ALBEDO RECONSTRUCTION FOR A NONUNIFORM SURFACE

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This paper considers a restoration algorithm for the albedo of the inhomogeneous orthotropic underlying surface from its brightness observed from an arbitrary point through the atmosphere with the given optical parameters. The Newton-Kantorovich method, each iteration of which is computed using the Monte-Carlo and below method, provides the basis for this algorithm. A model problem is solved to test the algorithm.

The problem of determining the albedo of an orthotropic underlying surface (US) observed through the atmosphere on the oasis of the known optical characteristics of the atmosphere and of the measured intensities of the upwelling radiation was studied most thoroughly in Refs. 1 and 2. A solution of the problem was sought based on the linear approximation of the influence of horizontal inhomogeneities in the albedo on the measured intensities. However, such an assumption is valid only for atmospheres with small optical thickness and US with small horizontal albedo variations.

In the present paper this problem is solved using the Newton-Kantorovich method, in which the Monte-Carlo method is used at each iteration to calculate the intensities and their derivatives with respect to the albedo. This approach to the solution of atmospheric optics inverse problems was suggested in Ref. 3 and was utilized in the problems of reconstructing the profiles of the aerosol scattering[4,9] and absorption coefficients[5] and the water vapor[6,7] and ozone[8] concentrations.

STATEMENT OF THE PROBLEM

A plane-layered horizontally homogeneous atmosphere above a nonuniform US is considered. A parallel flux of the monochromatic solar radiation is incident on the upper atmospheric boundary. The nonuniform US is divided into n regions $S_1,\ldots,S_n$ of the given shapes and sizes, The albedo values η_1 $(i = 1,\ldots,n)$ within each region S_1 are taken to be constant but unknown, i.e., the piecewise-constant US model is used.

The following designations are used below: $q(r, \vec{\omega})$ is a point of the phase space $Q = R \times \Omega$ of the collision point coordinates $r = (x, y, z) \in R$ and of the directions $\vec{\omega}=(a,b,c)\in\Omega$, $(a^2+b^2+c^2=1)$; $\sigma_m(\mathbf{r})\equiv\sigma_m(z)$, $\sigma_a(\mathbf{r})\equiv\sigma_a(z)$ are the coefficients of the total (molecular plus aerosol) absorption and molecular plus aerosol scattering, respectively; $\sigma_s(\mathbf{r}) = \sigma_m(\mathbf{r})+\sigma_a(\mathbf{r})$, $\sigma(\mathbf{r}) = \sigma_s(\mathbf{r})+\sigma_c(\mathbf{r})$; $g(\mathbf{r},\vec{\omega}',\vec{\omega}) \equiv g(z,\vec{\omega}',\vec{\omega})$ is the effective averaged scattering phase function; w' and w are the directions of the photon paths before and after scattering; $\tau(\mathbf{r}_1,\mathbf{r}_2)=\int_0^1\sigma(\mathbf{r}(t)dt$ is the optical path length of the segment $l=|\mathbf{r}_2-\mathbf{r}_1|$; $\vec{\omega}_F=(a_F,0,c_F)$ is the direction of the incident solar flux.

Suppose that a detector measuring the intensities I^*_k of the solar radiation in the directions $\vec{\omega}^*_k(k=\overline{1,N})$ is located at an arbitrary point $\mathbf{r}_k(x_k,y_k,0)$.

The observation lines intersect the US at the points $\mathbf{r}^*_k = (x_k, y_k, 0)$. The points $\mathbf{r}^*_k$ are chosen so that at least one of them lies within each region $S_1, \ldots S_n$. Thus, $N \geq n$, where N is the number of chosen points, and n is the number of regions. For the given optical parameters of the atmosphere the measured magnitudes of the intensities are functions of the quantities $\eta_1, \ldots, \eta_n$, i.e.,

$$I^*_k = I_k(\eta_1,\ldots,\eta_n); \quad k = \overline{1,N} \geq n \tag{1}$$

In this case the problem is to determine the quantities $\eta_1, \ldots, \eta_n$ from the known parameters $I^*_1, \ldots, I^*_N$.

SOLUTION METHOD

Nonlinear system (1) is solved by the Newton-Kantorovich method

$$\eta_i^{j+1}=\eta_i^j+\Delta\eta_i^j; \quad i=\overline{1,n}, \quad j=0,1,2,\ldots$$

Here j is the iteration order, $\eta^o_1, \ldots, \eta^o_n$ are prognostic albedo values and $\Delta\eta^j_i$ are the increments satisfying the system of equations

$$\sum_{i=1}^{n} \frac{\partial I_k(\eta_1^j,\dots,\eta_n^j)}{\partial \eta_i^j} \Delta\eta_i^j = I_k^* - I_k(\eta_1^j,\dots,\eta_n^j), \tag{2}$$

$$k = \overline{1,N} \geq n.$$

The iteration procedure stops when the following inequality is satisfied:

$$|I_k^* - I_k(\eta_1^j,\dots,\eta_n^j)| \leq \varepsilon; \quad k=\overline{1,N} \geq n,$$

where ε is a small positive quantity corresponding to the measurement error I_k^*r.

The values of the intensity $I_k(\eta_1^j, \dots, \eta_n^j)$ and its derivatives $\partial I_k / \partial \eta_1^j$ $(i = \overline{1, n})$ are calculated by the Monte-Carlo method in turn for each $k = 1, \dots, N$ in the adjoint scheme. Following Ref. 9, the expression for $I_k(\eta_1, \dots, \eta_n)$ is written in the form of a series:

$$I_k = \sum_{m=0}^{\infty} \int\!\!\int_{QQ} dq_0 \dots dq_m \psi(q_0)\ \varphi(q_m) \prod_{l=1}^{m} K(q_{l-1}, q_l) \tag{3}$$

Here q_l are the collision phase points, $\psi_k(q)$ is the density function of the initial collisions corresponding to the method of the adjoint trajectory simulation[9]:

$$\psi_k(q) = \begin{cases} \dfrac{\sigma(\mathbf{r})}{|\mathbf{r}-\mathbf{r}_0|^2} \exp(-\tau(\mathbf{r}_0,\mathbf{r})) \times \delta(\vec{\omega}^* - \omega_k) \\ \times \delta(\vec{\omega}_k^* - (\mathbf{r}-\mathbf{r}_0)/|\mathbf{r}-\mathbf{r}_0|);\ z>0 \\ \\ \dfrac{|c|}{|\mathbf{r}-\mathbf{r}_0|^2} \exp(-\tau(\mathbf{r}_0,\mathbf{r})) \times \delta(\vec{\omega}^* - \vec{\omega}_k) \\ \times \delta(\vec{\omega}_k^* - (\mathbf{r}-\mathbf{r}_0)/|\mathbf{r}-\mathbf{r}_0|);\ z=0 \end{cases}$$

$K(q', q)$ is the density function for the transition from q' to q (Ref. 11):

$$K(q',q) = \begin{cases} \dfrac{\sigma_s(\mathbf{r}')g(\mathbf{r}',\vec{\omega}',\vec{\omega})\exp(-\tau(\mathbf{r}',\mathbf{r}))\sigma(\mathbf{r})}{\sigma(\mathbf{r}')\ 2\pi\ |\mathbf{r}-\mathbf{r}'|^2} \\ \times\ \delta(\vec{\omega}-(\mathbf{r}-\mathbf{r}')/\ \mathbf{r}-\mathbf{r}'|);\quad z'>0,\ z>0, \\ \dfrac{\sigma_s(\mathbf{r}')g(\mathbf{r}',\vec{\omega}',\vec{\omega})\exp(-\tau(\mathbf{r}',\mathbf{r}))\ |c|}{\sigma(\mathbf{r}')\ 2\pi\ |\mathbf{r}-\mathbf{r}'|^2} \\ \times\ \delta(\vec{\omega}-(\mathbf{r}-\mathbf{r}')/\ \mathbf{r}-\mathbf{r}'|);\quad z'>0,\ z=0, \\ \dfrac{\eta(x,y)\ |c|\ \exp(-\tau(\mathbf{r}',\mathbf{r}))\sigma(\mathbf{r})}{\pi x|\mathbf{r}-\mathbf{r}'|^2} \\ \times\ \delta(\vec{\omega}-(\mathbf{r}-\mathbf{r}')/|\mathbf{r}-\mathbf{r}'|);\quad z'=0,\ z>0; \end{cases}$$

$\varphi(q)$ is the contribution from the m-th collision at the point q_m to the statistical estimate of I_k:

$$\varphi(q_m) = \begin{cases} \dfrac{\tau_s(\mathbf{r}_m)\ g(\mathbf{r}_m,\vec{\omega}_m,\vec{\omega}_F)}{\sigma(\mathbf{r}_m)\ 2\pi} \exp(-\tau_F),\ z_m>0 \\ \dfrac{\eta(x,y)\ |c_F|}{\pi} \exp(-\tau_F),\quad z_m=0 \end{cases}$$

where τ_F is the optical length of the path between the collision point q_m and the upper atmospheric boundary along the direction to the Sun.

To compute the derivatives $\partial I_k/\partial\eta_i$ ($i = \overline{1, n}$), series (3) is differentiated term by term:

$$\frac{\partial I_k}{\partial \eta_1} = \sum_{m=0}^{\infty} \int_Q \dots \int_Q dq_0 \dots dq_m \psi_k(q_0)\varphi(q_m)\ \times$$

$$\times \prod_{l=1}^{m} K(q_{l-1}, q_l) \left[\sum_{l=1}^{m} \frac{K_1'(q_{l-1}, q_l)}{K(q_{l-1}, q_l)} + \frac{\varphi_1'(q_m)}{\varphi(q_m)} \right], \tag{4}$$

Here $K_1' = \dfrac{\partial}{\partial\eta_1} K$ and $\varphi_1' = \dfrac{\partial}{\partial\eta_1}\varphi$.

As it is seen from Eq. (4), one can use the same trajectories to compute the functional I and its derivatives $\partial I_k/\partial\eta_i$ ($i = \overline{1, n}$). Every successive collision at the point q_m makes the contribution $\varphi(q_m)$ to the statistical estimates of the derivatives $\partial I_k/\partial\eta_i$, where $v_i(q_0, q_1, \dots, q_m)$ are the expressions in square brackets in Eq. (4), The function $K(q', q)$ depends on ηI only when the collision point $\mathbf{r}_i$ lies on the US inside the regions S_i with the albedo η_i and η_i is a linear term in $K(q', q)$. Consequently

$$i = \frac{K_1'(q',q)}{K(q',q)} = \begin{cases} 1/\eta_1, & \mathbf{r}\in S_1 \\ 0, & \mathbf{r}\in S_1 \end{cases} .$$

Analogously,

$$\frac{\varphi_1'(q)}{\varphi(q)} = \begin{cases} 1/\eta_1, & \mathbf{r}\in S_1 \\ 0, & \mathbf{r}\notin S_1 \end{cases}$$

Thus, $v_i(q_0, q_1, \dots, q_m) = p/\eta_i$, where p is the number of collisions on the US inside the region S_i on a segment of the trajectory $q_0 \to q_1 \to \dots \to q_m$. Finally, the derivatives $\partial I_k/\partial\eta_i$ are computed for the regions $S_1, \dots, S_n$ by the following algorithm.

Let $p_1, \dots, p_n$ be the collision counters on the US within the regions $S_1, \dots, S_n$, respectively. At each collision within S_i the counter p_i is incremented by one. Contributions equal to $p_i\varphi(q_m)/\eta_i$ are made to the statistical estimates in order to compute the derivatives $\partial I_k/\partial\eta_i$ for each collision (in the atmosphere or on the US), where p_i is the value of the i-th counter at the moment of the collision.

System (2) is in general over determined ($N \geq n$) and is solved by the method of the least squares involving a procedure of row and column scaling in order to decrease computer round-off errors.

NUMERICAL EXAMPLE

To test the proposed algorithm, we consider the following model problem.

Let the origin of the coordinate system (x, y, z) be located at some point of the US with the axis z directed upwards along a normal to the US, and let the axis x lie in a plane of incidence of the solar rays. The solar zenith angle is equal to 50°. A detector measuring the intensities I^*_k with 2% error is placed at the point (20 km, 0 km, 300 km). The US is assumed to be a rectangle (9 km×12 km) surrounded by a background with the albedo η_0. The rectangle is situated in the plane (x, y) so that the ends of its one side (9 km) are at the points (0 km, 0 km) and (9 km, 0 km), and those of the other side (12 km) are at the points (0 km, 0 km) and (0 km, 12 km). The rectangle is divided into twelve (3 km×3 km) squares, the observation points are at the centers of each square (i.e., $N = n$). The desired values of η_i are listed in the left column of the Table 1. For convenience the model of a single-layer (50 km thick) atmosphere is examined. The aerosol scattering phase function, effectively averaged with the molecular scattering phase function, was taken from Ref. 10 (for $\lambda = 0.55$ μm, the Elterman model); $\sigma_m = 0.002$ km^{-1}, $\sigma_c = 0$ km^{-1}.

TABLE 1.

η^*	$\sigma_a=0.002$ km^{-1}, $\eta_0=0.25$		$\sigma_a=0.01$ km^{-1}, $\eta_0=0.25$		$\sigma_a=0.002$ km^{-1}, $\eta_0=0.80$		$\sigma_a=0.01$ km^{-1}, $\eta_0=0.80$	
	$\delta\eta$	I^*	$\delta\eta$	I^*	$\delta\eta$	I^*	$\delta\eta$	I^*
0.45	3.1	0.289	3.0	0.310	3.2	0.334	3.1	0.159
0.20	3.2	0.167	-3.6	0.251	3.4	0.204	3.7	0.125
0.55	-3.0	0.336	-3.3	0.333	-3.1	0.384	-3.1	0.170
0.30	-3.0	0.214	3.3	0.276	-3.1	0.253	3.5	0.136
0.60	3.1	0.360	3.1	0.345	3.1	0.406	3.2	0.143
0.10	-3.4	0.117	3.3	0.227	3.1	0.151	3.9	0.136
0.50	-3.1	0.315	-3.3	0.324	-3.0	0.360	3.3	0.147
0.15	3.2	0.143	4.3	0.242	-3.5	0.177	3.6	0.150
0.35	3.1	0.239	3.3	0.287	3.0	0.281	-3.2	0.144
0.25	-3.1	0.188	3.7	0.264	3.3	0.227	3.3	0.152
0.40	-3.2	0.264	-3.0	0.298	3.1	0.308	-3.0	0.145
0.65	3.2	0.386	3.1	0.360	-3.1	0.437	-3.1	0.180

The calculations were carried out on a "BESM-6". computer by closed cycle for 4 alternative schemes: ($\sigma_a = 0.002$ km^{-1}, $\eta_0 = 0.25$); ($\sigma_a = 0.01$ km^{-1}, $\eta_0 = 0.25$); ($\sigma_a = 0.002$ km^{-1}, $\eta_0 = 0.80$); ($\sigma_a = 0.01$ km^{-1}, $\eta_0 = 0.80$). First, for the given values of $\eta^*_1, \ldots, \eta^*_n$, η_0, σ_m, σ_a, σ_c, and g_a the magnitudes of $I^*_1, \ldots, I^*_M$, simulating the results of measurements, which were compared with $\eta^*_1, \ldots, \eta^*_n$ with subsequent calculation of the errors $\delta\eta$ (in %). The magnitudes of I^*_a, rounded off to two significant digits, were used as the prognostic values. To determine η_i in all the schemes only one iteration was required when η^o_i was defined in this way. The results of the computations are given in Table 1. As seen from the Table, the albedo reconstruction algorithm proposed here ensures satisfactory accuracy for many problems of the US remote sounding.

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