PROBLEMS OF FORMATION OF A LASER REFERENCE STAR

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Some problems connected with the development of modern ground-based adaptive telescope are investigated, in particular, to its equipment with additional optical system of formation of a laser reference star. This paper is devoted to problems of determining of the type of a formed laser reference star. The matter is that the scientific literature monostatic and bistatic schemes of formation of a laser reference star are known. It is assumed in the monostatic scheme that the laser beam is focused in the atmosphere so, as the fluctuations of the laser reference star caused by the radiation that has passed the forward and back paths have maximum correlation. An opposite situation is observed for the bistatic scheme the fluctuations caused by the radiation that has passed the forward and back paths are assumed completely uncorrelated. In this paper, the results of calculations are presented for general scheme of formation of the laser reference star, when the arbitrary correlation between the random image angular shifts caused by the fluctuations on the forward and back paths can be obtained. The expressions for the monostatic and bistatic schemes are obtained as the limiting cases.

Equipment of a modern ground-based adaptive telescope with an additional optic system of laser reference star formation is one of the most promising tendencies for its development (Refs. 1–3). In my opinion, fairly extensive bibliography of modern works (unfortunately, the works carried out earlier in the USSR and Russia in the last few years were not included) on basic stages of the development of the laser star formation systems is presented by Rigazzoni.[3]

This paper is a logical continuation of works carried out in 1966–1986 in the USSR (Refs. 4–17) as well as recently in Russia (Refs. 18–22). Theoretical and applied investigations on the use of lasers for the laser reference star formation, which become very popular in the last few years, made me to return to some results obtained by me more than 18 years ago. These results were most completely presented in Refs. 7, 12 and 16. They become available for wide scientific community, especially western, after publication of Ref. 17. The forth and five chapter of this monograph were devoted to this problem. It should be mentioned that many problems being solved now have already been solved at that time.

PECULIARITIES OF REFLECTED OPTICAL WAVE FLUCTUATIONS

Importance of investigations of the efficiency of adaptive optics systems with the use of artificial reference sources have been understood in the early 70s (Refs. 14–17). Basic principles of operation of the systems using reference sources to obtain the information on the fluctuations in the channel of propagation of optical radiation have already been formulated at that period. Since the reciprocity principle[14,15,17] provides the basis for the adaptive optics systems the system using the independent reference source whose radiation propagates in the direction opposite to that of the radiation propagation is the most efficient adaptive optics system.

From the viewpoint of practical realization the systems using the back scattered radiation (from the atmospheric inhomogeneities or an object) are most simple for realization. In this case, an artificial (virtual) reference source is formed[12].

In the early 80s among the astronomers and designers of adaptive telescopes a new concept of *a laser reference star* (LRS) appeared (Refs. 2 and 3). There are two basic schemes of LRS formation: monostatic and bistatic. A ground-based laser is used for these aims and therefore the received optical radiation passes twice through one and the same atmospheric inhomogeneities. The first time along the path upward in order to form the LRS itself and for the second time along the path downward as a result of scattering (secondary emission and elastic aerosol or molecular scattering) from the atmospheric inhomogeneity. In both schemes it is necessary to consider the peculiarities of the fluctuations of light waves that has passed twice through the atmosphere.

About the early 70s the researchers dealing with the optics vision and laser beam formation systems in

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the atmosphere realized the importance of taking into account the peculiarities of the fluctuations of the reflected waves (Refs. 4 and 5). Double passage over along the atmospheric paths is always present in sounding systems in contrast with transmission systems. The researchers of the above-mentioned systems introduced such terms as the *effective scattering volume, monostatic optical scheme*, *bistatic laser sounding scheme*, and some others. In my opinion, such terms as the *effective scattering volume* and the *laser reference star* are scientific synonyms. In the coarse of this investigations in 1975–1983, the fluctuations of the displacement of the center of gravity of the image formed in a system of laser detection and ranging using a focused laser beam were considered. The fluctuations of the displacement of the atmospheric sounding volume image were investigated in Ref. 4. The monostatic and bistatic schemes were considered.

Relations for the dispersion of the fluctuations of the displacement of the optical image center of gravity in the photodetector plane in case of detection and ranging of the surface with arbitrary scattering properties were obtained (Ref. 4, pp. 84–85). For strongly scattering surface – in the approximation of the Lambert scattering – for the bistatic scheme it was obtained that the variance of the linear displacement of the image center of gravity $\rho_{\rm im}$ in case of reflection is given by the formula (Ref. 4, p. 92)

$$\langle\rho_{\rm im}^2\rangle = \frac{F^2}{x^2}\langle\rho_{\rm l.b}^2\rangle + F^2\langle(\rho_F^{\rm ss})^2\rangle, \tag{1}$$

where $\langle\rho_{\rm l.b}^2\rangle$ is the variance of the random displacement of the laser beam center of gravity in the plane of detection and ranging for upward a beam propagation; $\langle(\rho_F^{\rm ss})^2\rangle$ is the variance of the random angular displacement of a fixed secondary source (for downward propagation); F is the focal distance of a telescope; X is the distance between the laser source and the scattering volume.

Thus, it was shown that for the bistatic scheme (the limiting case was considered for the bistatic scheme, when the fluctuations for the forward and back paths are uncorrelated) the variance of the angular shift of the image is a sum of the variances of the sounding beam angular shifts and of the fixed secondary source image angular shifts. For strongly scattering medium and the focused beam, the secondary source is in fact a point source.

The extended source image jitter was also investigated at this period. References 6 and 10 should be especially mentioned here. Expression for the variance of the image jitter of the extended source in form of glowing fine filament was obtained in Ref. 6. Reference 10 was devoted to investigations of the correlation between jitters of centers of gravity of two arbitrary oriented laser beams.

Nevertheless, Orlov et al.[4] failed to account and to calculate cross correlation between the fluctuations of the focused beam and secondary source image shifts.

CROSS CORRELATION BETWEEN RANDOM SHIFTS OF BEAMS AND IMAGES

In 1978–1980 (Refs. 7 and 12) I studied the problem of stabilization of the laser beam propagation direction in the turbulent atmosphere. For its solution the measurements of the reference source image shift were considered (including a natural star) in the focal plane of a telescope. In particular, in Ref. 7 the cross correlation function $\langle\rho_{\rm l.b}\,\rho_F\rangle$ between the vector characterizing the random shift of the energetic center of gravity of an optical beam $\rho_{\rm l.b}$ propagating through the turbulent medium and vector specifying the center of gravity of the image of a star or any reference source formed by the same optical system ρ_F. Therewith it was assumed that it could be the image of a reference source – beacon or of the optical beam reflected from an object. As a particular case, they could be the images of a natural star, a specularly reflected laser beam or a point reference source.

The cross-correlation function between the random displacement of the Gaussian beam center of gravity and the image center of gravity of a plane wave was calculated in Ref. 7. The beam and the plane wave propagated along the same optical path. The random shift of the beam center of gravity are determined by the vector (Ref. 8)

$$\rho_{\rm l.b} = \frac{1}{P_0}\int_0^X {\rm d}\xi\,(X-\xi)\iint {\rm d}^2R\; I(\xi,\mathbf{R})\times\nabla_R\, n_1(\xi,\mathbf{R}), \tag{2}$$

$$P_0 = \iint {\rm d}^2R\; I(0,\mathbf{R}),$$

where $n_1(\xi, \mathbf{R})$ denote the fluctuations of the refractive index at the point $(\xi, \mathbf{R})$; $I(\xi, \mathbf{R})$ is the field intensity at the point $(\xi, \mathbf{R})$ from a laser source located at the origin of coordinates in the initial plane (for $\xi = 0$); X is the thickness of the atmosphere layer. The random image shifts in the focal plane of optical system (a telescope or equivalent thin lens with a focal distance F and an area $\Sigma = \pi R_0^2$) are given by the formula (Ref. 9)

$$\rho_F = -\frac{F}{k\,\Sigma}\iint_\Sigma \nabla_\rho\, S(x,\rho)\,{\rm d}^2\rho, \tag{3}$$

where k is the radiation wave number, $S(x, \rho)$ are the optical wave phase fluctuations on the aperture of the optical system (in the plane $\xi = X$) at the point ρ. The cross-correlation between the random vectors $\rho_{\rm l.b}$ and ρ_F is given:

$$K = \langle\rho_{\rm l.b}\,\rho_F\rangle/[\langle\rho_{\rm l.b}^2\rangle\langle\rho_F^2\rangle]^{1/2}. \tag{4}$$

Here and further in this paper $\langle\ldots\rangle$ denotes averaging over an ensemble of realization of the random function $n_1(\xi, \mathbf{R})$. Let us assume that the functions $\langle I(\xi, \mathbf{R})\rangle$ and $\Phi_n(\xi, \kappa)$ are isotropic, and the average intensity $\langle I(\xi, \mathbf{R})\rangle$ for a Gaussian beam is given by the equation (Ref. 8)

$$\langle I(\xi, \mathbf{R})\rangle = \frac{a^2}{a_{\rm ef}^2(\xi)} \exp\,(-\,R^2/a_{\rm ef}^2(\xi)), \qquad (5)$$

where

$$a_{\rm ef}^2(\xi) = a^2 \left[\left(1 - \frac{X}{f}\,\xi\right)^2 + \Omega^{-2} + \Omega^{-2}\left(\frac{1}{2}\,D_S(2a)\right)^{6/5}\right]$$

$\Omega = \dfrac{ka^2}{X\xi}$, a and f are the initial parameters of the Gaussian beam, $D_S(2a)$ in the structure phase function. As a result, we obtain (Ref. 7)

$$K = \int\limits_0^1 d\xi\,(1-\xi) \int\limits_0^\infty d\kappa\kappa^3\,\Phi_n(\kappa)\,\times$$
$$\times \exp\left(-\,\frac{\kappa^2\,(R_0^2 + a_{\rm ef}^2)}{4}\right) \cos\left(\frac{\kappa^2\,x\,(1-\xi)}{2k}\right) \times$$
$$\times \left(\int\limits_0^1 d\xi(1-\xi)^2 \int\limits_0^\infty d\kappa\kappa^3 \Phi_n(\kappa)\,\exp\left(-\,\frac{\kappa^2 a_{\rm ef}^2(\xi)}{2}\right)\right)^{-1/2} \times$$
$$\times \left(\int\limits_0^1 d\xi \int\limits_0^\infty d\kappa\kappa^3\,\Phi_n(\kappa)\,\exp\left(-\,\frac{\kappa^2\,R_0^2}{2}\right) \times\right.$$
$$\left.\times \cos^2\left(\frac{\kappa^2\,x\,(1-\xi)}{2k}\right)\right)^{-1/2}. \qquad (6)$$

In calculations, we use the following spectrum:

$$\Phi(\xi, \kappa) = 0.033\,C_n^2(\xi)\,(\kappa^2 + \kappa_0^2)^{-11/6}, \qquad (7)$$

which takes into account the deviation from the power-low dependence in the range of the external turbulence scale $L_0 = 2\pi\kappa_0^{-1}$, $C_n^2(\xi)$ is the structural parameter of the turbulent atmosphere.

The estimates were made out for a homogeneous path (the initial beam diameter was equal to the diameter of a receiving aperture telescope) for the following parameters of the problem:

$$\kappa_0^{-1} \gg (R_0,\, a_{\rm ef},\, \sqrt{x/k});\; kR_0^2 \gg x,$$
$$\Omega^{-2}\left(\frac{1}{2}\,D_S(2a)\right)^{6/5} \ll 1.$$

We obtained (for the focused beam $f = X$) the value $K = 0.84$.

Thus, in Ref. 7 a high positive correlation was found between the shifts of the Gaussian beam and of the plane wave center of gravity under condition that the laser beam and plane wave propagate over the same path and in the same direction.

Later in 1980 these results were generalized in Ref. 12 for the case in which the beam and image formation occur in the opposite directions. Therewith it was assumed that the reference image formation in the telescope focal plane occurs for the following scenarios: plane wave, spherical wave and arbitrary Gaussian beam reflected from a plane mirror. For the plane wave and homogeneous path we obtained: $K = -\,0.87$ for a collimated beam, $K = -0.82$ for the focused beam. For a reference spherical (and any other) wave $\langle\rho_{\rm l.b}\,\rho_F\rangle$ can be calculated directly from the formulas of Ref. 12.

Calculations of the efficiency of the correction of the extended object image observed through the turbulent atmosphere on the basis of an adaptive telescope with a reference star were first done in Ref. 16.

As a result, we established that in the Russian papers published in 1966–1982 all functions necessary for an analysis of the image random shifts of the sounded volume for laser sounding of the atmosphere were calculated not only for bistatic, but also for monostatic scheme. However, every time when we solve a particular problem, the question on a scattering (or reflecting) medium model remains open, when in its turn determines a secondary source model (see Eq. (1)). In this case, we may introduce a model of a scatterer or solve the scattering optical problem.

PROBLEMS OF THE USE OF THE LASER REFERENCE STAR

A rebirth of interest to this problem was caused by the suggestion to use a signal of the laser reference star for image correction in a ground-based telescope. In particular, Fugate[2] pointed out several serious problems connected with the use of the laser reference star in telescopes, namely the effect of focus isoplanatism and the practical impossibility (for the monostatic scheme) to separate out contributions of the upward and downward propagation to the laser reference star image jitter.

It should be mentioned that some papers (Refs. 23–25) appeared in which several ways of the solution of one of these problems were suggested. So, in Refs. 24 and 25 the scheme of formation of the laser reference star was suggested, in which the laser beam passing through the principal telescope was used together with two auxiliary telescopes used for measurements of the laser reference star image jitter. The optical scenario is such that for the principal telescope the laser reference star represents a point source, whereas for the auxiliary telescopes it is extended. Therefore, as pointed out by Belen'kii,[24] the monostatic laser reference star cannot be used for the correction of wavefront tilts in the principal telescope, however, the bistatic scheme (with the auxiliary telescopes) allows one to separate out the component of the laser reference star image jitter, corresponding to a directed laser beam, which is highly correlated with the total wavefront tilt for the nature star. Unfortunately, Belen'kii does not list all the references he used, and Eqs. (15) and (16) presented in Ref. 24 were incorrect. In his next paper (Ref. 25) Belen'kii used a narrow laser beam directed by the optical system located in front of the principal telescope and suggested to use the difference between the laser reference star image jitter measured simultaneously by the principle and auxiliary telescopes as a signal for the tilt correction. References 4, 6, 7, 10, and 12 on the papers published long before of Ref. 25 were also lacking.

In turn, I must also claim that while analyzing the Ragazzoni scheme in Refs. 20 and 21, I did not list all the references and omitted Refs. 23–25.

CORRELATION BETWEEN LASER BEAM SHIFTS AND NATURAL STAR IMAGE FOR THE BISTATIC SCHEME OF LASER REFERENCE STAR FORMATION

After Ragazzoni (Refs. 3 and 23), let us consider the following scheme of the laser reference star formation (Fig. 1). A laser reference star is formed with a laser system with a separate transmission aperture. Here, the following designations are introduced: R_0 is the radius of the principal telescope aperture, X is the altitude of the laser reference star formation (the receiving aperture of the telescope is placed in the plane $x = 0$); a_0 is the radius of the aperture of the auxiliary telescope that forms the laser reference star; $\boldsymbol{\rho}_0$ is the vector of the auxiliary telescope center shift relative to the principal telescope optical axis.

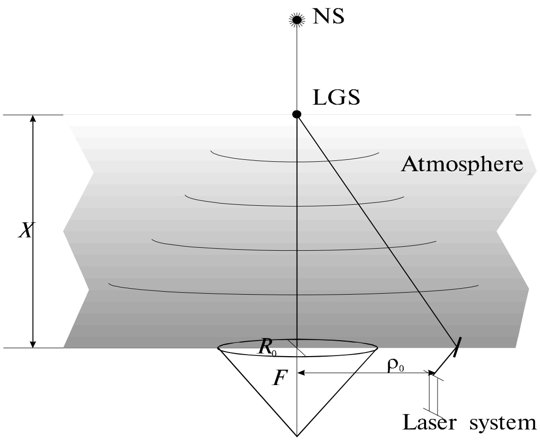

FIG. 1. Schemes of formation of laser reference star: monostatic, intermediate, and bistatic.

We assume that the principle telescope operates with the adaptive correction using the radiation of the laser reference star formed by laser system on the optical axis of the principal telescope at the distance (altitude) X from the receiving aperture. As shown in Fig. 1, the principal telescope is pointed exactly to the zenith. The weak natural star and the laser reference star are simultaneously observed on its optical axis. The laser beam zenith angle (under condition that $|\boldsymbol{\rho}_0| \ll X$) is equal to $|\boldsymbol{\rho}_0|/X$.

Suppose that the natural star is located in the infinity and forms the plane wavefront. The vector that characterizes the random tilt of this wavefront caused by the atmospheric turbulence is written as follows (Ref. 9):

$$\boldsymbol{\varphi}_F^{\rm pl} = -\frac{1}{\Sigma} \iint_{\Sigma} d^2\rho\, \nabla_\rho\, S^{\rm pl}(0, \boldsymbol{\rho}), \qquad (8)$$

where $S^{\rm pl}(0, \boldsymbol{\rho}) = k \int_0^\infty d\xi \iint d^2 n(\boldsymbol{\kappa}, x - \xi) \exp(i\boldsymbol{\kappa\rho})$ are the phase fluctuations of the plane wavefront on the receiving aperture, Σ is the area of the telescope receiving aperture, and k is the radiation wave number. The fact that the optical wave from the natural star is propagated downward through the atmosphere was accounted in the derivation of Eq. (8) and the following spectral expansion of the fluctuations of the atmospheric refractive index was used:

$$n_1(\xi, \boldsymbol{\rho}) = \iint d^2 n(\boldsymbol{\kappa}, \xi) \exp(i\boldsymbol{\kappa\rho}).$$

Random angular shifts of the center of gravity of the laser reference star formed by the laser system at the altitude X can be written (Ref. 10), using Eq. (2) and making substitution

$$I = I(\xi, \mathbf{R} + \boldsymbol{\rho}_0 (1 - \xi/X)).$$

In the last expression it was taking into account that the optical axis of the laser source is shifted by the vector $\boldsymbol{\rho}_0$ and tilted from the zenith by the angle $|\boldsymbol{\rho}_0|/X$.

Let us assume that the focused laser beam is fairly wide (such as $\Omega^{-1} = k a_0^2/X \gg 1$), and the additional laser beam broadening due to the effect of turbulence is insignificant. Here, from an input pupil of the principal telescope the laser reference star can be seen as a point source. In case of backward propagation, the additional angular image jitter of such spherical wave is given by the formula[9]:

$$\boldsymbol{\varphi}_F^{\rm sp} = -\frac{1}{\Sigma} \iint_{\Sigma} d^2\rho\, \nabla_\rho\, S^{\rm sp}(0, \boldsymbol{\rho}). \qquad (9)$$

Let us calculate the cross-correlation function between the random angular image shift of the natural star (Eq. (8)) formed by the telescope and the shift of the center of gravity of the laser beam formed by the tilted laser source (Eqs. (2) and (9)). This correlation was calculated many times in my papers (Refs. 7, 12, 14, and 17), including calculations with the use of the turbulence spectrum model considering the deviation from the power law in the range of the large scales of turbulence (Ref. 17, 26–31)

$$\Phi_n(\kappa, \xi) = 0.033\, C_n^2(\xi)\, \kappa^{-11/3} \times$$
$$\times \{1 - \exp(-\kappa^2/\kappa_0^2)\}, \qquad (10)$$

where $C_n^2(\xi)$ is the turbulence intensity on the propagation path, $\kappa_0^{-1}(\xi)$ is the outer scale of turbulence. Considering Ref. 7 and 12, this correlation function can be written as the follows:

$$\langle \boldsymbol{\varphi}_{\rm l.b.}(\boldsymbol{\rho}_0)\, \boldsymbol{\varphi}_F^{\rm pl} \rangle = \left(-2\pi^2\, 0.033\, \Gamma\left(\frac{1}{6}\right)\right) 2^{1/3} \times$$

$$\times R_0^{-1/3} \int\limits_0^X d\xi\, C_n^2(\xi)\,(1 - \xi/X) \times$$

$$\times \Bigg\{[1 + b^2 (1 - \xi/X)^2]^{-1/6} \times$$

$$\times {}_1F_1\left(\frac{1}{6}, 1; -\frac{d^2(1 - \xi/X)^2}{(1 + b^2 (1 - \xi/X)^2)}\right) -$$

$$- [1 + b^2 (1 - \xi/X)^2 + 4c^2]^{-1/6} \times$$

$$\times {}_1F_1\left(\frac{1}{6}, 1; -\frac{d^2(1 - \xi/X)^2}{(1 + b^2(1 - \xi/X)^2 + 4c^2)}\right)\Bigg\}. \qquad (11)$$

Here, the designations are used: $b = a_0/R_0$, $b = |\rho_0|/R_0$, $c = \kappa_0^{-1}R_0^{-1}$, a_0 is the initial radius of the focused laser beam, and ${}_1F_1(...)$ is the degenerate hypergeometric Gaussian function.

It can be seen from Eq. (11) that the second term in curly brackets describes the effect of the outer scale of turbulence. For the infinite outer scale ($c \to \infty$) the second term in Eq. (11) can be neglected. Then the correlation function assumes the following form:

$$<\varphi_{\text{l.b.}}(\rho_0)\ \varphi_F^{\text{pl}}> = \left(- 2\pi^2\ 0.033\ \Gamma\left(\frac{1}{6}\right)\right) 2^{1/3} \times$$

$$\times R_0^{-1/3} \int\limits_0^X d\xi\, C_n^2(\xi)\,(1 - \xi/X) \times$$

$$\times [1 + b^2 (1 - \xi/X)^2]^{-1/6} \times$$

$$\times {}_1F_1\left(\frac{1}{6}, 1; -\frac{d^2(1 - \xi/X)^2}{(1 + b^2 (1 - \xi/X)^2)}\right). \qquad (12)$$

The case $d = 0$ corresponds to the monostatic scheme of laser reference star formation. In the opposite case (for the bistatic scheme) the condition $d \gg 1$ corresponds to the asymptotic for the hypergeometric function ${}_1F_1(...)$ then

$$<\varphi_{\text{l.b.}}(\rho_0)\ \varphi_F^{\text{pl}}> = \left(- 2\pi^2\ 0.033\ \Gamma\left(\frac{1}{6}\right)\right) 2^{1/3} R_0^{-1/3} \times$$

$$\times \Gamma^{-1}\left(\frac{5}{6}\right) d^{-1/3} \int\limits_0^X d\xi\, C_n^2(\xi)\,(1 - \xi/X)^{2/3}. \qquad (13)$$

It can be concluded from an analysis of the last expression that the correlation between the plane wave and the beam decreases approximately to the level 0.1 when $d \ge 10^3$. This, in fact, corresponds to the limiting case of the bistatic scheme of laser reference star formation.

As numerous experimental data (Refs. 17, 27, 29, and 32–35) shows, the outer scale of turbulence $\kappa_0^{-1}(\xi)$ in the atmosphere is known to be the finite quantity. Numerical estimates made by us for different models of the vertical profiles of $C_n^2(\xi)$ and $\kappa_0^{-1}(\xi)$ (Ref. 37) showed that for the optical radiation propagating through the entire atmosphere *an effective outer scale of turbulence* can be introduced for the atmosphere as a whole (in analogy with Ref. 36). It turned out (see Ref. 37) that for intermediate conditions of vision (Ref. 38) the value of such *an effective outer scale* is 5–60 m. Then for the telescope with $R_0 = 4$ m the parameter $c = \kappa_0^{-1}R_0^{-1} = 10$.

Let us analyze asymptotically the effect of the outer scale of turbulence on the correlation function (Eq. (11)). We consider the variable d to be argument of function (11) and b, c, and X to be its parameters. Simple estimates demonstrates that the initial correlation (for $d = 0$), due to the effect of the finite outer scale of turbulence (for $c < 5$), decreases 2–3 times in comparison with its value for the infinite outer scale. With the increase of the argument d (for $d > 1$), correlation (11) already does not exceed 0.2. For $d > 2c$ this correlation is about 17 times smaller then its value for the infinite outer scale. And finally, for $d \gg c$ the correlation $<\varphi_{\text{l.b.}}(d)\ \varphi_F>$ changes its sign and the dependence $\approx d^{-7/3}$ is observed.

Thus, our asymptotic analysis shows that for $c < 5$ when the axis of the principal and auxiliary telescopes are separated at $\rho_0 \ge 2\kappa_0^{-1}$, we obtain practically limiting case of the bistatic scheme.

To confirm the conclusion of our asymptotic analysis, let us do numerical calculations of the correlation coefficient

$$K(d, b, c, X) = \frac{<\varphi_{\text{l.b.}}(\rho_0)\ \varphi_F^{\text{pl}}>}{\sqrt{<(\varphi_{\text{l.b.}}(\rho_0))^2>\ <(\varphi_F^{\text{pl}})^2>}}, \qquad (14)$$

which is expressed in terms of the correlation function (11) and the corresponding variances

$$<(\varphi_F^{\text{pl}})^2> = \left(2\pi^2\ 0.033\ \Gamma\left(\frac{1}{6}\right)\right) 2^{1/6} R_0^{-1/3} \times$$

$$\times \int\limits_0^\infty d\xi\, C_n^2(\xi)\,[1 - [1 + 4c^2]^{-1/6}], \qquad (15)$$

$$<(\varphi_{\text{l.b.}}(\rho_0))^2> = \left(2\pi^2\ 0.033\ \Gamma\left(\frac{1}{6}\right)\right) 2^{1/6} R_0^{-1/3} \times$$

$$\times \int\limits_0^X d\xi\, C_n^2(\xi)\,\{(b^2 (1 - \xi/X)^2)^{-1/6} -$$

$$- (b^2 (1 - \xi/X)^2 + 4c^2)^{-1/6}\}. \qquad (16)$$

The calculations were done for the model $C_n^2(\xi)$ (Ref. 38) corresponding to the intermediate conditions of vision. To the represent the results, the variable d characterizing the relative separation of the axes of the principal and auxiliary telescopes was chosen as a function argument. The calculations were done for two most typical altitudes $X = 10$ and 100 km, that is correspond approximately to the positions of Rayleigh and natrium laser reference stars. The values of the parameter b were chosen equal to 0.1, 0.3, 0.7, 1.0, 3.0, and 5.0.

Practically, the value of the parameter $b > 1$ corresponds to the reference star formed by the large telescope for the small one. It can be realized in the observatories where the telescopes of different sizes are located, for example, in the observatory Mauna Kia, where the 10-m Kekk telescope forms in the atmosphere the reference star for the telescope of smaller size.

The values of the parameter c in our calculations took the following values: 1, 3, 5, 10, 100, and 1000. The case $c = 1000$ practically corresponded to the case of the Kolmogorov turbulence. The results of calculations are shown in Figs. 2 and 3. Each of the figures is represented by six fragments (a, b, c, d, e, and f) that correspond to the values 1, 3, 5, 10, 100, and 1000 of the parameter c.

It is interesting to note that our numerical results confirm the conclusions of analytical analysis presented above. In particular, the following conclusions can be drawn based on the results of our calculations:

For large values of the outer scale ($c = 100$ and 1000) transition from the monostatic ($d = 0$) to the limiting bistatic scheme takes place when the separation of the axes of the principal and auxiliary telescopes is $(200-1000)R_0$, i.e. when $d > 200$.

For finite outer scale ($c < 5$) although for the separations of the order of two-three outer scales the transition to the limiting bistatic scheme takes place.

The smaller separations of the telescope axes ($d < 200R_0$ or $d < c$, respectively) yield the intermediate case.

It should be noted here that our calculations differ from the calculations done in Refs. 23 and 40, because in these papers the angular dependence of the cross-correlation between two plane waves coming from the infinity at different angles was studied. In particular, in our case the correlation coefficient K for the argument $d = 0$ (see Eqs. (11)–(13)) is not equal to −1. Only with the increase of the parameters X and c the value of K (for $d = 0$) asymptotically approaches −1. First such calculations were done as long ago as 1980 (Ref. 12), and I should also mention the calculations done in Ref. 10.

It is interesting to analyze the peculiarities in the behavior of the correlation coefficient K for small values of the parameter c ($c = 1$, 3, and 5) and for large values of the parameter b ($b = 3$ and 5). This case is observed when a larger telescope forms the reference star for a smaller one. The results (Figs. 2 and 3) show that in this case the correlation for arguments $d < c$ remains practically constant (equal to 0.4, 0.5, and 0.7, respectively).

With the increase of the parameter c the correlation function K becomes larger-scale: the increase of the correlation radius is observed. It should be note that analogous effect was established in Ref. 25, where the increase of the angular correlation radius was recorded with the increase of the outer scale.

Gradual saturation of the increase of the correlation radius also occurs. For small c (small value of the outer scale of turbulence) the correlation $K(d)$ decreases down to 0.1 at $d = c$. However, for $c = 100$ the correlation decreases to 0.1 at $d = c/2$ and for $c = 1000$ at $d = c/10$. This happens for both altitudes of 10 and 100 km.

The change of the sign of the correlation K predicted on the basis of the asymptotic analysis (it is necessary to return and to computer Eqs. (11) and (12)) is caused by the finite outer scale. For small values of the outer scale of turbulence ($c = 1$, 3, and 5) and $d > (2-3)c$ the correlation K changes its sign. For large values of the parameter c this was not established in practice. For the infinite outer scale (see Eq. (12)) the correlation coefficient K does not change its sign.

It is very important to found the relation between the angular correlation radius of two plane waves (this value was calculated in Refs. 25 and 40) and the cross-correlation radius between the plane wave and the Gaussian beam (this characteristic is shown in Figs. 2 and 3). Then we could use the data of the direct astronomic observation of the image jitter of two stars seen at different angles predict the cross-correlation coefficient for the system telescope − laser reference star and to make correct conclusions about the transition of the laser reference star formation system to the bistatic regime. Now the simple comparison between the curves in Figs. 2 and 3 from the presented paper and, for example, the curves in Fig. 2 of Ref. 25 is impossible, because the significantly different models of the turbulent atmosphere were used in these papers.

ALGORITHM OF OPTIMAL CORRECTION OF TOTAL WAVEFRONT TILT

It is well known that the use of the laser reference star expands the range of stable operation of the adaptive system. However, because the laser star is formed at a finite distance, the correction of the data of optical measurements of the laser star is necessary to provide efficient distortion correction for the natural astronomic objects. I consider the investigations of the possible of the improvement of the correction of atmospheric distortions using the atmosphere models to be very important (Refs. 20, 22, and 39). Using the atmospheric turbulence models, we can:

a) to estimate the value of the turbulent distortions above the reference star and hence to determine the optimal altitude of the star formation,

b) to compensate partially the focus isoplanatism for the system operating with the reference star located at a finite distance in the atmosphere,

c) to choose the optimum scheme of forming the laser reference star,

d) and finally, to improve the estimation of the reference star total wavefront tilt.

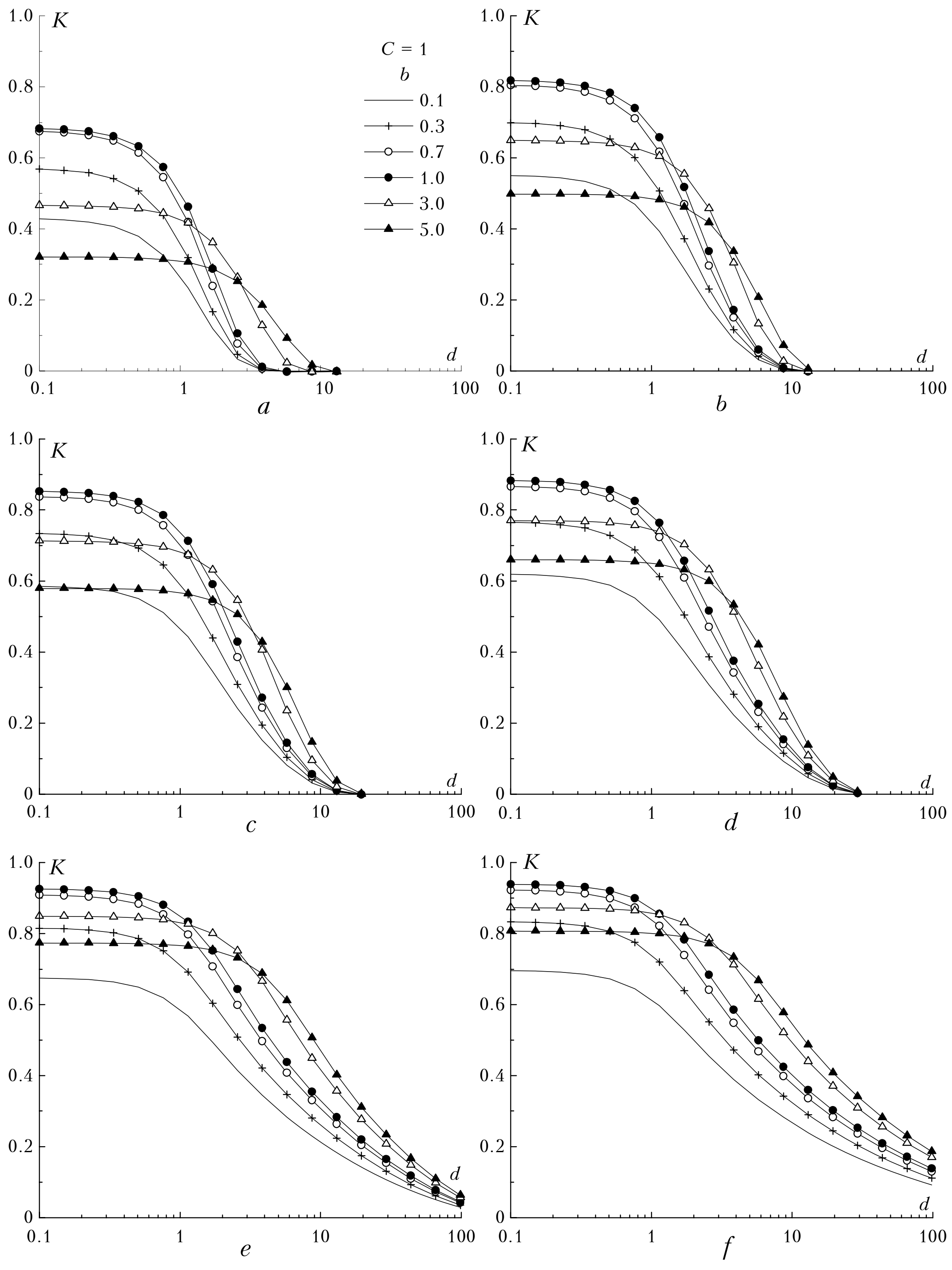

FIG. 2. Correlation index (without sign accounting) of the random angular shifts of the image center of gravity of the normally falling plane wave and angular shifts of the center of gravity of the laser focused beam tilted relatively to the telescope axis for the height of formed laser reference star u = 10 km.

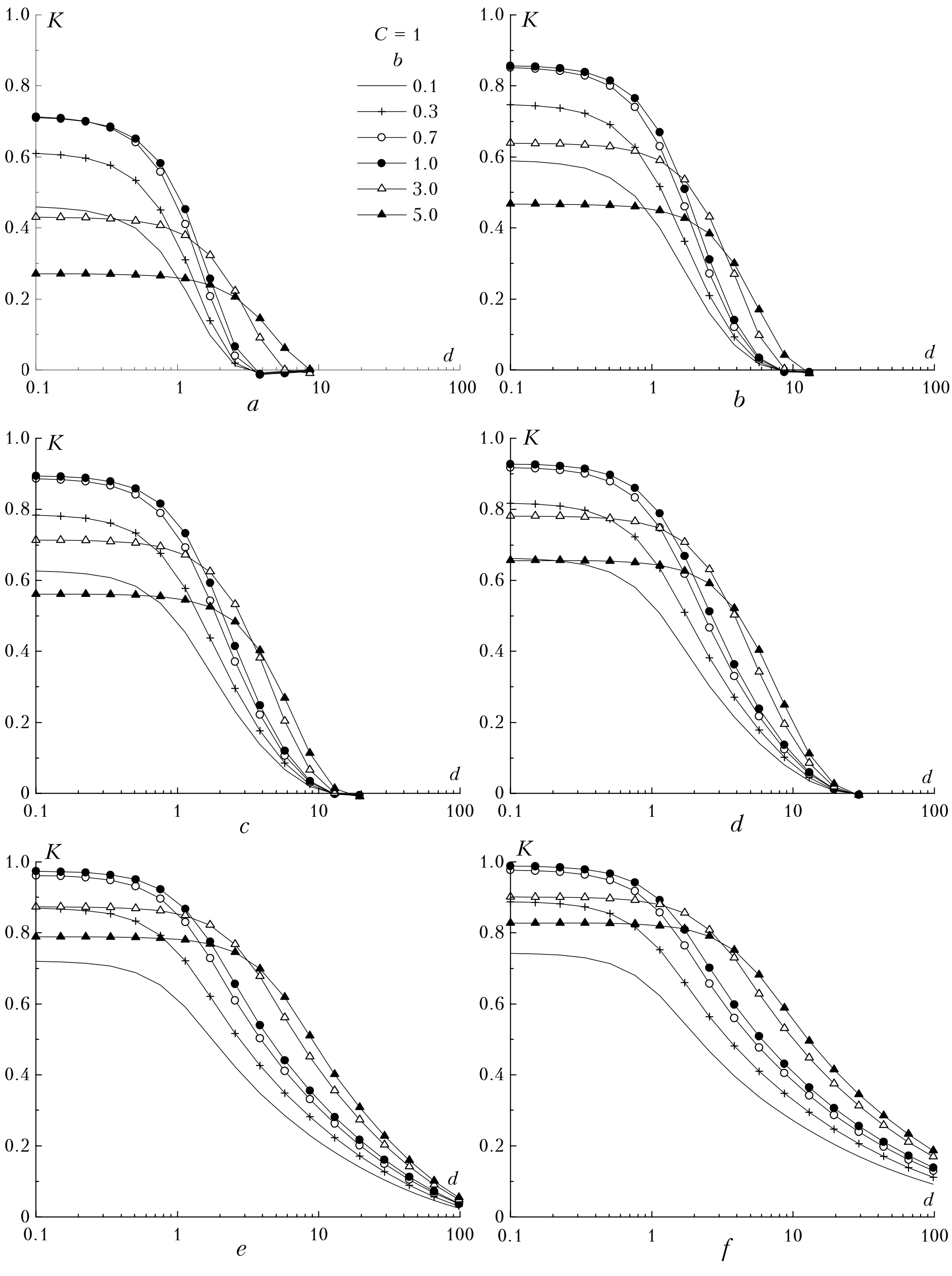

FIG. 3. Correlation index (without sign accounting) of the random angular shifts of the image center of gravity of the normally falling plane wave and angular shifts of the center of gravity of the laser focused beam tilted relatively to the telescope axis for the height of formed laser reference star u = 100 km.

Undoubtedly, the use of the laser reference star formed in the atmosphere on the basis of a backscattered signal is connected with the problem of the choosing the optimal algorithm of using the data of optical measurements for the correction of the random jitter of the star images. We suggest to develop up the algorithm for the correction of the star image jitter in the form (Ref. 20)

$$\varphi_F^{\rm pl} - A\varphi_m, \tag{17}$$

providing by the choice of the coefficient A the minimum variance of the residual distortions

$$\langle\beta^2\rangle = \langle(\varphi_F^{\rm pl} - A\varphi_m)^2\rangle = \langle(\varphi_F^{\rm pl})^2\rangle + $$
$$+ A^2 \langle(\varphi_m)^2\rangle - 2A \langle\varphi_F^{\rm pl}\ \varphi_m\rangle. \tag{18}$$

Calculating the minimum for the variance given by (Eq. 18),

$$\langle\beta^2\rangle_{\min} = \langle(\varphi_F^{\rm pl})^2\rangle - \langle\varphi_F^{\rm pl}\ \varphi_m\rangle^2/\langle(\varphi_m)^2\rangle, \tag{19}$$

we have correction coefficient A is expressed only in terms of deterministic functions in the following way:

$$A = \langle\varphi_F^{\rm pl}\ \varphi_m\rangle/\langle\varphi_m^2\rangle. \tag{20}$$

From the form of this correcting coefficient A, we can conclude that this optimization factor can be obtained directly in optical experiment using the data of direct measurements. The calculations of the correction coefficient for turbulent atmospheric models using Eqs. (20), (9), (12), and (13) is another alternative way.

It should be noted that the conventional correction algorithm (17), when the coefficient $A = -1$, naturally, does not provide the minimum of variance (18). In order to confirm this, let us compare the residual variance for the optimal and nonoptimal correction algorithms.

In the experiment, as a rule, we have only the data of measurements φ_m, because the vector $\varphi_F^{\rm pl}$, characterizing the angular jitter of the natural star, whose image should be corrected, cannot be measured so far, because the star illuminance is insufficient for measurements with a wavefront sensor. In this situation, this optimal coefficient A can be calculated using Eq. (20) for the model vertical profile of the turbulence (Ref. 39).

In our designations the minimum dispersion of the residual fluctuations of the angular shifts of the star images for the scheme shown in Fig. 1 can be estimated using the following formula:

$$\langle\beta^2\rangle_{\min} = \langle(\varphi_F^{\rm pl})^2\rangle \times$$
$$\times\left\{1 - \frac{2^{1/3} f(X, b, d, C_n^2)}{\left[1 + b^{-1/3} - 2^{7/6}(1+b^2)^{-1/6}\ {}_1F_1\left(\frac{1}{6}, 1; -\frac{d^2}{(1+b^2)}\right)\right]}\right\}, \tag{21}$$

where the function

$$f(X, b, d, C_n^2) = \left(\int\limits_0^X d\xi\ C_n^2(\xi)\ (1 - \xi/X) \times\right.$$
$$\times\left\{[1 + (1 - \xi/X)^2]^{-1/6} - \right.$$
$$- [1 + (b^2\ (1 - \xi/X)^2]^{-1/6} \times$$
$$\left.\left.\times\ {}_1F_1\left(\frac{1}{6}, 1; -\frac{d^2(1 - \xi/X)^2}{(1 + b^2\ (1 - \xi/X)^2)}\right)\right\}\right)^2 \times$$
$$\times\left[\int\limits_0^X d\xi\ C_n^2(\xi)\ (1 - \xi/X)^{5/3} \int\limits_0^\infty d\xi\ C_n^2(\xi)\right]^{-1} \tag{22}$$

depends on the parameters of optical experiment and the atmospheric model (these formulas are written under assumption of the infinite outer scale). As numerical analysis of the last formulas shows (Ref. 21), the optimal correction allows to decrease slightly the residual angular distortions in comparison with the conventional scheme.

To demonstrate the effect from the use of the optimal algorithm, we give here only Table I that demonstrates the values of the residual angular distortions for the telescope operating with the bistatic reference star. Therewith it was assumed that the reference star can be considered as a point source. For optimal and nonoptimal correction algorithms, in the third and forth columns of the table the values of the normalized variance of the residual angular distortions $\langle\beta^2\rangle_{\min}/\langle(\varphi_F^{\rm pl})^2\rangle$ are given. The values of the coefficient A calculated for the turbulence model (Ref. 38) are given in the fifth column. It can be vividly seen from the table that the optimal correction using the optimizing coefficient calculated for the turbulent atmospheric model allows to decrease more then twice the value of the residual distortions, whereas the conventional algorithm (i.e., when $A = -1$) in some cases even increases the residual distortions.

Thus, the optimal correction based on the data about the vertical profiles of the turbulence is efficient, whereas the conventional (nonoptimal) correction in some cases may even increase the distortions. However, it should be noted that the obtained levels of the residual distortions are too large to recommend this correction for the experiments.

In this connection, we can conclude that the use of various hybrid schemes may cardinally increase the quality of the total wavefront tilt correction (Refs. 3, 25, and 41).

TABLE 1. Comparison of the algorithms for optimal and nonoptimal correction of the random wavefront shifts for the limiting bistatic scheme of formation of the laser reference star.

X, km	b	Residual level of angular distortions		A
		Optimal algorithm	Nonoptimal algorithm	
8	0.3	0.640	1.291	− 0.427
	0.5	0.603	1.105	− 0.471
	0.7	0.578	0.999	− 0.500
	1	0.552	0.899	− 0.532
	2	0.500	0.736	− 0.593
	3	0.471	0.656	− 0.628
	5	0.434	0.570	− 0.671
20	0.3	0.612	1.354	− 0.420
	0.5	0.572	1.148	− 0.463
	0.7	0.545	1.030	− 0.492
	1	0.516	0.918	− 0.523
	2	0.461	0.736	− 0.583
	3	0.429	0.647	− 0.618
	5	0.330	0.551	− 0.660
40	0.3	0.602	1.406	− 0.413
	0.5	0.561	1.187	− 0.456
	0.7	0.533	1.062	− 0.484
	1	0.504	0.944	− 0.515
	2	0.447	0.751	− 0.574
	3	0.414	0.657	− 0.608
	5	0.374	0.556	− 0.650
80	0.3	0.600	1.446	− 0.407
	0.5	0.558	1.22	− 0.450
	0.7	0.531	1.091	− 0.478
	1	0.501	0.969	− 0.508
	2	0.443	0.769	− 0.567
	3	0.410	0.672	− 0.600
	5	0.370	0.567	− 0.641
100	0.3	0.599	1.455	− 0.406
	0.5	0.588	1.227	− 0.448
	0.7	0.530	1.097	− 0.477
	1	0.501	0.974	− 0.507
	2	0.443	0.774	− 0.565
	3	0.410	0.676	− 0.598
	5	0.370	0.570	− 0.639

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