STATISTICAL CHARACTERISTICS OF BRIGHTNESS OF RADIATION REFLECTED BY THE ATMOSPHERE–GROUND SYSTEM

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A method for calculating the average values and correlation functions of brightness of the radiation reflected by the system ground–atmosphere with an account of the topographic roughness and the occurrence of stochastic cloud field in the atmosphere has been developed.

In order to solve a great number of practical problems it is necessary to know the fluctuation statistical characteristics of brightness of the radiation reflected by the ground and by the atmosphere.[1,2] Many factors are responsible for these fluctuations, namely, the stochastic variations of the optical properties of the atmospheric aerosol[3,4] and of the ground[5,6] in time and space, the occurrence of cumulus clouds in the atmosphere, and the topographic roughness.

The purpose of this paper is to develop methods for calculating the average values and the correlation functions of brightness of the reflected radiation with an account of all these factors.

The stochastic cloud layer of the atmosphere is illuminated by the solar radiation incident in the direction of the unit vector $\mathbf{\Omega}_0$ (Fig. 1). We will describe the optical properties of the atmosphere by the horizontally averaged attenuation index $\langle\varepsilon(z)\rangle$ and of differential index of light scattering at the angle $\gamma\langle\sigma(z;\gamma)\rangle$ and by the correlation functions of these characteristics,

$$R_{\varepsilon\varepsilon}(\mathbf{r}_1;\mathbf{r}_2)=\langle\tilde{\varepsilon}(\mathbf{r}_1)\tilde{\varepsilon}(\mathbf{r}_2)\rangle,\ R_{\varepsilon\sigma}(\mathbf{r}_1;\mathbf{r}_2;\gamma)=\langle\tilde{\varepsilon}(\mathbf{r}_1)\,\tilde{\sigma}(\mathbf{r}_2;\gamma)\rangle,$$

$$R_{\sigma\sigma}(\mathbf{r}_1;\mathbf{r}_2;\gamma_1;\gamma_2)=\langle\tilde{\sigma}(\mathbf{r}_1;\gamma_1)\,\tilde{\sigma}(\mathbf{r}_2;\gamma_2)\rangle.$$

Here and below, the angular brackets denote averaging and the tilde stands for the deviation of random parameter from its average value. We assume that the horizontal scale of the fluctuations of the parameters of scattering of the atmosphere $L_\perp$ satisfies the condition of the local homogeneity,[7,9] namely, $L_\perp > R_\perp$, where $R_\perp$ is the width of the Green's function of the atmosphere. We assume that the lower boundary of the atmosphere is a stochastic quasidiffusely reflecting surface. Let us assume that the stochastic character of the ground is due to two factors, namely, the topographic roughness and the random character of variations of the luminance factor of the ground. The requirement of the quasidiffuse reflection corresponds to an assumption that the luminance factor varies insignificantly within the width of angular brightness distribution of the radiation incident on the ground. For the earth's atmosphere, the width of the angular distribution of the brightness field is $\sim 10^\circ$, so most surfaces, with the exception of specularly reflecting ones, satisfy the requirement of quasidiffusive reflection.[5,6]

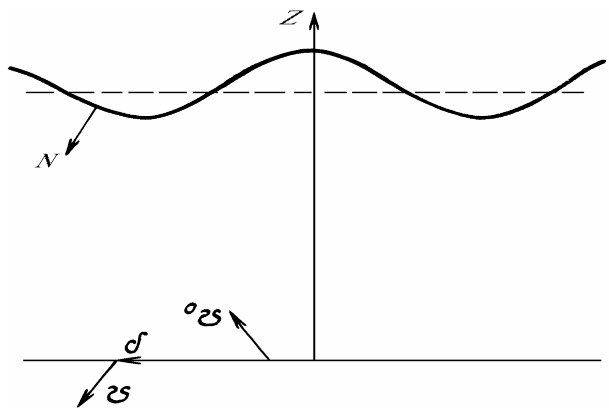

FIG. 1. The geometry of the problem.

The relation for the brightness $B(\boldsymbol{p};\,\Omega)$ of radiation reflected at the point $\boldsymbol{p}$ of the stochastic surface in the direction Ω, was written in Ref. 10:

$$B(\boldsymbol{p};\,\Omega)=\int\limits_{2\pi}\frac{d\Omega'}{\pi}\,\beta(\boldsymbol{p};\,\Omega;\,\Omega')\,|\Omega'\cdot\mathbf{N}(\mathbf{p})|\,B_0(\boldsymbol{p};\,\Omega'),$$

where $\beta(\boldsymbol{p};\,\Omega;\,\Omega')$ is the luminance factor of the ground $B_0(\boldsymbol{p};\,\Omega)$ is the brightness of the incident radiation, and $\mathbf{N}(\boldsymbol{p})$ is the normal to the ground surface at the point $\boldsymbol{p}$ (Fig. 1). In a small–angle approximation, the brightness of the quasidiffusely reflecting surface can be written down in a much simpler form. Since $B_0(\boldsymbol{p};\,\Omega')$ has a peak in the direction Ω_0, we have for the quasi–diffusely reflecting surface $\beta(\boldsymbol{p};\,\Omega;\,\Omega')\simeq\beta(\boldsymbol{p};\,\Omega;\,\Omega_0)$. However, owing to a strong angular anisotropy of $B_0(\boldsymbol{p};\,\Omega')$ we obtain

$$\int\limits_{2\pi}d\Omega'\,|\Omega'\cdot\mathbf{N}(\boldsymbol{p})|\,B_0(\boldsymbol{p};\,\Omega')=$$

$$=\int\limits_{2\pi}d\Omega'\frac{|\Omega'\cdot\mathbf{N}(\boldsymbol{p})|}{\mu'}\mu' B_0(\boldsymbol{p};\,\Omega')\simeq\frac{|\Omega'\cdot\mathbf{N}(\boldsymbol{p})|}{\mu_0}E_0\,,$$

where $E_0=\int\limits_{2\pi}d\Omega'\mu' B_0(\boldsymbol{p};\,\Omega')$ is the solar illuminance at the point $\boldsymbol{p}$ in the case of the plane ground surface; μ' and μ_0

are the direction cosines of the vectors $\boldsymbol{\Omega}'$ and $\boldsymbol{\Omega}_0$ with the z axis. Thus, for the quasi–diffusely reflecting surfaces in the small–angle approximation we have the following formula:

$$B(\boldsymbol{p};\ \boldsymbol{\Omega}) \simeq \frac{\beta(\boldsymbol{p};\ \boldsymbol{\Omega};\ \boldsymbol{\Omega}_0)C(\boldsymbol{p})E(\boldsymbol{p})}{\pi}\ , \tag{1}$$

where

$$C(\boldsymbol{p}) = \frac{|\boldsymbol{\Omega}_0 \cdot \mathbf{N}(\boldsymbol{p})|}{\mu_0}\ . \tag{2}$$

With account of Eq. (1), the brightness of radiation at the top of the atmosphere

$$I(\boldsymbol{p};\ \boldsymbol{\Omega}) = D(\boldsymbol{p};\ \boldsymbol{\Omega}) + \int \mathrm{d}\boldsymbol{p}' f(\boldsymbol{p}';\ \boldsymbol{\Omega}_0;\ \boldsymbol{\Omega})E(\boldsymbol{p}')G_0(\mathbf{r};\ \boldsymbol{p}';\ \boldsymbol{\Omega})\ , \tag{3}$$

where $f(\boldsymbol{p};\ \boldsymbol{\Omega}_0;\ \boldsymbol{\Omega}) = \dfrac{\beta(\boldsymbol{p};\ \boldsymbol{\Omega}_0;\ \boldsymbol{\Omega})C(\boldsymbol{p})}{\pi}$; $G_0(\mathbf{r};\ \boldsymbol{p}';\ \boldsymbol{\Omega})$ is the Green's function of the atmosphere for the diffuse sources, and $D(\boldsymbol{p};\ \boldsymbol{\Omega})$ is the brightness of atmospheric haze.

In the case, in which the condition of local homogeneity in the atmosphere is satisfied, the horizontal scale of variation of the illuminance $E(\boldsymbol{p}')$ is of the order of $L_\perp$ and is much greater than the width of the Green's function $G_0(\mathbf{r};\ \boldsymbol{p}';\ \boldsymbol{\Omega})$. Based on this, Eq. (3) can be simplified as follows:

$$I(\boldsymbol{p};\ \boldsymbol{\Omega}) = D(\boldsymbol{p};\ \boldsymbol{\Omega}) + E_0(\boldsymbol{p})U(\boldsymbol{p}), \tag{4}$$

where

$$U(\boldsymbol{p}) = \int \mathrm{d}\boldsymbol{p}' f(\boldsymbol{p}';\ \boldsymbol{\Omega}_0;\ \boldsymbol{\Omega})G_0(\mathbf{r};\ \boldsymbol{p}';\ \boldsymbol{\Omega}), \tag{5}$$

and

$$E_0(\boldsymbol{p}) = E(\boldsymbol{p} - \mathbf{b}z_s), \tag{6}$$

where $\mathbf{b} = \boldsymbol{\Omega}_\perp / |\mu|$, $\boldsymbol{\Omega}_\perp$ is the projection of $\boldsymbol{\Omega}$ onto the top of the atmosphere, and μ is the direction cosine of the vector $\boldsymbol{\Omega}$ with the z axis.

From Eq. (4), it is easy to derive the relations for the average brightness of the reflected radiation $\langle I\rangle$ and for the second moment of this quantity

$M_{ii}(\boldsymbol{p}_1;\ \boldsymbol{p}_2) = \langle I(\boldsymbol{p}_1;\ \boldsymbol{\Omega})I(\boldsymbol{p}_2;\ \boldsymbol{\Omega})\rangle$:

$$\langle I(\boldsymbol{p})\rangle = \langle D(\boldsymbol{p})\rangle + M_{eu}(\boldsymbol{p};\ \boldsymbol{p}),$$

$$M_{ii}(\boldsymbol{p}_1;\ \boldsymbol{p}_2) = M_{dd}(\boldsymbol{p}_1;\ \boldsymbol{p}_2) + M_{deu}(\boldsymbol{p}_1;\ \boldsymbol{p}_2) +$$

$$+ M_{deu}(\boldsymbol{p}_2;\ \boldsymbol{p}_1) + M_{eueu}(\boldsymbol{p}_1;\ \boldsymbol{p}_2), \tag{7}$$

where

$M_{eu}(\boldsymbol{p}_1;\ \boldsymbol{p}_2) = \langle E_0(\boldsymbol{p}_1)U(\boldsymbol{p}_2)\rangle$,
$M_{deu}(\boldsymbol{p}_1;\ \boldsymbol{p}_2) = \langle D(\boldsymbol{p}_1)\,E_0(\boldsymbol{p}_2)\,U(\boldsymbol{p}_2)\rangle$,
and $M_{eueu}(\boldsymbol{p}_1;\ \boldsymbol{p}_2) = \langle E_0(\boldsymbol{p}_1)U(\boldsymbol{p}_1)E_0(\boldsymbol{p}_2)U(\boldsymbol{p}_2)\rangle$ are the moments of the second, third, and fourth order of the functions $E_0(\boldsymbol{p})$, $U(\boldsymbol{p})$, and $D(\boldsymbol{p})$. The relation for $E_0(\boldsymbol{p})$ and $D(\boldsymbol{p})$ have been obtained in Refs. 7 and 8 and have the following form:

$$E_0(\boldsymbol{p}) = E_0 \exp\left[-\int\limits_0^{z_s} k^*(\mathbf{r}_u)\frac{\mathrm{d}u}{\mu_0}\right],$$

$$D(\boldsymbol{p}) = \int\limits_0^{z_s} \frac{\mathrm{d}z_1}{|\mu|}\sigma(\boldsymbol{p} - \mathbf{b}z_1;\ z_1;\ \boldsymbol{\Omega};\ \boldsymbol{\Omega}_0) \times$$

$$\times E(\boldsymbol{p} - \mathbf{b}z_1;\ z_1)T(\boldsymbol{p};\ \boldsymbol{\Omega};\ z_1),$$

where E_0 is the illuminance at the top of the atmosphere, z_s is the thickness of the atmospheric layer, $\mathbf{r}_u = \{\boldsymbol{p} - \boldsymbol{\alpha}(z_s - u);\ u\}$, $\boldsymbol{\alpha} = \boldsymbol{\Omega}_{\perp 0}/\mu_0$, $\boldsymbol{\Omega}_{\perp 0}$ is the projection of $\boldsymbol{\Omega}_0$ onto the top of the atmosphere, $\sigma(\boldsymbol{p};\ z;\ \boldsymbol{\Omega}\cdot\boldsymbol{\Omega}_0)$ is the differential index of light scattering at the point $\mathbf{r} = \{\boldsymbol{p};\ z\}$ of the medium, which describes the scattering of radiation propagating in the direction $\boldsymbol{\Omega}_0$ in the direction $\boldsymbol{\Omega}$, $E(\boldsymbol{p};\ z)$ is the illuminance of the medium at the point $\{\boldsymbol{p};\ z\}$, $T(\boldsymbol{p};\ \boldsymbol{\Omega};\ z_1)$ is the stochastic transmittance of the medium, which determines the brightness of radiation at the point $\{\boldsymbol{p};\ z\}$ in the direction $\boldsymbol{\Omega}$ produced by the diffuse spatially unbounded source located in the plane $z = z_1$, and $k^*(\mathbf{r})$ is the effective absorption index of the medium.[8] Let us perform the further analysis for the Gaussian model of stochastic atmosphere presented in Refs. 7–9 which can be employed for the description of the propagation of radiation through the cloudless atmosphere, fog, and cirrus clouds when the vertical scales of inhomogeneities $L_{||} < z_s$. In this case, based on the central limiting theorem,[11] it is possible to state that the values $U(\boldsymbol{p})$ and $D(\boldsymbol{p})$ are approximately Gaussian ones. For the Gaussian model, it is not difficult to show that the fluctuations of the quantity $E_0(\boldsymbol{p})$ are described, strictly speaking, by a lognormal distribution function. However, as the estimates show, for the optical thicknesses of the atmosphere $\tau \le 5$, the value $\tau_k^* = \int\limits_0^{z_s} k^*(\mathbf{r}_u)\mathrm{d}u$, which determines the values of illuminance, is much smaller than the unity. In this case, the lognormal distribution is close to the normal one and so $E_0(\boldsymbol{p})$ can be considered to be a normally random quantity. For the normal values $E_0(\boldsymbol{p})$, $U(\boldsymbol{p})$, and $D(\boldsymbol{p})$ the moments of the third and fourth orders are easy expressed in terms of the moments of the second order.[11] As a result, in the case of horizontally homogeneous fluctuations of the optical parameters of the atmosphere and of the reflecting surface, we obtain

$$M_{deu}(\Delta) = \langle E_0\rangle M_{du}(\Delta) +$$

$$+ \langle U\rangle M_{de}(\Delta) + \langle D\rangle M_{eu}(0) - 2\langle D\rangle\langle E_0\rangle\langle U\rangle, \tag{8}$$

$$M_{eueu}(\Delta) = M_{eu}(0)M_{eu}(0) + M_{ee}(\Delta)M_{uu}(\Delta) +$$

$$+ M_{eu}(\Delta)M_{eu}(-\Delta) - 2\langle U\rangle^2\langle E_0\rangle^2, \tag{9}$$

where $M_{de}(\Delta) = \langle D(\boldsymbol{p}_1)E_0(\boldsymbol{p}_2)\rangle$, $M_{ee}(\Delta) = \langle E_0(\boldsymbol{p}_1)E_0(\boldsymbol{p}_2)\rangle$; $M_{du}(\Delta) = \langle D(\boldsymbol{p}_1)U(\boldsymbol{p}_2)\rangle$, $\Delta = \boldsymbol{p}_1 - \boldsymbol{p}_2$.

It is obvious that the random function $f(\cdot)$, which enters into Eq. (3), is uncorrelated with the other random functions $D(\boldsymbol{p})$, $E_0(\boldsymbol{p})$, and $G_0(\mathbf{r};\ \boldsymbol{p}';\ \boldsymbol{\Omega})$. With an account of this, it is not difficult to verify that

$$<U> = <f><T(\boldsymbol{p};\ \Omega)>,\ M_{du}(\Delta) = <f>M_{dt}(\Delta),$$

$$M_{eu}(\Delta) = <f>M_{et}(\Delta), \tag{10}$$

where $T(\boldsymbol{p};\ \Omega) = \int \mathrm{d}\boldsymbol{p}' G_0(\mathbf{r};\ \boldsymbol{p}';\ \Omega)$ is the diffuse transmittance of the atmosphere, $M_{dt}(\Delta) = <D(\boldsymbol{p}_1)T(\boldsymbol{p}_2;\ \Omega)>$, and $M_{et}(\Delta) = <E_0(\boldsymbol{p}_1)T(\boldsymbol{p}_2;\ \Omega)>$. It is also simple enough to find the moment $M_{uu}(\boldsymbol{p}_1;\ \boldsymbol{p}_2)$. It follows from the definition of the random function $U(\boldsymbol{p})$ that

$$M_{uu}(\boldsymbol{p}_1;\ \boldsymbol{p}_2) =$$

$$\int\int \mathrm{d}\boldsymbol{p}'\mathrm{d}\boldsymbol{p}'' M_{ff}(\boldsymbol{p}';\ \boldsymbol{p}'')M_{gg}(\boldsymbol{p}';\ \boldsymbol{p}'';\ \boldsymbol{p}_1;\ \boldsymbol{p}_2), \tag{11}$$

where $M_{gg}(\boldsymbol{p}';\ \boldsymbol{p}'';\ \boldsymbol{p}_1;\ \boldsymbol{p}_2) = <G_0(\boldsymbol{p}_1;\ \boldsymbol{p}';\ z)G_0(\boldsymbol{p}_2;\ \boldsymbol{p}'';\ z)>$, and $M_{ff}(\boldsymbol{p}';\ \boldsymbol{p}'') = <f(\boldsymbol{p}')f(\boldsymbol{p}'')>$ are the second moments of the Green's functions and of the function $f(\boldsymbol{p})$, which enters into Eq. (3). When the conditions of local homogeneity are satisfied and the fluctuations of the scattering parameters of the atmosphere and of the reflecting properties of the ground are horizontally homogeneous,

$$M_{uu}(\Delta) = \int \frac{\mathrm{d}\omega}{4\pi^2} \hat{M}_{ff}(\omega)M_{tt}(\omega;\ \Delta)\exp(i\omega\cdot\Delta)\ , \tag{12}$$

where $\hat{M}_{ff}(\omega)$ is the Fourier spectrum of the function $M_{ff}(\Delta)$ and $M_{ff}(\omega;\ \Delta) = <T(\omega;\ \boldsymbol{p}_1)T(\omega;\ \boldsymbol{p}_2)>$ is the second moment of the local unnormalized optical transfer function (OTF) of the cloudy medium $T(\omega;\ \boldsymbol{p})$ (see Ref. 9).

The analysis performed shows that, in order to calculate the statistical characteristics of the radiation brightness, it is necessary to know the first moments, namely, $<f>$, $<E_0>$, $<D>$, and $<T(\boldsymbol{p};\ \Omega)>$ as well as the second moments, namely, $M_{dt}(\Delta)$, $M_{et}(\Delta)$, $M_{ee}(\Delta)$, $M_{dd}(\Delta)$, $M_{de}(\Delta)$, $M_{gg}(\boldsymbol{p}';\ \boldsymbol{p}'';\ \Delta)$, and $M_{ff}(\Delta)$. A technique for calculating the moments of the first order $<E_0>$, $<D>$, and $<T(\boldsymbol{p};\ \Omega)>$ and of a number of the moments of the second order, for instance, $M_{ee}(\Delta)$ and $M_{dd}(\Delta)$, has been described in Refs. 7–9. The rest of the moments, which are required for the calculation, can be found in a similar way. Whereas the final relations have a cumbersome form we will not give them here and restrict ourselves by consideration of a more particular case, in which the data already given here and those published in the literature are sufficient for calculating the statistical characteristics.

The results of calculations of the light fields in the atmosphere[7–9,12] show that the fluctuations of the illuminance, diffuse transmittance, and brightness of atmospheric haze are determined by the magnitude of fluctuations of the effective optical thickness of the absorption of the atmosphere $\tau_k^* = \int_0^{z_s} k^*(\mathbf{r})\mathrm{d}z$, where $k^* = k + \sigma\Phi$ is the effective absorption index, k and σ are the absorption and scattering indices of the cloudy medium, and Φ is the relative fraction of light that is scattered backwards in a single scattering act. At the same time, the fluctuations of the local OTF are determined by the fluctuations of the optical thickness of the atmosphere $\tau = \int_0^{z_s} \varepsilon(\mathbf{r})\mathrm{d}z$. For the cloudy medium in the visible range the value of is smaller than τ by a factor of ~ 20. Therefore, in the case of occurrence of the semi–transparent clouds in the atmosphere, for which $\tau_k^* \ll 1$, it is possible to ignore the fluctuations of brightness of the atmospheric haze, illuminance, and diffuse transmittance compared to the fluctuations of the local OTF. In this case, replacing the random quantities E, $T(\boldsymbol{p};\ \Omega)$, and D by their average values we obtain

$$<I> = <D> + <f><E_0><T(\boldsymbol{p};\ \Omega)>, \tag{13}$$

$$M_{ii}(D) = <D>^2 + 2<E_0><D><U> + <E_0>^2M_{uu}(\Delta)\ . \tag{14}$$

A simple relation for the correlation function of brightness of the reflected radiation $R_{ii}(\Delta) = M_{ii}(\Delta) - <I>^2$ follows from this

$$R_{ii}(\Delta) = <E_0>^2R_{uu}(\Delta), \tag{15}$$

where $R_{uu}(\Delta) = M_{uu}(\Delta) - <u>^2$ is the correlation function of $U\,(\boldsymbol{p})$.

We have given here the relation for horizontally homogeneous fluctuations of the parameters of the atmosphere and underlying surface. Its generalization to the case of horizontally inhomogeneous fluctuations is obvious and is not met with any difficulties.

For $\tau_k^* \ll 1$, it is simple enough to find the variance of fluctuations of the brightness σ_i^2 within the scope of a Poisson model of the stochastic atmosphere as well.[12] It then follows from Eq. (4) that

$$\sigma_i^2 = <E_0>^2\sigma_u^2\ , \tag{16}$$

where σ_u^2 is the variance of the random quantity $U(\rho)$. It is obvious that

$$\sigma_u^2 = \int \frac{\mathrm{d}\omega}{4\pi^2} \hat{M}_{ff}(\omega)\hat{M}_{tt}(\omega;\ 0) - <U>^2. \tag{17}$$

The functions $<f>$ and $M_{ff}(\Delta)$, which describe the statistical properties of the reflecting surface, can be found based on Eq. (1). It is obvious that in this case the random functions $\mathrm{b}(\cdot)$ and $C(\cdot)$, which describe two statistically independent processes, namely, the variation of the luminance factor as functions of the position at the ground and the local topography are not correlated, therefore,

$$<f> = \frac{<\beta><C>}{\pi},\ \text{and}\ M_{ff}(D) = \frac{M_{\beta\beta}(\Delta)\ M_{cc}(D)}{\pi^2}\ ,$$

where $M_{\beta\beta}(\Delta)$ and $M_{cc}(\Delta)$ are the second moments of the luminance factor of the ground and of the function $C(\boldsymbol{p})$, which describes the ground profile.

In the scope of a Poisson model of clouds, the second moment of the unnormalized local OTF $\hat{M}_{tt}(\omega,0)$ can be found following the technique of Ref. 12.

$$\hat{M}_{tt}(\omega;\,0) =$$

$$= \exp\left\{\frac{\bar{M}}{z}\int_0^z \mathrm{d}t\left[\chi_\varepsilon\left(2i\int_0^z \eta_0(t-t')F_\omega(t')\mathrm{d}t'\right)-1\right]\right\}, \qquad (18)$$

where $\bar{M}$ is the mean number of individual clouds on a segment of length z, $\chi_\varepsilon(\cdot)$ is the characteristic function that describes the fluctuations of the attenuation index of the cloud ε, $\eta_0(x)$, which is equal to unity for $x < \Delta_0$ and to zero for $x > \Delta_0$, is the indicator function, Δ_0 is the cloud size along the z axis,

$$F_\omega(t) = 1 - \Lambda^* Q(\omega t),$$

$$Q(\omega t) = \frac{1}{2}\int_0^\infty \gamma i(\gamma) J_0(\omega t\gamma)\mathrm{d}\gamma;\ \Lambda^* = \Lambda(1-\Phi)\ ;$$

Λ is the survival probability of the light quantum in a single scattering act, and $i(\gamma)$ is the scattering phase function of the medium. For example, for the scattering phase function, which is described by the small–angle Henyey–Greenstein approximation[12] $i(\gamma) = 2\alpha(\alpha^2+\gamma^2)^{-3/2}$, where $\alpha = (1-\bar{\mu})\bar{\mu}^{-1/2}$, $\bar{\mu}$ is the mean cosine of the scattering angle, and

$$Q(\omega t) = \exp(-\alpha p)\ . \qquad (19)$$

As an example, let us consider a cloudy medium, which has the exponential distribution of the scattering index of the clouds. The characteristic function $\chi_\varepsilon(v)$ for the model has the following form:

$$\chi_\varepsilon(v) = \frac{1}{1 - i\varepsilon_0 v}\ , \qquad (20)$$

where $i = \sqrt{-1}$ and ε_0 is the mean attenuation index of the clouds. For the model of the scattering medium described by relations (19) and (20) and for the spatial frequencies satisfying the condition $\alpha\omega z \ll 1$, it is possible to obtain a simple relation for the moment $\hat{M}_{tt}(\omega;\,0)$

$$\hat{M}_{tt}(\omega;\,0) =$$

$$= \exp\left\{\frac{\bar{M}b_2}{1+b_2}\left[1 - \frac{1}{b_2 g}\ln\left[1 + \frac{b_2\Lambda^*}{S_2}(1-e^{-g})\right]\right]\right\}, \qquad (21)$$

where $b_n = n\varepsilon_0\Delta$, $S_n = 1 + b_n(1 + b_n(1-\Lambda^*)$, and $g = a\omega z$. The range of applicability of formula (21) used to calculate the fluctuations of brightness can be easily found from relation (17). Since the width of the Fourier spectrum $\hat{M}_{ff}(\omega)$ is of the order of the scale of the fluctuations of the reflecting properties of the surface δ_f, it is obvious that formula (21) can be employed for analyzing the brightness fluctuations of the radiation reflected by the ground with large–scale inhomogeneities of the reflecting properties satisfying the condition $p^* = \dfrac{\alpha z}{\delta_f} \le 1$.

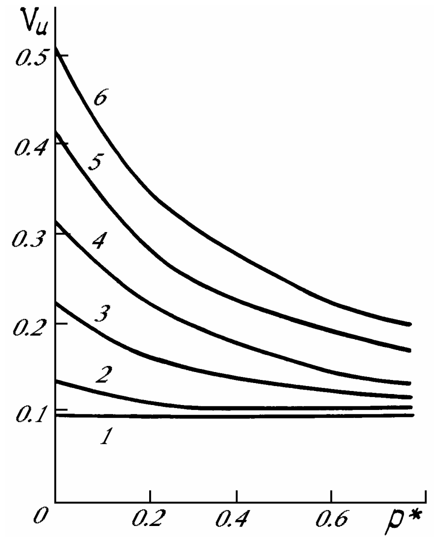

FIG. 2. The relative fluctuations of brightness of the radiation reflected by the system atmosphere–ground calculated for $b_1 = 5$, $b_1 = 0.5$, *and* $V_f = 0$ (*1*), 0.1 (*2*), 0.3 (*3*), 0.4 (*4*), *and* 0.5 (*5*).

For model (20), some simple relations for the first two moments of the diffuse transmittance can be written down as well

$$\langle T\rangle = \exp\left[-\frac{\bar{M}b_1}{1+b_1}\left(1 - \frac{\Lambda^*}{S_1}\right)\right]; \qquad (22)$$

and

$$\langle T^2\rangle = \exp\left[-\frac{\bar{M}b_2}{1+b_2}\left(1 - \frac{\Lambda^*}{S_2}\right)\right]. \qquad (23)$$

The methods of determining the function $M_{ff}(\Delta)$ depending on the statistical characteristics of the local topography and the structure of the landscape are described in the monograph *Optical Image for Remote Observation* indicated above. In this paper, we will consider a simple model of the ground, which admits an analytical solution of the problem and permits one to consider qualitatively the main characteristic features of formation of the radiation field

$$M_{ff}(\Delta) = \frac{\sigma_f^2\sigma_f^3}{(\Delta^2+\delta_f^2)^{3/2}}\ , \qquad (24)$$

where   is the variance of $f(p)$. In this case, based on Eqs. (17), (21), and (24) for the relative fluctuations of the brightness V_u, we obtain

$$V_u = \left[V_f^2\frac{\langle T^2\rangle}{\langle T\rangle^2}\frac{1}{(1+\Psi p^*)^2} + V_t^2\right]^{1/2}, \qquad (25)$$

where V_f are the relative fluctuations of the quantity $f(p)$,

$$V_t = \left[\frac{\langle T^2\rangle - \langle T\rangle^2}{\langle T\rangle^2}\right]^{1/2}$$

are the relative fluctuations of the diffuse transmittance of the cloud layer, and

$$\Psi = \frac{\bar{M}b_2}{1 + b_2} \frac{\Lambda^*}{2S_2} .$$

The dependence of the relative fluctuations of brightness of the reflected radiation on the dimensionless parameter p^* is illustrated in Fig. 2. It is evident from the figure that the values of V_u monotonically decrease as the values of p^* increase. This means that the magnitude of the relative fluctuations of brightness decreases as the angular scale of fluctuations of the ground and the degree of elongation of the scattering phase function decrease. With increase of V_f , the quantity V_u monotonically increases. Physical interpretation of these dependences is clear enough and is associated with the scattering of the light reflected from the ground by the cloudy medium. It is obvious that, when the angular scale of the fluctuations of the inhomogeneities of the ground and the elongation of the scattering phase function decreases the "smearing" of the image of the surface elements become more pronounced, while the variance of the brightness decreases.

REFERENCES

1. Yu.G. Yakushenkov, V.N. Lukantsev, and M.P. Kolosov, *Methods of Noise Control in Opto–Electronic Devices* (Radio i Svyaz', Moscow, 1981), 180 pp.
2. M.M. Miroshnikov, *Theoretical Principles of Opto–Electronic Devices* (Mashinostroenie, Leningrad, 1983), 696 pp.
3. G.M. Krekov, S.I. Kavkyanov, and M.M. Krekova, *Interpretation of Lidar Returns in Optical Sounding of the Atmosphere* (Nauka, Novosibirsk, 1987), 184 pp.
4. M.S. Malkevich, *Optical Sensing of the Atmosphere from Satellites* (Nauka, Moscow, 1973), 303 pp.
5. P.P. Fedchenko and K.Ya. Kondrat'ev, *Spectral Reflectivity of Some Soils* (Gidrometeoizdat, Leningrad, 1981), 231 pp.
6. Yu.S. Tolchel'nikov, *Optical Properties of Landscape as Applied to Air Survey* (Nauka, Leningrad, 1974), 252 pp.
7. A.N. Valentyuk, in: *Transfer of Images through the Earth's Atmosphere,* Tomsk Affiliate of the Siberian Branch of the Academy of Sciences of the USSR, Tomsk (1988), pp. 11–18.
8. A.N. Valentyuk, Izv. Vyssh. Uchebn. Zaved. , Ser. Radiofiz. **31,** No. 7, 881–883 (1988).
9. A.N. Valentyuk, Issled. Zemli iz Kosmosa, No. 3, 91–99 (1987).
10. Yu.–A.R. Mullamaa, M.A. Sulev, V.K. Tyldmaa, et al., *Stochastic Structure of Cloud and Radiation Fields,* Tartu, (1972), 281 pp.
11. S.M.Rytov, *Introduction to Statistical Radiophysics* (Nauka, Moscow, 1976), Pt. 1, 494 pp.
12. A.N. Valentyuk, Atm. Opt. **2**, No. 1, 80–82 (1989).