SHOT NOISE REDUCTION IN THE PHOTODETECTION OF RADIATION FROM A LASER WITH INTRACAVITY GENERATION OF THE SECOND HARMONIC

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The possibility of construction of a macroscopic source of radiation with decreased natural fluctuations (squeezed light) based on a laser with intracavity generation of the second harmonic is validated.

It is shown that the dip in the photocurrent power spectrum below the shot noise level in the course of detection of frequency-doubled light exceeds the corresponding value for the primary wave by a factor of four.

The sensitivity of some devices, for example, laser detectors,[1–3] intracavity laser spectrometers,[4] etc., and optical communication lines,[5] reaches a limiting value and is limited only by the spontaneous noise of the laser source (quantum laser noise). In the detection process, quantum noise with Poisson photon statistics results in shot noise, the lower limit of photodetection noise.[6] At present, various ways of constructing light sources with a decreased level of quantum fluctuations, sub-Poisson lasers,[7–8] or, in general, sources of squeezed states of the electromagnetic field,[9–11] have been thoroughly studied. One of the main properties of squeezed light, which is manifested in photodetection, is a complete or partial reduction in the shot noise by a negative excess.[6] In the end result this results in an increase of the limiting value of the signal-to-noise ratio.[12] Important experimental results in the construction of a squeezed light source have been achieved using optical parametric frequency dividers[11,13–14] and a semiconductor laser with sub-Poisson pumping.[8] In this paper, we theoretically investigate another scheme of squeezed light sources. The emission is formed within the general cavity, in which, along with the active laser medium, is placed a transparent nonlinear crystal that transforms the field at the laser source frequency ω – the primary wave (PW) into the second harmonic (SH). It is shown that, first, such a source can generate squeezed light both of the PW and the SH, and, second, in the photodetection of both the PW and the SH, a dip in the low-frequency region of the photocurrent power spectrum is produced. The depth of this decrease in the photocurrent noise in the detection of SH can in principle be four times greater than the corresponding value for PW.

THE PHOTOCURRENT POWER SPECTRUM

The noise that limits the accuracy of measurement is described by the formula derived in the plane wave approximation[6]:

$$\langle i_\Omega^2 \rangle = q\gamma\left(n_0 + 2q\gamma\mathrm{Re}\right) \times \int_0^\infty e^{i\Omega t}\left[\langle a^+(0)\, a^+(t)\, a(t)\, a(0)\rangle - \langle a^+ a\rangle^2\right] dt \qquad (1)$$

where q is the quantum efficiency of the photodetector, γ is the cavity width of the radiation source, and a^+ and a are the photon creation and annihilation operators. Under conditions of stationary generation $n = |a|_N^2 = n_0 + v$, N denotes normal ordering, α is the complex amplitude (the eigenvalue of the operator a in the coherent states representation). Taking this into account, as well as the correlations $\langle v(0)v(t)\rangle = \langle v^2(0)\rangle e^{-\Gamma t}$, where Γ is the rate of decay of the amplitude fluctuations, Eq. (1) takes the form

$$\langle i_\Omega^2 \rangle = q\gamma n_0\left[1 + \frac{2q\delta\gamma\Gamma}{\Gamma^2 + \Omega^2}\right], \qquad (2)$$

where $\delta = \dfrac{\langle n^2\rangle - \langle n\rangle^2 - \langle n\rangle}{\langle n\rangle}$ is the Fano factor, which has the meaning of a relative variance of the intensity fluctuations. The first term in Eq. (2) is the shot noise. It does not depend on the frequency and is due to natural field fluctuations in the coherent state. The shot noise is the minimum noise that can arise in the detection of radiation from a single-mode laser in the regime of multiple excess over the generation threshold. The second term in Eq. (2) is called the excess noise and is proportional to a parameter of statistics. It is known that $\delta = 0$ for the coherent state, $\delta = 1$ for radiation with Gaussian statistics, $0 < \delta < 1$ for light squeezed in intensity, and $\delta > 1$ for light squeezed in phase.[6] Thus, in the course of direct photodetection of the intensity-

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squeezed light, reduction of the shot noise by a negative excess can occur completely or partially in the low-frequency region of the spectrum. This, in turn, results in a growth of the limiting accuracy of measurement of the light intensity (power). In the same experiment, for phase-squeezed light, on the contrary, an anomalous augmentation of the excess noise occurs. In this paper, we calculate δ and Γ in order to be able to analyze Eq. (2) applied to the detection of the primary wave and the secondary harmonic.

A SEMICLASSIC DESCRIPTION OF SH GENERATION IN THE LASER CAVITY

We shall describe the process of generation of the SH in the laser cavity (or intracavity secondary harmonic generation) (intracavity SHG) by the following system of the equations[15]:

$$\dot{\alpha}_1 = -\frac{\gamma_1}{2}\alpha_1 + \frac{k/2}{1+\beta|\alpha_1|^2}\alpha_1 - g\alpha_1^*\alpha_2;$$

$$\dot{\alpha}_2 = -\frac{\gamma_2}{2}\alpha_2 + \frac{g}{2}\alpha_1^2 \tag{3}$$

where α_1 and α_2 are the dimensionless amplitudes of the PW and the SH in the cavity, k and β are the linear amplification and saturation coefficients of the active medium in which generation of the PW takes place, g is the coefficient of nonlinear coupling between the PW and the SH proportional to the square of the susceptibility of the crystal, γ_1 is the width of the cavity at the frequency ω (without nonlinear losses in intracavity SHG), and γ_2 is the cavity width at the frequency 2ω. For system of equations (3), the following approximations are assumed to hold: the SH does not interact with the atoms of the active medium, the operation of the laser is described by the Lamb model,[17] the laser generation is single-frequency, and the frequency tuning is central. The system of nonlinear equations (3) describes the interaction of two modes, one of which — the one at the frequency ω — has a positive coefficient of the linear term, while the SH term has a negative one. We will carry out an analysis of the dynamics of this nonlinear system based on the principle of subordination.[16] The formal solution for a has the form

$$\alpha_2(t) = \frac{g}{2}\int_{-\infty}^{t}\exp\left[-\frac{\gamma_2}{2}(t-\tau)\right](\alpha_1^2)_\tau d\tau. \tag{4}$$

After integrating Eq. (4) by parts, we have

$$\alpha_2 = \frac{g}{\gamma_2}\alpha_1^2 - \frac{g}{\gamma_2}\int_{-\infty}^{t}\exp\left[-\frac{\gamma_2}{2}(t-\tau)\right]2(\alpha_1\dot{\alpha}_1)_\tau\, d\tau. \tag{5}$$

In Eq. 5 we substitute the right side of the equation for α_1 of Eqs. (3), where, instead of the unknown α_2, we make use of the solution $\alpha_2^0 = \frac{g}{\gamma_2}\alpha_1^2$ as a zeroth approximation, and again integrate by parts to obtain

$$\alpha_2 = \frac{g}{\gamma_2}\alpha_1^2\left[1 + \left(\frac{k}{2\gamma_2}\left(1+\beta|\alpha_1|^2\right)^{-1} - \frac{\gamma_1}{\gamma_2} - \frac{2g^2}{\gamma^2}|\alpha_1|^2\right]_2 + \Pi; \tag{6}$$

$$\Pi = \frac{-2g}{\gamma_2}\int_{-\infty}^{t}\left\{\exp\left[-\frac{\gamma_2}{2}(t-\tau)\right]\left[(\alpha_1\dot{\alpha}_1)\varepsilon(\tau) + (\alpha_1^2)_\tau(\dot{\varepsilon})_\tau\right]\right\}d\tau. \tag{7}$$

The following notation was used in Eq. (7):

$$\varepsilon = \frac{k}{2\gamma_2}\left(1+\beta|\alpha_1|^2\right)^{-1} - \frac{\gamma_1}{\gamma_2} - \frac{2g^2}{\gamma_2^2}|\alpha_1|^2. \tag{8}$$

Note that under the condition $0 < \varepsilon \ll 1$, a solution for α_2 in the form

$$\alpha_2^1(t) = \frac{g}{\gamma_2}\alpha_1^2(1+\varepsilon) \tag{9}$$

is a good approximation. In Ref. 18, the solution for α_2 when $\varepsilon = 0$ was used to describe the quantum fluctuations of the PW and the SH in the squeezed light source model being analyzed. This results in an exaggerated value of the Fano factor, since the part of the solution of Eq. (9) proportional to ε is of the same order of smallness as the fluctuations. The condition $0 < \varepsilon \ll 1$ has a clear physical meaning: the cavity Q at 2ω must distinctly exceed the difference between the linear field amplification in the active laser medium and the PW losses involving nonlinear ones. He must take care that a breakdown of laser generation does not occur under the conditions of intracavity SHC; therefore, the left part of the inequality has been introduced. The condition $0 < \varepsilon \ll 1$ makes it possible for us to evaluate Π, the remainder (7) of the series (6), which we write in the form

$$\Pi = \varepsilon_{max}\frac{g}{\gamma_2}\int_{-\infty}^{t}\exp\left[-\frac{\gamma_2}{2}(t-\tau)\right]2(\alpha_1\dot{\alpha}_1)_\tau d\tau, \tag{10}$$

where $\varepsilon_{max} = \max[\varepsilon(\tau)]$. Comparing Eq. (10) with the second term in Eq. (5) and taking into account that

$0 < \varepsilon_{\max} \ll 1$, we see that any subsequent term of the series α_2^n is less than the term of the smallness in εmax. Thus, $\alpha_2 = \frac{g}{\gamma_2}\alpha_1^2(1+\varepsilon+\varepsilon^2+\ldots\varepsilon^n+\Pi^{(n)})$, where

$$\Pi^{(n)} = \varepsilon^n_{\max}\frac{g}{\gamma_2}\int\limits_{-\infty}^{t}\exp\left[-\frac{\gamma_2}{2}(t-\tau)\right]2(\alpha_1\dot{\alpha}_1)_\tau\,d\tau,$$

n denotes the number of operations of taking the integral (10) by parts and corresponds to the obtained remainder of the series. Substituting Eq. (9) in the right side of Eq. (3), we obtain for α_1

$$\dot{\alpha}_1 = -\left[\frac{\gamma_1}{2} - \frac{k/2}{1+\beta|\alpha_1|^2} + \frac{g^2}{\gamma_2}(1+\varepsilon)|\alpha_1|^2\right]\alpha_1. \tag{11}$$

Under the conditions of stable simultaneous generation of the laser and SH, a nontrivial solution for $n_0 = |\alpha_1|^2$ is given by the relation

$$\gamma_1 + \frac{2g^2(1+\varepsilon)}{\gamma_2}n_0 = \frac{k}{1+\beta n_0} \tag{12}$$

for $\Psi = 0$, where $\Psi = 2\phi_1 - \phi_2$. Let us introduce the following notation: $\eta = \frac{g^2}{\gamma_2^2}n_0$ is the coefficient of transformation to the SH, $I_0 = \beta n_0$ is the dimensionless PW intensity within the cavity. It is known that an optimal cavity width $\gamma_1^{opt} = \frac{k}{1+I_0}$ exists in the absence of intracavity SHG.[3] The coefficient of transformation to the SH is related to the system parameters in the following way:

$$\eta = \frac{1}{2}\frac{1}{1+\varepsilon}\left[\frac{\gamma_1^{opt}}{\gamma_2} - \frac{\gamma_1}{\gamma_2}\right]. \tag{13}$$

Let us consider the case in which the radiation at the frequency ω is locked up within the cavity ($\gamma_1 = 0$), and all of it is transformed into SH. In order that the above-described scheme generate the SH in the optimum regime, we require that $\gamma_2 = \gamma_1^{opt}$. It then follows that $\eta^{\max} = \frac{1}{2}(1+\varepsilon)^{-1}$ and, at $\varepsilon = 0$, $\eta^{\max} = 0.5$. Indeed, if all the photons of the PW generated by the laser are transformed into the SH, then the maximum number of photons at the doubled frequency will be less by a factor of two. Based on Eq. (11), we write an equation for v ($n = n_0 + v$, $v \ll n_0$)

$$\dot{v} = -\Gamma_1 v; \tag{14}$$

$$\Gamma_1 = \gamma_1\frac{I_0}{1+I_0}\left[1 + \frac{2\eta\gamma_2}{\gamma_1}\right]. \tag{15}$$

At $\eta = 0$. Eq. (15) transforms into a well-known formula for the rate of decay of fluctuations of the number of photons generated by the laser.[7]

A QUANTUM DESCRIPTION OF INTRACAVITY SHG

Let us analyze the statistical properties of the considered source on the basis of the Fokker-Planck equation (FPE) for the positive definite Glauber phase density[19–20] $\rho = \langle\alpha_{1,2}|\rho^F|\alpha_{1,2}\rangle$:

$$\frac{\partial\rho}{\partial t} = \Bigg\{\left[\frac{\partial}{\partial\alpha_1}\left(\frac{\gamma_1}{2} - \frac{k/2\ \alpha_1}{1+\beta|\alpha\ |^2} + g\,\alpha_1^*\alpha_2\right) + k.c.\right] + \left[\frac{\partial}{\partial\alpha_2}\left(\frac{\gamma_2}{2}\alpha_2 - \frac{g}{2}\alpha_1^2\right) + k.c.\right] + \hat{D}_{1az} + \gamma_1(\langle n_1^T\rangle + 1)\frac{\partial^2}{\partial\alpha_1\partial\alpha_1^*} + \left(\frac{g}{2}\alpha_2^*\frac{\partial^2}{\partial\alpha_1^{*2}} + k.c.\right)\Bigg\}\rho$$

$$i = 1,\ 2. \tag{16}$$

Analysis of the semiclassical system (3) makes it possible for us to apply the procedure of adiabatic exclusion of variables in Eq. (16).[16] Let us transform to polar coordinates $\alpha = \sqrt{n}e^{i\varphi}$ in the derived equation. Under the conditions of stationary generation, stable values of n_0 and Ψ_0 are set up. Therefore, the fluctuations v and $\delta\Psi$ given by ρ are small. Taking the abovesaid into account, and assuming the independence of the amplitude and phase fluctuations, i.e., $\rho = R\cdot\Phi$, on the basis Eq. (16) we obtain a linearized equation for R

$$\dot{R} = \Gamma_1\left[\frac{\partial vR}{\partial v} + \langle v^2\rangle_A\frac{\partial^2 R}{\partial v^2}\right], \tag{17}$$

where Γ_1 has the form (15). Taking into consideration the relation $\langle v^2\rangle_A = \langle v^2\rangle_N + 2n_0 + 1$, where $\langle v^2\rangle_N = n_0\delta_1$, we obtain a formula for δ_1 which is needed for the calculation of $\langle i_\Omega^2\rangle$:

$$\delta_1 = I_0^{-1} - \frac{1}{(1-\varepsilon)}\cdot\frac{1+I_0^{-1}}{2+\gamma_1/2\gamma_2\eta}. \tag{18}$$

Note that in the absence of intracavlty SHG, i.e., $\eta = 0$, Eq. (18) transforms into a well-known equation for a laser without instrumentation noise.[7] In the opposite case, i. e., $\gamma_1 / 2\gamma_2\eta \ll 1$ — the case of highly effective intracavity SHG, squeezing of the PW within the resonator is achieved. Let us turn our attention to the fact that a laser with Poisson photon statistics and intracavity SHG as $\gamma_1 - 0$ (for any nonzero value of the transformation coefficient $\eta \neq 0$, and $I_0 \gg 1$), generates PW photons within the cavity with sub-Poisson statistics.

REDUCTION OF THE SHOT NOISE OF PHOTODETECTION OF RADIATION GENERATED IN THE INTRACAVITY SHG PROCESS

It follows from Eq. (2) that suppression of the shot noise by a negative excess ($\delta < 0$) is possible in the low-frequency region of the photocurrent power spectrum, i. e., for $\Gamma^2 \ll \Omega$. If $\Omega - 0$, then

$$\langle i^2 \rangle_\Omega = q\gamma n_0 \left[1 - \frac{2q\gamma\delta}{\Gamma} \right]. \tag{19}$$

If we substitute $\Gamma = \Gamma_1 + 2\eta\gamma_2$ and given by formulas (15) and (16) into Eq. (19), we obtain the following equation for the depth of the decrease below the shot noise level K_1:

$$K_1 = q \frac{1 - \varepsilon}{\left[2 + \dfrac{\gamma_1}{2\eta\gamma_2} \right] \left[1 + 4\eta \dfrac{\gamma_2}{\gamma_1} \right]}. \tag{20}$$

Note that $K_1 = 0$ corresponds to maximum squeezing of the SH in the cavity. When the transmission coefficient for the PW is selected so that the linear losses are equal to twice the nonlinear ones (due to intracavity SHG), i. e., if $\gamma_1 = 4\eta\gamma_2$, then $K_1 = -q\dfrac{1-\varepsilon}{8}$; ($q \leq 1$, $\varepsilon \ll 1$).

In the photocurrent spectrum of the SH radiation, there will have three contours defined by the correlators $\langle u(0)u(\tau)\rangle_\Omega$, where $u = f + 2\eta\bar{n}_1 v$, f is the source of the SH natural fluctuations. When $n_1 \gg 1$, the main contribution to the photocurrent from the SH will be produced by the contour defined by the term $4\eta\overline{n_1^2}\langle v(0)\cdot v(\tau)\rangle_\Omega$. Taking this into account, K_2 has the following form:

$$K_2 = q \frac{4\gamma_2\eta\delta_1}{(\Gamma_1 + 2\eta\gamma_2)(1 + 2\varepsilon)}. \tag{21}$$

For $I_0 \gg 1$ formula (21) reduces to the expression

$$K_2 = -\frac{q}{1 + 2\varepsilon} \frac{4\eta\gamma_2}{\gamma_1} \left[\left[1 + 4\eta \frac{\gamma_2}{\gamma_1} \right] \left[2 + \frac{\gamma_1}{2\eta\gamma_2} \right] \right]^{-1}. \tag{22}$$

When $\gamma_1 \ll 4\pi\eta\gamma_2$, $K_2 = \dfrac{-q}{2}(1 + 2\varepsilon)^{-1}$.

The depth of the decrease in the photocurrent power spectrum of the SH is thus four times greater than the corresponding value for PW.

CONCLUSION

On the basis of the foregoing theoretical study we may conclude that it is possible to build an effective macroscopic source of squeezed light based on a laser with intracavity SHG. Radiation from such a source can result in the reduction of the shot noise of photodetection of both the PW and the SH. We call special attention to the result of an almost fourfold increase in the suppression of shot noise in the course of SH detection in comparison with PW. An incomplete description of the behavior of quantum noise during intracavlty SHG[21] has resulted in the fallacious view of the inefficiency of the given model of a squeezed radiation source.

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