ON THE EFFICIENCY OF FILTRATION OF PRIMARY SPECTROPHOTOMETRIC INFORMATION

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Received November 29, 1988

A technique for quasi-optimal filtration of spectrophotometric information is considered. For an additive mixture of a signal and white noise, the frequency characteristic of a filter is determined by the signal spectrum. The analysis makes use of a model of the signal, which is based on the extraterrestrial solar spectrum and the instrument function of an actual spectrometer. Estimation of the frequency spectral density of this signal model allows one to conclude that the Gaussian filter is a quasi-optimal filter in the class of linear systems. A relationship between the filter parameter and standard deviation of the noise is obtained. The efficiency of this filter is estimated from experimental data.

In some cases when carrying out spectrophotometric measurements of solar radiation in the atmosphere (or hydrosphere) we have to deal with weak light fluxes, which result in a small signal-noise ratio in the output of a spectral device and causes some additional difficulties in interpreting the results obtained. We face analogous problems if the parameters, which are to be determined experimentally have values comparable to the random errors of measurement (e.g. , spectral influxes of radiant energy in the atmosphere which are small secondary differences of the measured fluxes). In this paper we try to reduce the random errors in the data by means of a priori known statistic characteristics of the signal and the noise of the spectrophotometric instrumentation.

SPECTRAL MEASURING SYSTEM FOR INVESTIGATING THE SPECTRAL SHORT-WAVELENGTH CHARACTERISTICS OF THE ATMOSPHERE AND HYDROSPHERE (SMS)

A spectral measuring system has been developed and manufactured in the atmospheric shortwavelength radiation laboratory of the Scientific Research Institute at Leningrad State University with the direct participation of the authors of this work[1]. The basic elements are the K-3 spectrophotometer and the microcomputer Electronika D3-28. An enlarged diagram of the SMS is reproduced in Fig. 1.

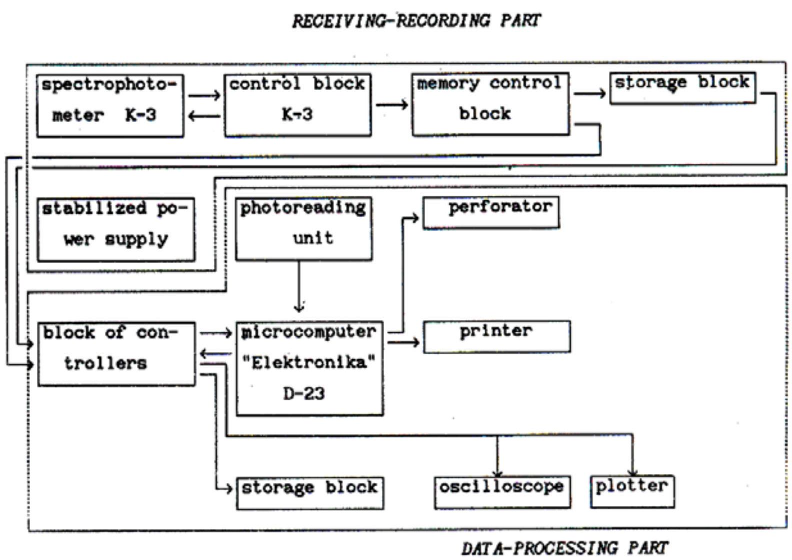

FIG. 1. A structural diagram of the spectral measuring system.

The K-3 spectrophotometer enables one to measure solar radiation in the range 0.3 to 1.0 μm. The field of view is equal to 2°10′ for at the measurements of sky brightness and the underlying surface and 180° for measurements of radiation fluxes. The scanning time of the entire spectral range is 7 sec.

The monochromator of the spectrophotometer was built according to a vertically symmetric design. The dispersive element is a diffraction grating (660 grooves/mm). The monochromator dispersion at the output slit is 55 Å/mm. The effective spectral width of the monochromator slit is of the order of 18 Å. The possibility exists of attenuating the signals to be recorded by a factor of nearly ~1000 using a ten-stage attenuator. The automated circuit enables one to record one cycle of measurements and also to record alternately the outer signal or the signal from the inner standard source. Besides the modular peripheral equipment, the SMS includes controllers which allow one to load the data into the microcomputer directly from the spectrophotometer or from the memory used to record the data under field conditions. These controllers also allow one to extract the initial processed data to be recorded on magnetic or punched tape (for further processing) and on an oscillograph or a plotter (for visual control or a graphic documentation).

TECHNIQUE FOR INITIAL DATA PROCESSING

An initial processing technique has been developed which assumes that the signal and noise are statistically independent and interact additively. To use methods of optimum linear filtration in processing the received data, it is necessary to have estimates of the signal and noise spectra[2] (from the radio-engineering viewpoint).

To estimate the statistical characteristics of signal typically encountered in atmospheric investigations, we propose a model formed according to the expression

$$I(\lambda)=\{[I_0(\lambda)\times T(\lambda)]*A\}\times\kappa(\lambda), \qquad (1)$$

where $I(\lambda)$ is the value of the signal at the spectrophotometer output during the recording of solar radiation; $I_0(\lambda)$ is the extraterrestrial value of the solar radiation; $T(\lambda)$ is the atmospheric transmission function; A is the instrument function of the spectrometer, which is approximated by the Gaussian curve $A = e^{-\lambda/2\alpha^2}$, where α is some parameter, and $\kappa(\lambda)$ is the spectral sensitivity of the spectrophotometer.

The extraterrestrial values of the solar radiation given in Ref. 3 at the wavelengths which correspond to the calibration of the K-3 spectrophotometer in the range 310 to 430 nm are used for the signal model. This spectral range has been chosen for the following reasons: 1) it corresponds to the UV interval of the spectrophotometer; 2) this structure is much more developed compared to other spectral regions, wherefore its energy spectrum should be more powerful at higher frequencies, which, in turn, enables one to apply methods which are used for signal extraction in this range to the remaining spectral regions; 3) the spectrophotometer noise in this spectral interval has a rather high σ.

In practice the function $T(\lambda)$ has not been introduced into the model of the signal. Thus, the investigated model represents values of the solar radiation as if they were measured by the actual spectrophotometer K-3 in the upper atmospheric boundary. The molecular absorption in the O_3 bands (and also the O_2 and the H_2O bands in other spectral intervals) but, can be added to the model of signal absorption, first the values of the absorption coefficients depend on atmospheric parameters which, as a rule, are not known a priori, and, second, allowing for absorption can change the statistical characteristics of the signal (in particular, the energy spectrum) only in the lower range of frequencies.

The spectral (frequency) density of the signal model is estimated by the formula

$$S(f)=2\Delta T\sum_{K=0}^{m} R(K\Delta T)W(K\Delta T)\times\cos 2\pi f K\Delta T, \qquad (2)$$

where $R(K\Delta T)$ is the autocorrelation function; $W(K\Delta T)$ is a spectral window[4] used for smoothing the selected spectrum in order to reduce the variance of the estimate. The spectral density of the noise of the entire measuring system recorded in the absence of an input signal at all amplifications is estimated according to formula (2).

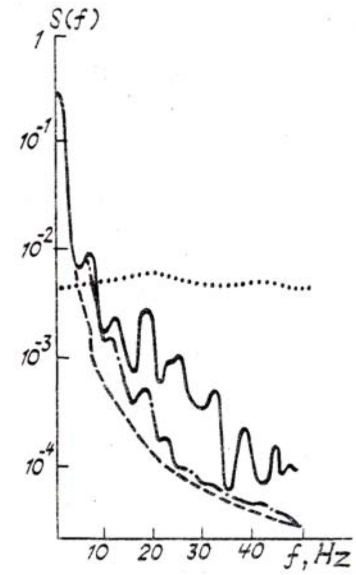

FIG. 2. Spectral densities of the signal model (at a different halfwidth of the instrument function) and noise of the spectral measuring system: the solid curve is the extraterrestrial solar radiation I_0; the dashed curve is $I_0*A(A/2 = 7$ nm); the dash-dotted line is $I_0*A(A/2=2.5$ nm); and the dotted line is the noise.

Figure 2 represents the values of the spectral density of the noise and the model of the signal at a different halfwidth of the instrument function. It can be seen from Fig. 2 that the noise in the frequency range where the spectral density of the signal decreases practically to zero has a constant spectral density and, hence, can be classified as white noise.

It is known[5] that when receiving an additive mixture of a signal plus white noise the frequency characteristic of the filter is entirely determined by the input signal spectrum. Since it is impossible to obtain an analytic expression for the spectral density of the signal, it is reasonable to discuss a quasi-optimal filtration. Judging from the spectral density of the signal, one can assume that the optimal filter should be sought among linear systems.

Table 1 displays the characteristics of some filters which can be used to smooth the signal.

The final signal model which was used to check the smoothing properties of the various signals has the following form:

$$I(\lambda)=\{[I_0(\lambda)\times T(\lambda)]*A\}\times\kappa(\lambda)+n(\lambda), \qquad (3)$$

where $n(\lambda)$ is the actual noise of the spectrophotometer with its statistic characteristics (variance, rms deviation). One should note that, the applied filters[4] being linear systems, the smoothing process (a digital narrow-band filtration) can be realized by convolution, thus:

$$I(\lambda)=I(\lambda)*\Phi(\lambda). \qquad (4)$$

The results of the smoothing achieved by the Gaussian filter prove to be somewhat better (according to the criterion of least-square error) compared to those made by the other filters (see Fig. 3a), although we can hardly consider this difference statistically important. It should be noted that a lower criticality also contributes to the Gaussian filter's merit.

Figure 3b shows a plot of the experimentally obtained data dependence of the optimal (according to the criterion of least-square error) parameter of the Gaussian filter d on the initial rms deviation of the noise. The curve in Fig. 3b allows one to introduce concrete filter parameters into the initial data program.

TABLE 1.

LINEAR SYSTEM	ANALITICAL EXPRESSION
Low-frequency RC-filter	$\exp(\alpha\|\tau\|)\times\alpha = 1/RC$
Two low-frequency RC-filters	$(1+\alpha\|\tau\|)\exp(-\alpha\|\tau\|)$
Three low-frequency RC-filters	$(1+\alpha\|\tau\|+\alpha^2\tau^2/3)\exp(-\alpha\|\tau\|)$
Gaussian low-frequency RC-filter	$\exp(-\alpha\tau^2)$
Standart low-frequency RC-filter	$\frac{\sin\Delta\omega t}{\Delta\omega t}$

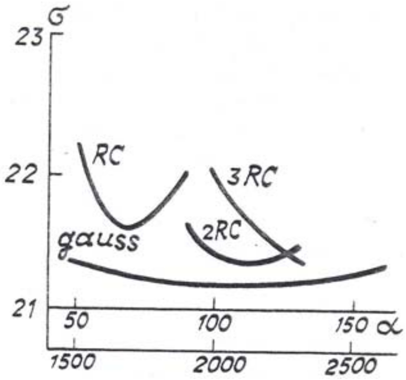

FIG. 3a. The efficiency of smoothing of the data by different filters for σ_n = const.

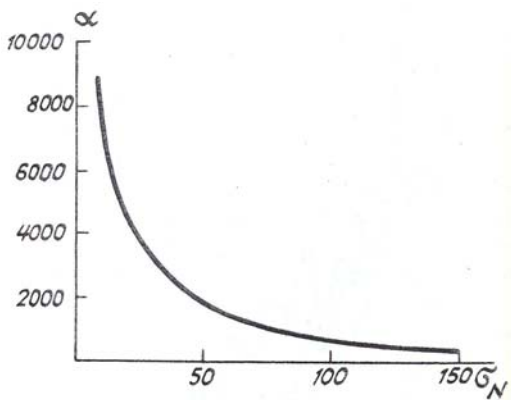

FIG. 3b. Dependence of the optimal parameter of the Gaussian filter a on the standard deviation of the noise σ_n .

EFFICIENCY OF APPLICATION OF PROCESSING TECHNIQUE TAKING INTO ACCOUNT THE STATISTICAL CHARACTERISTICS OF THE SIGNAL AND NOISE

To evaluate the practical efficiency of the suggested technique on an actual K-3 spectrophotometer, registergrams of a radiant flux from an SI-10 standard lamp were obtained. To evaluate the optical characteristics, a few wavelengths were arbitrarily chosen at different portions of the range and at different amplifications. Ten recordings were made at each amplification, i.e., at different signal-to-noise ratios. Recordings were made at 2580 sampling points within the entire range of wavelengths. Further processing of the data was carried out in two ways. First, 640 points were chosen from the entire dataset, i.e., a registergram was reduced to the standard variant (without smoothing the experimental data by any filter). The rms value, variance, and the standard were calculated from these data.

In the second variant before a registogram was reduce to the standard variant (640 points) the Gaussian filter was applied whose parameter was determined by the curve in Fig. 3b, depending on the amplification coefficient (rms deviation of noise). The same procedure as in the first variant was then carried out, i.e., 640 points were chosen from the filtered signal and the rms value, variance, and standard were determined for each chosen wavelength. Comparative results are given in Table 2.

From analysis of the data in Table 2, one can see that the application of digital filtration gives a rather noticeable advantage, especially in the center of the range, where one finds larger values of a signal (a larger signal-to-noise ratio). At the wavelength $\lambda = 735$ nm the rms deviation is reduced by nearly a factor of two at $K = 6$, and nearly a factor of four at $K = 9$; and at $\lambda = 823$ nm it is reduced by nearly a factor of 2 and 6, respectively.

TABLE 2.

λ, μm	$\sigma/\bar{F}$, %	K=6	$\sigma/\bar{F}$, %	K=6
	Initial signal	Smoothed signal	Initial signal	Smoothed signal
0.735	13.5	7.8	24.7	5.9
0.755	12.8	3.1	20.4	5.8
0.823	7.2	4.8	20.3	4.9
0.920	13.1	7.2	24.8	7.5

In July, 1987 at Krasnoye Lake (Leningrad oblast) we carried out an experiment on measuring the downwelling ($F_{\downarrow}$) and upwelling ($F^{\uparrow}$) fluxes of solar radiation at different depths H in the lake. In the experiment we used an optical fiber waveguide as the input unit (which was submerged) with the receiving surface manufactured from milky glass of type MS-13. The enquiry system of the spectrophotometer made it possible to receive 2560 readings over the entire range of wavelengths (350 + 950 nm). Further increase of the number of points by a factor of four makes it possible to obtain more, precision since it is well known[6] that obtaining additional measurements in a homogeneous series makes it possible to decrease the random error of the result. The number of readings is limited by the technical possibilities of the fundamental design of the K-3 spectrophotometer. As an illustration we chose two series of observations (July 22, 1987) of the downwelling flux $F_{\downarrow}$. The measurements were made cloudless conditions and without strong disturbance on the water surface, i.e., external conditions for appearance of additional random error were excluded.

The results at each observation point were smoothed by the Gaussian filter with the parameter α, determined by the curve of the dependence $\alpha = f(\sigma)$(Fig. 3b). The values corresponding to the wavelengths, λ_1 which were determined by the spectral calibration, were selected from the initial signal and the filtered data.

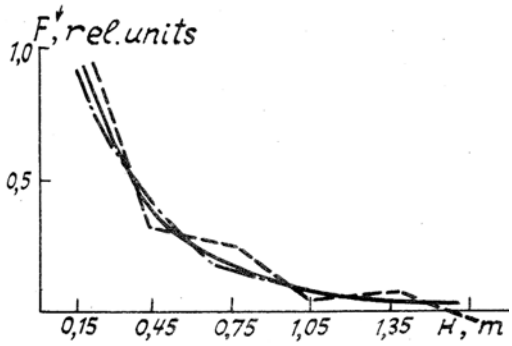

FIG. 4. Vertical profile of the incident flux in a water medium for $\lambda = 735$ nm: the dashed curve represents the measured values; the solid curve represents the values smoothed by the Gaussian filter; the dash-dotted curve is an approximating curve.

Figure 4 shows curves of $F_{\downarrow}(H)$ for a few arbitrarily selected wavelengths λ_1. .To evaluate the results of the technique for measuring the variance (or the rms deviation), it is necessary that the resultant dependences be approximated by a smooth curve.

It is apparent from their shape that the resultant curves can be referred to an exponential dependence. However, when they are plotted on a semilog plot, they give some deviation from a straight line towards a parabola. Therefore, we should add one more term to the equation of a straight line

$$y=a+bH+cH^2, \quad (5)$$

where $y = I_n$, $a = I_nF_0$ (F_0 is the value of F at $H = 0$).

The approximating functions obtained are shown in Fig. 4.

Table 3 presents the values $F(H)$: F are the values of the incident flux after filtration: F' are the initial (observed) values: F'' are the values calculated by the approximating function.

TABLE 3.

H, m	λ, nm								
	735			755			823		
	F	F'	F''	F	F'	F''	F	F'	F''
0.15	204	243	175	211	242	183	186	243	161
0.45	70	67.8	82	67	87.2	70.4	65.4	69.8	79.5
0.75	42	49.3	38.7	20.8	5.2	27.2	40.3	39.3	39.2
1.05	15.1	11.3	18.1	10.8	11.2	10.5	20.6	37.3	19.4
1.35	8.4	10.9	8.5	5.2	-3.9	4.0	8.4	12.6	9.5
1.65	4.2	-6.6	3.9	1.4	-3.9	1.6	5.1	-7.9	4.7

After fitting the formulas to the experimentally obtained dependences, the variances D and standard deviations σ of the initial and filtered values relative to those calculated by the approximating functions were determined. The results are given in Table 4 where D_1 and σ_1 are the initial values; D_2 and σ_2 are the smoothed ones. The values of D and σ are given in relative units.

TABLE 4.

λ, nm	D_1	D_2	σ_1	σ_2
735	1028	207	32.1	14.4
755	880	173	29.7	13.1
823	1475	165	38.4	12.9

It can be seen from Table 4 that even for a rough choice of the approximating function the use of digital filtration allows one to reduce the variance of the observed data more them fourfold.

CONCLUSION

We have evaluated the efficiency of digital filtration of the initial spectrophotometric data in two ways. For a specific unit of the concrete K-3 spectrophotometer produced line. These evaluations show that at certain levels of amplification one can reduce the random errors of the measurements about twofold. This means, that when carrying out field experiments in the atmosphere we can, first, measure influxes of radiant energy in the atmosphere with twice as small an error. Second, we can investigate these influxes in still "clearer" air masses than was previously possible. Third, we can reconstruct their vertical profiles, which was previously impossible because of the large errors of the experimental data. The technique of digital filtration of the initial spectrophotometric data described in this paper is now being introduced in practice of in atmospheric-optical and underwater-optical measurements.

ACKNOWLEDGEMENT

The authors are grateful to Candidate of Phys. Math. Sci. V.S Grishechkin and Senior Engineer I.A. Tovstenko for their interest and helpful discussions in this work.

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