)[g_{\pi,a} - \tilde{g}_{\pi,a}]}{[\beta_{\pi,m}(r) + \varepsilon_a(r)\tilde{g}_{\pi,a}]^2}, \qquad (10)$$

where $\beta_{\pi,m}(r) = g_{\pi,m}\varepsilon_m(r)$ is the molecular backscattering coefficient.

As seen from Eq. (10), at the change of $\varepsilon_a(r)$ by the value $d\varepsilon_a$, the corresponding change in $|dQ(r)|$ is significantly greater if $\tilde{g}_{\pi,a} < g_{\pi,a}$, that explains the larger distortion of the reconstructed profile $\varepsilon_a(r)$.

It is also seen from Eq. (10) that the smaller is $\tilde{g}_{\pi,a}$, the larger is the rate of $Q(r)$ change. The maximum of the $Q(r)$ variation rate is reached, as follows from Eq. (10), under the condition that $\beta_{\pi,m}(r) = \tilde{g}_{\pi,a}\varepsilon_a(r)$. Thus, the reconstructed profile in a clean or weakly turbid medium, where this condition holds most likely, is more sensitive to the errors in setting $g_{\pi,a}$ and hence the choice of its value becomes more critical.

Numerical simulation (Fig. 2*a*) was performed in order to estimate the effect of variability of $g_{\pi,a}(r)$ on the efficiency of reconstruction of $\varepsilon_a(r)$ by iterative method using the functional relation between $g_{\pi,a}(r)$ and $\varepsilon_a(r)$ proposed[8] for conditions of a clean or weakly turbid atmosphere:

$$g_{\pi,a} = 0.02[\varepsilon_a + 0.000415]^{-0.23+0.03\sqrt{\varepsilon_a}}. \qquad (11)$$

As the value $\varepsilon_a(r_k)$ was set precisely, the error in reconstruction of $\varepsilon_a(r)$ was determined only by the error in setting the profile $\tilde{g}_{\pi,a}(r)$.

As is seen from Fig. 2*a*, the use of $g_{\pi,a}(r) =$ const for this medium leads to essential errors along the entire sounding path (the path-mean error is 38%), appearance of negative values $\varepsilon_a(r)$ is observed in the path segment to the right from the reference point. At the same time, the use of iteration algorithm leads to exact reconstruction of the model parameters if the functional relation between $\varepsilon_a(r)$ and $g_{\pi,a}(r)$ is known.

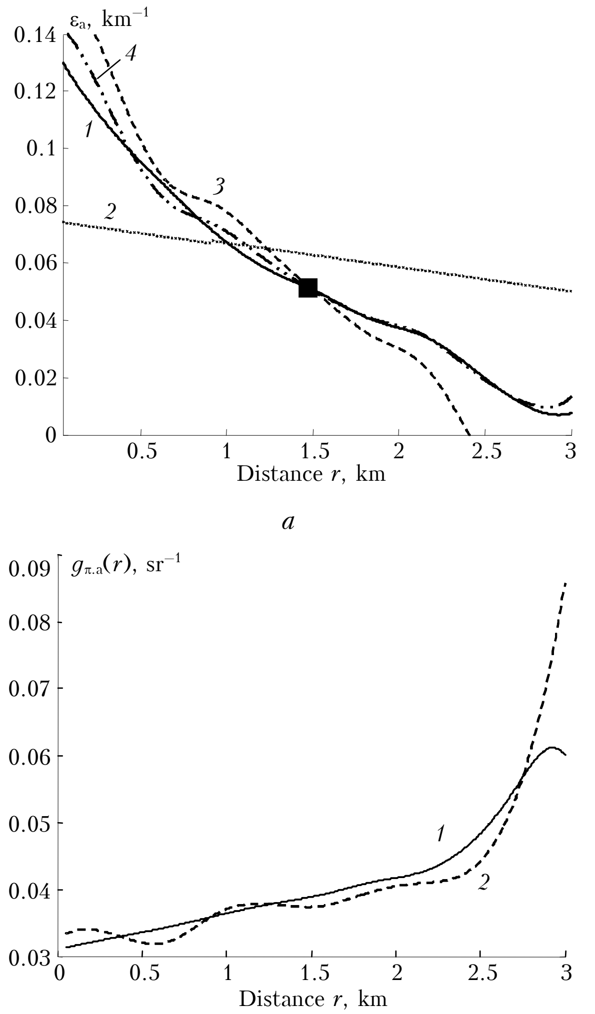

Fig. 2. Examples of reconstruction of the aerosol extinction coefficient profile in an inhomogeneous two-component medium: (*a*) curves *1* and *2* are model profiles of $\varepsilon_a(r)$ and $\varepsilon_m(r)$, respectively; curve *3* is $\varepsilon_a(r)$ profile reconstructed using constant value of the scattering phase function equal to its path-average value; curve *4* is $\varepsilon_a(r)$ profiles, obtained applying the iteration algorithm at reliably known functional relation (11) and at superposing deviations on this dependence, respectively; (*b*) curve *1* is profile of $g_{\pi,a}(r)$, satisfying the relationship (11); curve *2* is disturbed profile of $g_{\pi,a}(r)$.

To estimate the sensitivity of the iteration algorithm to the accuracy of the functional relation between $\varepsilon_a(r)$ and $g_{\pi,a}(r)$ used, deviations of no more than 40% were superposed on the profile $g_{\pi,a}(r)$ satisfying Eq. (11). As is seen in Fig. 2*a*, the quality of reconstruction of $\varepsilon_a(r)$ by iterative method (curve *4*) even in this case remains quite satisfactory (the path-mean error is 7%).

4. Sensitivity of the solution to the error in measuring the backscattering signal

In reconstructing the profile $\varepsilon(r)$ of the atmosphere, the presence of noise in $P(r)$ most strongly affects the calibration constant I_k calculated, especially if local reference point has been placed at the end of the sounding path, where the signal is comparable with noise.[9–11] In this case, even at a precise setting of $\varepsilon(r_k)$, I_k can be calculated with large error equal to $\delta I(r_k) = \delta S(r_k)$. Assuming the white noise, $\delta I(r_k, r) \approx 0$ due to averaging, so, as follows from Eq. (7)

$$\delta I(r) = V^2(r,r_k)\delta S(r_k). \qquad (12)$$

It is easy to derive, from Eqs. (6) and (12), a formula for calculating $\varepsilon(r)$ taking into account noise

$$\delta\varepsilon(r) = \frac{\delta S(r) - V^2(r,r_k)\delta S(r_k)}{1 + V^2(r,r_k)\delta S(r_k)}. \qquad (13)$$

As is seen from Eq. (13), the largest errors in reconstruction appear when the reference point is at a far distance from the lidar. In recalculating to the left from the reference point r_k, the error in calibration will less affect the accuracy of reconstruction of the profile $\varepsilon(r)$ due to decreasing $V^2(r,r_k)$. At setting the local reference point in the beginning of the path, the calibration constant is calculated quite precisely, due to small value of $\delta S(r_0)$, and, as follows from Eq. (13), $\delta\varepsilon(r) = \delta S(r)$.

It is important to note that if the value of the integral transmission of the sounding path $V^2(r_0, r_{\max})$ is used as the reference value, the accuracy of calculation of the profile $\varepsilon(r)$ improves, because in this case, as is seen from Eq. (8), $\delta\varepsilon(r_{\max}) \approx \delta S(r_{\max})$, that, according to Eq. (6), corresponds to the precise determination of the calibration constant.

As follows from Eq. (13), the effect of noise in the backscattering signal becomes less essential at increasing the optical thickness of the path's segment $[r, r_k]$. Hence, for optically dense media, neither the presence of noise in the backscattering signal, nor the errors in setting the reference value (as was noted in section 1) strongly affect the accuracy of reconstruction of the profile $\varepsilon(r)$ with the reference point at the end of the path. It is also confirmed by the results of modeling shown in Fig. 3*a* (optical situation "haze – cloud – haze").

The model backscattering phase functions in the haze and in the cloud are equal, respectively, to 0.03 and 0.05 sr^{-1}. In reconstructing the profile $\varepsilon_a(r)$, the reference value of the aerosol extinction coefficient was set at the point $r_k = 3$ km with the error of 100%, the backscattering phase function in the haze and in the cloud was assumed constant and equal to 0.05 sr^{-1}. Figure 3*b* shows the dependence of the error in reconstruction $\varepsilon_a(r)$ obtained using Eqs. (6) and (7) under the same conditions, for which the calculation of $\varepsilon_a(r)$ presented in Fig. 3*a* was made.

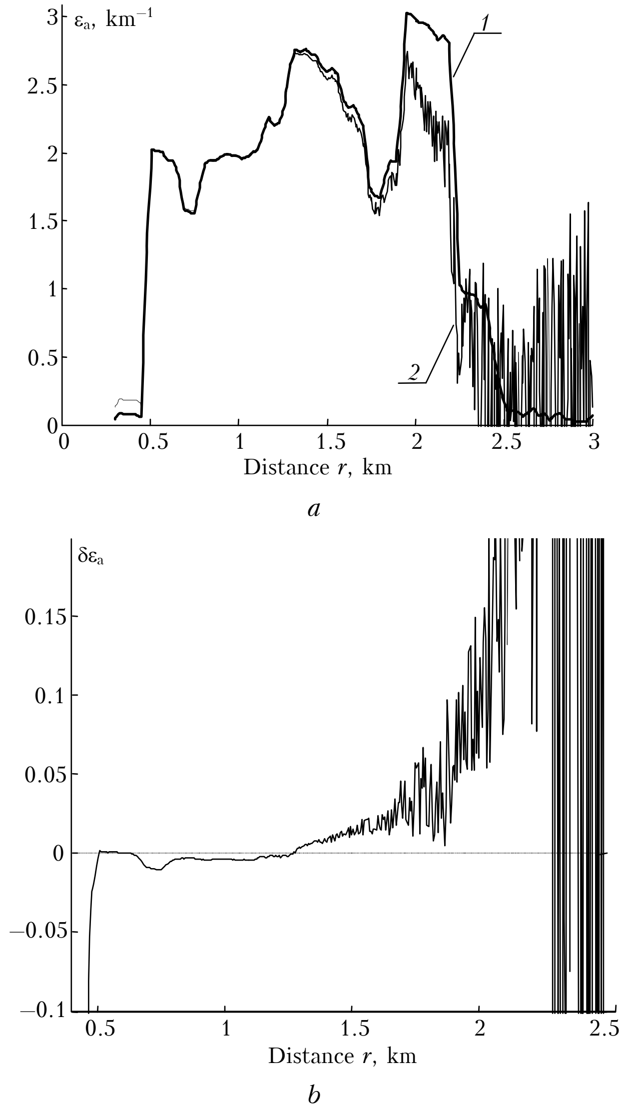

Fig. 3. Results of reconstruction of the extinction coefficient profile in the optically dense atmosphere (the reference point was set with the error of 100% at the end of the path, the signal-to-noise ratio in the beginning of the path is 10^4: (*a*) curves *1* and *2* are model and reconstructed profiles of $\varepsilon_a(r)$, respectively); (*b*) the dependence of the relative error in reconstruction $\delta\varepsilon_a(r)$ on the distance to the lidar obtained using Eq. (6) and from the results of numerical modeling.

The error in reconstructing $\varepsilon_a(r)$ was also determined from the results of numerical modeling $\varepsilon_a(r)$ shown in Fig. 3*a* and was presented in Fig. 3*b*. As the presented results coincide with each other, we see the only curve in Fig. 3*b*. Thus, knowing *a priori* the atmospheric situation and characteristics of the measurement instrumentation, one can estimate the limiting accuracy characteristics of measuring $\varepsilon(r)$ and $\varepsilon_a(r)$ in the given medium using the analytical formulas derived.

Conclusions

In clean and weakly turbid media, in the presence of errors in the backscattering signals, it is preferable to place the reference point in the beginning of the path. The accuracy of reconstruction of the profile $\varepsilon(r)$ for the points situated farther is worse than for the case with the reference point in the beginning of the path, even at decreasing $\delta S(r)$ and $\delta I(r)$ due to averaging over the set of realizations. One can also improve the accuracy of reconstruction by using the value of the integral transmission of the sounding path as the reference value.

However, in weakly turbid media one should be careful in choosing $\tilde{g}_{\pi,a}$ value because divergence of the solutions and appearance of negative $\varepsilon(r)$ values are possible at unsuccessful choice even if precisely setting the reference point and total absence of measurement errors in the backscattering signal. So, as it is impossible to set the precise value of the parameter $\tilde{g}_{\pi,\mathrm{a}}$, it is necessary to use known functional relations between $\varepsilon_\mathrm{a}(r)$ and $g_{\pi,\mathrm{a}}(r)$ in reconstructing the profile $\varepsilon_\mathrm{a}(r)$ in the clean atmosphere. Neither position of the reference point, nor the value of the parameter $\tilde{g}_{\pi,\mathrm{a}}$ used as the initial approximation affect the accuracy of reconstruction of $\varepsilon_\mathrm{a}(r)$ (it is determined only by the accuracy of setting the reference value and the functional relation between $g_{\pi,\mathrm{a}}(r)$ and $\varepsilon_\mathrm{a}(r)$).

In optically dense media, like clouds and fogs, the reconstructed profile $\varepsilon(r)$ is most sensitive to the position of the reference point (the solution may rapidly converge as well as to diverge), and the most optimal is the way of placing the reference point deep into the medium. Neither accuracy of it setting nor the presence of noise in $S(r)$ practically affect the quality of reconstruction of the profile in the beginning of the path, because the effect of the error in calibration on the obtained profile quickly decreases with the increase of the optical thickness between r and r_k. Besides, as in optically dense media $\varepsilon_\mathrm{a}(r) \gg \varepsilon_\mathrm{m}(r)$, the errors related to inaccurate choice of the scattering phase function do not affect the accuracy of reconstruction at slowly changing or quickly oscillating $g_{\pi,\mathrm{a}}(r)$. In this case $Q(r) = g_{\pi,\mathrm{a}}(r)/\tilde{g}_{\pi,\mathrm{a}}(r)$ does not depend on the profiles of aerosol and molecular extinction coefficients, and, to improve the accuracy of reconstruction, it is necessary to know not the path-average value of the scattering phase function, but its relative behavior, because in this case one can correct $S(r)$ to the constancy of the scattering phase function along the path.[12]

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