THE EFFICIENCY OF LIDAR MEASUREMENTS OF WIND VELOCITY BY A CORRELATION LIDAR

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A suboptimal estimate of the wind velocity based on the spectral processing of lidar signals is constructed. The error of this estimate is calculated and its calculations are performed for different atmospheric conditions and instrumental parameters for the experimentally confirmed models of the correlation functions of lidar signals. Some recommendations are given on the choice of parameters of a two-path method of sounding with an account of evolution time of the aerosol inhomogeneities.

The speed and the direction of wind are the important parameters needed, e.g., in the ecological investigations. Since lidars are in a wide use for ecological monitoring the development of techniques for measuring wind parameters by laser, which could unite in one and the same facility (i.e., in lidar) both ecological and meteorological monitoring functions is an urgent problem.

The method of laser sounding is most promising for measuring the wind parameters. Among the advantages of this method are relative simplicity of data acquisition and processing, low requirements to laser (compared to a coherent–Dopler lidar), fast operation, and high spatial and temporal resolution of measurements. However, the potentialities of a correlation technique are considerably limited by the variations in the transmission function along the paths to scattering volumes, weak contrast of aerosol inhomogeneities and their variability.[1] Just the influence of these factors, and return signal fluctuations and noise on the efficiency of wind measurements by a correlation technique using spectral processing of signals make the subject of this paper. By the efficiency we understand the relations between the error of wind velocity estimate and the factors of atmospheric (the parameters of spatio–temporal variations of aerosol scattering coefficients, fluctuations of wind velocity field, variations of the transmission, and the background noise) and of instrumental (noise the observational time, pulse repetition frequency, and pulse energy of laser transmitter) origin.

We shall carry out our analysis for the case of dual path sounding scheme, which, on the one hand, makes a part of a three path lidar arrangement[1] that provides for wind velocity profiling, and on the other hand, has an independent application to sounding wind velocity along the horizontal and slightly elevated paths what is typical of ecological studies. Two scattering volumes on these paths with the centers at the points $\mathbf{R}_1$ and $\mathbf{R}_2$ are illuminated alternatively each at the frequency F. For determining the wind velocity the correlation or spectral processing of lidar returns is used. The latter is based on the analysis of the phase $\Theta_u(\xi, f)$ of the cross–spectrum $W_u(\xi, f)$ of signals. The phase spectrum has the form

$$\Theta_u(\xi, f) = 2\pi|\xi|\, f\cos\varphi/|\langle\mathbf{V}\rangle| , \qquad (1)$$

where $\xi = \mathbf{R}_2 - \mathbf{R}_1$ is the spacing between the centers of scattering volumes, $|\langle\mathbf{V}\rangle|$ is the absolute value of the average wind velocity, and φ is the angle between the direction of the average wind velocity and the spacing ξ. It should be noted that the cross–correlation functions and the cross–spectrum densities are interpreted similarly. However, the latter provide the desirable result in the form of the function of frequency f, i.e., of the size of aerosol inhomogeneities rather than of the point moments. This essentially increases the number of possible interpretations of the data obtained with the use of a lidar. Thus, for example, the estimate of the mean wind velocity component $|\langle\hat{\mathbf{V}}\rangle|$ along the direction ξ

$$|\langle\hat{\mathbf{V}}\rangle|/\cos\varphi = 2\pi|\xi|\, f/\Theta_u(\xi, f) , \qquad (2)$$

derived from Eq. (1) in contrast to the correlation analysis is independent of the variance of the wind velocity σ_v^2. This is explained by the fact that the phase spectrum is insensitive to the fluctuations of wind velocity.[3] This spectrum is also insensitive to the evolution of the aerosol inhomogeneities caused by the turbulent diffusion.[4]

The diagram shown in Fig. 1 explains the forming of lidar signal and the structure of noise for the correlation lidar. Similar representation for a single–path method of sounding without an account of noise structure has been used in Ref. 5.

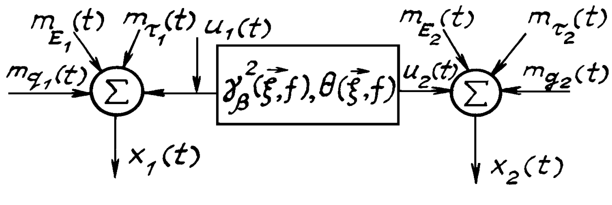

FIG. 1.

Alternative illumination of the scattering volumes makes it possible to perform the temporal selection of signals inside the lidar recording channel. The coherence function

$$\gamma_u^2(\xi, f) = |W_u(\xi, f)|^2/[W_{u1}(0, f)\, W_{u2}(0, f)]$$

and the phase $\Theta_u(\xi, f)$ describe the relation between the lidar signals $u_1(t)$ and $u_2(t)$ due to the transportation of

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aerosol inhomogeneities across the scattering volumes ($W_{ui}(0, f)$ is the autospectrum of signal $u_i(t)$). The noise components represent the background shot noise, the dark–current noise, and the noise induced by a signal from the ith scattering volume ($i = 1, 2$); $m_{Ei}(t)$ is the noise due to the fluctuations of sounding pulse energy E, and $m_{\tau i}(t)$ is the noise due to variations of the optical thickness $\tau(R_i)$ along the ith path. Such an additive representation follows from the lidar equation under condition that variations of the aerosol backscattering coefficient $\beta_a(R_i)$ averaged over the scattering volume and also of $\tau(R_i)$ and E are small. Normally, short–period variations of β_a are less than or equaled to 0.1,[1] while the output energy fluctuations of solid–state lasers are about several percent.[6]

We shall analyzehe the correlation technique accuracy assuming Gaussian character of signals and noise. Fluctuations of the sounding pulse energy are assumed to be uncorrelated

$$B_{Ei,j}(\tau) = \overline{m_{Ei}(t)\, m_{Ei}(t+\tau)} = \overline{P}(R_i)\, \overline{P}(R_j)\, \Delta_E^2 \delta(\tau)\, \delta_{ij} ,$$

where $\Delta_E = \sqrt{D(E)}/E$ is the relative rms error of E and $\delta(\tau)$ is the delta–function,

$$\delta_{ij} = \begin{cases} 1 , & i = j \\ 0 , & i \neq j \end{cases}$$

is the Kronecker symbol, i and j refer to the channel numbers, $\overline{P}(R_i)$ is the average power of lidar return signals. The shot noise is assumed to be a white noise with the spectral power density

$$G_{gi} = 2eM_l^2\, [\overline{P}(R_i)\, \kappa\eta + P_b \kappa\eta + I_{\rm d}] .$$

Here e is the electron charge, κ is the coefficient of optical losses M_1, η, and $I_{\rm d}$ are the multiplication coefficient, quantum efficiency, and dark current of photomultiplier, respectively, and P_b is the background radiation power recorded by the detector. Within the scope of this model the cross–spectrum of signals and noise is

$$W(\xi, f) = W_u(\xi, f) + W_s(\xi, f) .$$

The component $W_u(\xi, f)$ may successfully be used for estimating by Eq. (1) mean wind velocity along the direction ξ. The cross–spectrum $W_\tau(\xi, f)$ of noise $m_{\tau 1}(t)$ and $m_{\tau 2}(t)$ is, in turn, the noise that distorts the phase $\Theta_u(\xi, f)$ of lidar signals

$$\Theta(\xi, f) = \Theta_u(\xi, f) + \Delta\Theta_\tau(\xi, f) .$$

As a result, the estimate of the wind velocity by Eq. (1) becomes biased. The value of the bias at $\Delta\Theta_\tau/\Theta_u \ll 1$ is

$$\Delta|\langle\hat{\mathbf{V}}\rangle|/\cos\varphi = -\, 2\pi|\xi|\, f\, \Delta\Theta_\tau(\xi, f)/\Theta_u^2(\xi, f) .$$

Orlovet al.[1] have estimated the error which is of the same origin but occurrs in the correlation technique of processing of lidar signals. They showed that this error should be taken into consideration in situations when the atmospheric regions under study are at large distances R and at high turbidity of the atmosphere. For this reason we will neglect this error component in our further consideration.

For constructing estimate (1) it is advisable to use the whole set of spectral data $\{\hat{\Theta}(i\Delta f)\}$, $i = \overline{1, N}$ being obtained. Here $\Delta f = 1/t_{\rm ob}$ is the spectral resolution, and $t_{\rm ob}$ is the observational time. The procedure of choosing the parameter N will be considered later. As can be seen from Eq. (1), the phase is linearly dependent on frequency

$$\Theta(\xi, f) = af , \tag{3}$$

where $a = 2\pi|\xi|\cos\varphi/|\langle\mathbf{V}\rangle|$. Thus, by estimating the coefficient a from the phase spectrum we obtain the estimate of the wind velocity

$$|\langle\hat{\mathbf{V}}\rangle|/\cos\varphi = -\, 2\pi|\xi|/\hat{a} . \tag{4}$$

For constructing a suboptimal estimate of the coefficient a from the set of $\{\hat{\Theta}(i\Delta f)\}$ values we shall use the least–squares method.[7] It can be shown that in the case of linear dependence given by Eq. (3) we have

$$\hat{a} = \frac{6 \sum\limits_{i=1}^{N} i\, \hat{\Theta}(i\, \Delta f)}{\Delta f N(N+1)\,(2N+1)}$$

In addition, in this case the relative rms error of estimate (4) is

$$\delta_V = \delta\left(\frac{|\langle\hat{\mathbf{V}}\rangle|}{\cos\varphi}\right) = \frac{3|\langle\mathbf{V}\rangle| \sqrt{\sum\limits_{i=1}^{N} i^2 D[\hat{\Theta}(i\, \Delta f)]}}{\pi|\xi|\cos\varphi\Delta f N(N+1)\,(2N+1)} .$$

Within the scope of the accepted Gaussian model of signals and noise the variance $D[\hat{\Theta}(i\Delta f)]$ of the estimate is given by the equation[8]

$$D[\hat{\Theta}(i\, \Delta f)] = \frac{1 - \gamma^2(i\, \Delta f)}{2M\, \gamma^2(i\, \Delta f)} , \tag{5}$$

where $\gamma^2(f)$ is the function of mutual coherence of arrays of signals and noise, corresponding to different scattering volumes and M is the number of independent pairs of samplings used in the spectral analysis. With increase of $\gamma^2(f)$ the variance of the phase estimate decreases. The function of the mutual coherence entering into Eq. (5) is

$$\gamma^2(f) = \gamma_u^2(\xi, f) \prod_{i=1}^{2} \left\{ \frac{\Delta_E^2\, \overline{P}^2(R_j)\, \eta k}{W_{ui}(0, f)} + \frac{G_{gi}(f)}{2\, W_{ui}(0, f)} + 1 \right\}^{-1} .$$

For making calculations one needs for a model of cross–spectrum of lidar signals which is related to the cross–correlation function $B_u(\xi, \tau)$ by means of Fourier transformation. Balin et al.[2] have derived the formula for $B_u(\xi, \tau)$ within the assumption that the spectrum of fluctuations of the aerosol particles concentration is described by the Kolmogorov–Obukhov power law and the wind velocity components are distributed normally with the equal values of the variance σ_v^2. For the case of the frozen turbulence this relation has the form

$$B_u(\xi,\tau)=\frac{\eta^2\kappa^2\prod_{i=1}^{2}\bar P(R_j)\,\varepsilon_i}{(a_V^2+\nu_0^2)^{1/3}-a_V^{2/3}}\left\{\left(a_V^2+\nu_0^{-2}+\frac{\sigma_V^2\tau^2}{2}\right)^{1/3}\times\right.$$

$$\times\,{}_1F_1\left(-\frac{1}{3};\frac{3}{2};-\frac{(\xi-\langle\mathbf{V}\rangle\tau)^2}{4\left(a_V^2+\nu_0^{-2}+\frac{\sigma_V^2\tau^2}{2}\right)}\right)-$$

$$\left.-\left(a_V^2+\frac{s_V^2t^2}{2}\right)^{1/3}{}_1F_1\left(-\frac{1}{3};\frac{3}{2};-\frac{(\mathbf{x}-\langle\mathbf{V}\rangle t)^2}{4\left(a_V^2+\frac{s_V^2t^2}{2}\right)}\right)\right\},$$

where

$$\varepsilon_i=\Delta_{\beta i}/\left(1+\beta_m(R_i)/\beta_a(R_i)\right),$$

$\Delta_{\beta i}$ is the relative rms error of $\beta_a(R_i)$, and $\beta_m(R_i)$ is the molecular backscattering coefficient.

Here a_v is the size of equivalent isotropically scattering volume, ${}_1F_1$ is the confluent hypergeometric function, $\nu_0=2\pi/L_0$, L_0 is the outer scale of turbulence, whose vertical stratification is given by the relation

$$L_0=\min(2\sqrt{H},\,120),$$

H is the height above the Earth's surface in meters. The fluctuations of the wind velocity and the evolution of aerosol inhomogeneities are statistically independent.[4] Therefore, the effect of the evolution time t_1 may be accounted for by multiplying $B_u(\xi,\tau)$ by the coefficient[2]

$$q(\xi,t_1)=\left[(a_V^2+\nu_0^{-2})^{1/3}{}_2F_1\left(-\frac{1}{3};\frac{3}{2};\frac{5}{2};-\frac{|\xi|^2\tau_m^2}{2(a_V^2+\nu_0^{-2})t_1^2}\right)-\right.$$

$$\left.-\,a_V^{2/3}{}_2F_1\left(-\frac{1}{3};\frac{3}{2};\frac{5}{2};-\frac{|\xi|^2\tau_m^2}{2a_V^2t_1^2}\right)\right]\left[(a_V^2+\nu_0^{-2})^{1/3}-a_V^{2/3}\right]^{-1},$$

where $\tau_m=|\xi|\cos\varphi/\left(|\langle\mathbf{V}\rangle|+3\sigma_V^2/|\langle\mathbf{V}\rangle|\right)$ is the position of $B_u(\xi,\tau)$ maximum. The asymptotics can be shown to be valid: $q\to0$ as $t_1\to0$ and $q\to1$ as $t_1\to\infty$.

The model calculations were performed for lidar with the following parameters: diameter of the lidar receiver $d=0.3$ m, $\bar E=0{,}3$ J, $\Delta_E=0{,}05$, receiver's field of view $\Theta=10$ mrad, $\kappa=0{,}6$, wavelength $\lambda=1.06$ μm, photodetector FEU–83, spectral bandwidth of the optical filter is 10 Å, angle between the sounding paths $\alpha=3.9°$, spatial resolution $\Delta R=200$ m and the interval of spatial quantization $\Delta r=10$ m, $(M=\Delta R/\Delta r=20)$, and $F_r=10$ Hz, $t_{ob}=180$ s. The atmospheric optical model used in the calculations is taken from Ref. 9, the angular and frequency spectral density of the background radiation power taken in calculations is 10^{-10} W/m²·Å·sr, what corresponds to nighttime conditions of sounding,[10] $\Delta_\beta=0{,}06$, $\varphi=0$, $\sigma_v/v=0.05$ ($v=\langle|\mathbf{V}|\rangle$). Dependences of the relative error on the height for different values of wind velocity and evolution time for vertical sounding (the bisectrix of the angle formed by the paths normal to the Earth's surface) are shown in Fig. 2. Comparing the dependence for "frozen" turbulence (dashed lines) $t_1=\infty$ with that for $t_1=43$ s (solid lines) allows one to draw the conclusion that the evolution time affects considerably the error of measurements especially at small values of speed and large distances. It is explained by the fact that with increase of H the value of measurement base increases

$$|\xi|=2H\tan(\alpha/2)$$

that, in turn, leads to an increase of time during which aerosol inhomogeneities travel along it.

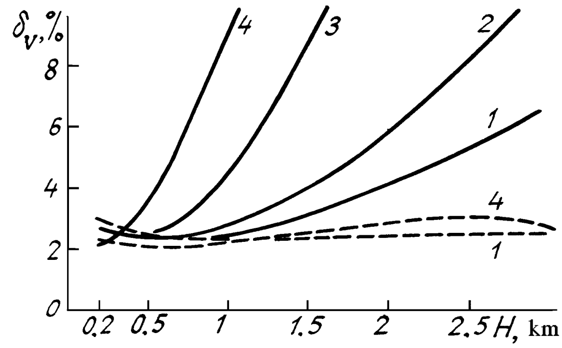

FIG. 2. The dependence of relative error δ_v on the altitude for evolution time $t_1=43$ s (solid lines) and for frozen turbulence (dashed lines). $v=30$ m/s (1), 20 (2), 10 (3), and 50 (4).

It is necessary to be careful when choosing N value for determining the value $\hat a$ from the set $\{\hat\Theta(i\Delta f)\}$. As the calculations show, there exists an optimal N value which minimizes the error δ_v. The range of phase $\Theta(\xi,f)$ lies within the limits $[-\pi,\pi]$. As a result, at the point $f_\pi=v/2|\xi|$ the phase spectrum has the discontinuity that essentially complicates the procedure of its processing (f_π is the frequency at which $\Theta(f_\pi)=\pm\pi$). Therefore, in calculations of the error the value N was assumed to be equal to the number of estimates of the phase on the interval $(0, f_\pi)$.

TABLE I.

H, km	$v=1$ m/s		$v=30$ m/s		l_{min}, m
	f_π, Gz	N	f_π, Gz	N	
0.2	0.03	5	1	180	29.7
0.4	0.015	3	0.5	90	59.3
0.6	0.01	2	0.33	60	88.9
0.8	0.0084	1	0.26	47	118
1.0	0.0067	1	0.20	36	148
1.4	0.0048	1	0.14	26	207
1.8	0.0038	–	0.11	20	267
2.2	0.0031	–	0.092	16	326
2.6	0.0026	–	0.078	14	385
2.8	0.0024	–	0.072	13	415
3.0	0.0023	–	0.067	12	445

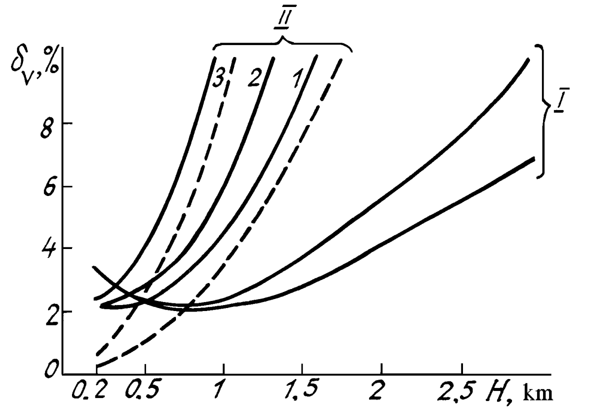

FIG. 3. The dependence of relative error δ_v *on the altitude at* $\Delta_E = 0.05$ *(solid curves) and at* $\Delta_E = 0$ *(dashed cures).* $v = 10$, $\alpha = 2.3°$ *(I),* $\alpha = 3.9°$ *(II),* $t_1 = 43$ s, 30 *(2) and* 15 *(3).*

The values of boundary frequency f_π for $v = 1$ m/s and $v = 30$ m/s, the numbers of estimates corresponding to these values of f_π, and the minimum dimensions of the aerosol inhomogeneities $l_{\min} = v/f_\pi = 2|\xi|$ are listed in Table I. If is obvious from the table that the measurements of small values of the wind velocity at high altitudes are impossible for the given geometry. One can avoid this difficulty by decreasing the measurement base. This is illustrated by Fig. 3, in which the dependences of δ_V on the altitude are compared at $\alpha = 3.9$ and $2.3°$ for $v = 10$ m/s and for different values of evolution time. In addition, the dependences at $\Delta_E = 0$ are shown by dashed curves what demonstrates a considerable contribution of the energy fluctuations to the error. The values of other parameters are the same as in Fig. 2. The minimum in δ_V value at the altitude of about 700 m is associated with increase of the measurement base, which is compensated for by the predominant effect of the evolution time of aerosol inhomogeneities and by the decrease of power of lidar signals at higher altitudes.

An increase of t_{ob} resulting in an increase of N is another way of decreasing the error of measurements. If the amount of information is fixed, the increase of t_{ob} has to be accompanied by simultaneous decrease of the frequency F_r. Since the Nyquist frequency at $F_r = 10$ Hz is 5 Hz, while the maximum value of $f_\pi = 1$ Hz (see Table I) such a way of improving the accuracy is quite realistic. The dependence of the relative error on the altitude at $t_{ob} = 180$ s, $F_r = 10$ Hz (solid curves) and $t_{ob} = 360$ s, $F_r = 5$ Hz (dashed curves) at different values of the wind velocity, $t_1 = 43$ s, and other parameters being unchanged are shown in Fig. 4.

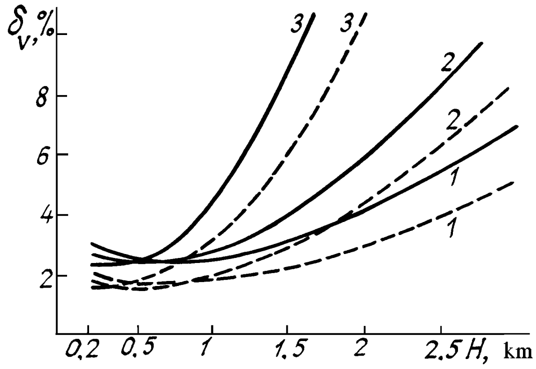

FIG. 4. The dependence of relative error δ_v *on the altitude at* $t_r = 180$ s, $F_r = 10$ Hz *(solid curves), and at* $t_{ob} = 360$ s, $F_r = 5$ Hz *(dashed curves).* $v = 30$ m/s *(1)*, 20 *(2) and* 10 *(3).*

Thus, for improving the accuracy of measurements the power of lidar signals should be normalized by the energy of sounding pulses. The error of energy measurement should be larger than Δ_E, since otherwise the normalization of $P(t)$ becomes senseless. In the case of a wide range of altitudes the measurements at different angles α between the paths could be advisable. This improves the accuracy of measurements and weaken the effect of evolution time at long distances. The obtained results also enable one to formulate reasonable requirements to the parameters of correlation lidars based on the accuracy characteristics.

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