

# Specific features of photophoretic motion of moderately large spherical aerosol particles

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We use Stokes approximation to describe theoretically the photophoretic motion of a solid moderately large aerosol particle of spherical shape at small relative drops of temperature in its neighborhood. By solving equations of gas dynamics, analytical expressions have been obtained for the photophoretic force and rate of photophoresis, taking into account the effect of medium motion.

## Introduction

The phenomenon of photophoresis in gas consists in the motion of aerosol particles in the field of electromagnetic radiation under the impact of radiometric force. Photophoresis can play a significant role in atmospheric processes,<sup>1–3</sup> when applied to purification of industrial gases, creation of installations designed for selective size segregation of particles, etc.

The mechanism of photophoresis can be briefly described as follows. Interaction of electromagnetic radiation with a particle yields a release of thermal energy with a certain volume density  $q_i$ , which nonuniformly heats the particle. Gas molecules, surrounding the particle, collide with the particle surface and upon reflection from warmer side of the particle they move faster than after reflection from its colder side. As a result, the particle acquires non-compensated impulse directed from warm to cold particle side. Depending on the size and optical properties of the particulate matter, any particle side, either illuminated or shadowed one, may get warmer. Therefore, there may be both positive (particle motion along the direction of incident light propagation), and negative photophoresis. Moreover, if the radiative flux is nonuniform over cross section, there may appear particle motion in gas, perpendicular to the direction of propagation of incident electromagnetic radiation.<sup>4</sup>

Other authors, Oseen<sup>5</sup> and Praudman and Pearson<sup>6</sup> for hydrodynamical problem, and Acrivos and Taylor<sup>7</sup> for thermal problem, have shown that, far away from the sphere, the inertial and convective terms are of the same order of magnitude as terms of molecular transfer. Therefore, usual method of series expansion over small parameter gives the known error, because already in the second approximation it does not provide satisfactory fulfillment of the boundary conditions at infinity and, hence, an exact unique solution, uniformly valid in the entire flow region.

So far, the published works in the theory of photophoretic motion for small relative temperature drops<sup>8–15</sup> ignored the influence of convective terms of heat conduction (motion of the medium) on photophoresis. In this paper, the method of matched

asymptotic expansions is used to estimate this influence.

## 1. Statement of the problem

Consider a solid, moderately large, spherical aerosol particle with the radius  $R$  suspended in a gas with temperature  $T_\infty$ , density  $\rho_e$ , and viscosity  $\mu_e$ . To classify aerosol particles by their sizes, Knudsen criterion  $\text{Kn} = \lambda/R$  is used, where  $\lambda$  is average free path of molecules of the gas mixture. Particles are called large if  $\text{Kn} \leq 0.01$ , moderately large if  $0.01 \leq \text{Kn} \leq 0.3$ , and small when  $\text{Kn} \gg 1$ . Let an electromagnetic radiation be incident on the particle and nonuniformly heat it. Gas interacting with nonuniformly heated surface begins to move along the surface along the direction of temperature increase. This phenomenon is called thermal gas sliding. The thermal sliding brings about the appearance of photophoretic force and viscous drag force of the medium, and particle begins to move uniformly. The velocity of uniform particle motion is called the photophoretic velocity ( $U_{\text{ph}}$ ).

In a theoretical description of the process of photophoretic particle motion, we will assume that, in virtue of smallness of thermal and diffusion relaxation times, the process of heat transfer in the particle – gas medium system is stationary. Particle motion takes place for small Peclet and Reynolds numbers and small relative temperature drops in the particle neighborhood, i.e.,  $(T_e - T_\infty)/T_\infty \ll 1$ . When this condition is satisfied, the coefficients of heat conductivity and dynamic and kinetic viscosity can be considered constant. The problem is solved by hydrodynamic method, i.e., the equations of hydrodynamics are solved with the corresponding boundary conditions, and the particle is considered to have uniform composition.

The particle motion is conveniently described in spherical coordinate system  $r, \theta, \varphi$ , associated with the center of mass of aerosol particle. The  $OZ$ -axis is oriented along the direction of propagation of uniform radiative flux with the intensity  $I_0$ . In this

case, the volume density of internal heat sources has the standard form

$$q_i(r) = 2\pi k k_0 I_0 B(\mathbf{r}), \tag{1}$$

where

$$B(r, \theta, \varphi) = \frac{1}{2\pi} \int_0^{2\pi} \frac{|E(r, \theta, \varphi)|^2}{E_0^2} dy = B\left(r, \theta, \frac{\pi}{4}\right)$$

is dimensionless source function of electromagnetic energy in the case of nonpolarized incident radiation;  $E(r, \theta, \varphi)$  is the local directionality of electrical field inside the particle;  $E_0$  is the electric field strength of the incident wave;  $k_0 = 2\pi/\lambda_0$  is the wave number;  $\lambda_0$  is the wavelength;  $m = n + ik$  is the complex refractive index of the particulate matter. Usually, to calculate the dimensionless source function  $B(\mathbf{r})$ , Mie solution for the internal field is used (see, e.g., Ref. 16). Since the coordinate system is associated with the center of moving aerosol particle, all we have to do is to analyze streamlining of particle by an infinite plane-parallel flow; for which it is assumed that the gas velocity at infinity equals the photophoresis rate, taken with the opposite sign, i.e.,  $U_\infty = -U_{ph}$ .

Under the above assumptions, the equations of hydrodynamics and heat conductivity and boundary conditions take the following form

$$\mu_e \Delta \mathbf{U}_e = \nabla P_e, \quad \text{div } \mathbf{U}_e = 0; \tag{2}$$

$$\rho_e c_{pe} (\mathbf{U}_e \nabla) T_e = \lambda_e \Delta T_e, \quad \Delta T_i = -q_i/\lambda_i; \tag{3}$$

$$r = R; \quad T_e - T_i = K_T^T \frac{\partial T_e}{\partial r};$$

$$-\lambda_e \frac{\partial T_e}{\partial r} + \lambda_i \frac{\partial T_i}{\partial r} = -C_q^T \text{Kn} \frac{\lambda_e}{R \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial T_e}{\partial \theta} \right);$$

$$U_r^e = C_V^T \text{Kn} \frac{v_e}{RT_e \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial T_e}{\partial \theta} \right),$$

$$U_\theta^e = C_m^* \left[ r \frac{\partial}{\partial r} \left( \frac{U_\theta^e}{r} \right) - \frac{1}{r} \frac{\partial U_r^e}{\partial \theta} \right] +$$

$$+ K_{TS} \left[ 1 + \text{Kn} (\beta_{RT}^* + \sigma_T \beta_{RT}) \right] \frac{\partial T_e}{\partial \theta} -$$

$$- K_{TS} \beta_{RT}^B \frac{v_e}{2T_e} \left[ r \frac{\partial}{\partial r} \left( \frac{1}{r^2} \frac{\partial T_e}{\partial \theta} \right) + \frac{1}{r} \frac{\partial^2 T_e}{\partial r \partial \theta} \right], \tag{4}$$

$$r \rightarrow \infty; \quad U_r^e = U_\infty \cos \theta, \quad U_\theta^e = -U_\infty \sin \theta; \tag{5}$$

$$P_e = P_\infty; \quad T_e = T_\infty,$$

$$r \rightarrow 0, \quad T_i \neq \infty. \tag{6}$$

Here,  $U_r^e$  and  $U_\theta^e$  are the components of gas mass velocity  $U_e$ ;  $c_{pe}$  is the heat capacity under constant

pressure;  $C_m^* = C_m \lambda$ ;  $K_T^T = C_T \lambda$ ;  $K_{TS}$  and  $C_m$  are the coefficients of thermal and isothermal sliding;

$$\beta_{RT}^{*B} = \beta_{RT}^B \lambda; \quad \sigma_T = \left( \frac{\partial^2 T_e}{\partial r \partial \theta} \right) \left( \frac{1}{R} \frac{\partial T_e}{\partial \theta} \right)^{-1};$$

$\lambda_e$  and  $\lambda_i$  are the heat conductivity coefficients of gas and particle;  $\mu_e$  and  $\nu_e$  are the coefficients of dynamical and kinematic gas viscosity; and from here on the subscripts e and i stand for gas and particle, respectively, while subscript  $\infty$  is for values of physical quantities characterizing the external medium in the unperturbed flow.

In formulation of boundary conditions for moderately large particles, the entire volume occupied by the gas is conventionally divided into two parts: (1) Knudsen layer, representing the portion of gas with the thickness of the order of free path in the immediate vicinity of a particle surface; and (2) the rest gas. Flow in the gas volume outside the kinetic layer is described by usual hydrodynamical equations; while in the Knudsen layer boundary conditions for hydrodynamic equations are specified. To describe motion of gas in this layer, it is necessary to solve the kinetic equation.<sup>17,18</sup> At the same time, it is required to take into account all corrections to velocity of aerosol particle, linear relative to Knudsen number. Boundary conditions (4) on the surface of aerosol particle assume that: (1) temperature jump is present on particle surface, proportional to the coefficient  $C_T$ ; and (2) for the heat flux and radial velocity component  $U_r^e$  there exists a discontinuity of radial heat flux due to spread of the part of the flux over the Knudsen layer, proportional, respectively, to coefficients  $C_q^T$  and  $C_V^T$ .

The latter boundary condition suggests that there are four effects contributing to the velocity, with which gas slides along spherical surface of small curvature. First effect is caused by nonuniformity of gas temperature distribution over the spherical surface (thermal sliding). The associated characteristics are the coefficient  $K_{TS}$  of thermal sliding over the plane surface and coefficient  $\beta_{RT}^*$  of  $K_{TS}$  correction, introduced to take into account the presence of curved surface. The second effect constitutes isothermal sliding of gas and is characterized by the coefficient  $C_m$ . Third effect results from sliding of gas due to nonuniformity of temperature gradient in the Knudsen layer caused by the presence of curved surface. The associated characteristic of this effect is  $\beta_{RT}$  coefficient. Fourth effect accounts for the Barnett gas sliding and is proportional to the coefficient  $\beta_{RT}^B$ . Expressions for the coefficients  $K_{TS}$ ,  $C_m$ ,  $C_T$ ,  $C_q^T$ ,  $C_V^T$ ,  $\beta_{RT}^*$ ,  $\beta_{RT}$ , and  $\beta_{RT}^B$  are derived by the methods of kinetic theory of gases and can be found in Refs. 17 and 18.

At large distance from a particle ( $r \rightarrow \infty$ ), boundary conditions (5) are valid, while the finiteness

of physical quantities characterizing the particle at  $r \rightarrow 0$  is accounted for in expression (6).

Let us transform equations (2) and (3) and the boundary conditions (4)–(6) to a dimensionless form, by introducing dimensionless coordinate, velocity, and temperature as follows:  $y_k = x_k/R$ ,  $t = T/T_\infty$ ,  $\mathbf{V}_e = \mathbf{U}_e/U_\infty$ .

For  $Re = (\rho_e U_\infty R)/\mu_e \ll 1$ , the incoming flow produces only disturbing effect; therefore, the solution of hydrodynamic equations should be sought in the form

$$\mathbf{V}_e = \mathbf{V}_e^{(0)} + \varepsilon \mathbf{V}_e^{(1)} + \dots, \tag{7}$$

$$\mathbf{P}_e = \mathbf{P}_e^{(0)} + \varepsilon \mathbf{P}_e^{(1)} + \dots (\varepsilon = Re).$$

The solution of equation, describing temperature distribution outside the particle, will be determined by the method of matched asymptotic expansions.<sup>19,20</sup> The inner and outer asymptotic expansions of dimensionless temperature are represented as

$$t_e(y, \theta) = \sum_{n=0}^{\infty} f_n(\varepsilon) t_{en}(y, \theta), \quad (f_0(\varepsilon) = 1), \tag{8}$$

$$t_e^*(\xi, \theta) = \sum_{n=0}^{\infty} f_n^*(\varepsilon) t_{en}^*(\xi, \theta),$$

where  $\xi = \varepsilon y$  is the “compressed” radial coordinate<sup>19</sup> ( $y = x/R$ ).

Moreover, it is required that

$$\frac{f_{n+1}}{f_n} \rightarrow 0, \quad \frac{f_{n+1}^*}{f_n^*} \rightarrow 0 \quad \text{for } \varepsilon \rightarrow 0. \tag{9}$$

The missing boundary conditions for inner and outer expansions follow from condition of identity of the extension of both asymptotic expansions into certain intermediate region, i.e.,

$$t_e(y \rightarrow \infty, \theta) = t_e^*(\xi \rightarrow 0, \theta). \tag{10}$$

Asymptotic expansion of solution inside the particle, as seen from the boundary conditions on the particle surface (4), should be sought in the form analogous to expression (8), namely:

$$t_i(y, \theta) = \sum_{n=0}^{\infty} f_n(\varepsilon) t_{in}(y, \theta). \tag{11}$$

As to the functions  $f_n(\varepsilon)$  and  $f_n^*(\varepsilon)$ , the order of their smallness in  $\varepsilon$  increases with the growth of  $n$ .

Taking into consideration the compressed radial coordinate, for the dimensionless temperature  $t_e^*$  we have the following equation:

$$\text{Pr} \left( V_r^* \frac{\partial t_e^*}{\partial \xi} + \frac{V_\theta^*}{\xi} \frac{\partial t_e^*}{\partial \theta} \right) = \Delta^* t_e^*, \quad t_e^* \rightarrow 1 \quad \text{for } \xi \rightarrow \infty \tag{12}$$

and

$$\mathbf{V}_e^*(\xi, \theta) = \mathbf{n}_z + \varepsilon \mathbf{V}_e^*(\xi, \theta) + \dots \tag{13}$$

Here  $\Delta^*$  is the axially symmetric Laplace operator obtained from  $\Delta$  by replacing  $y$  with  $\xi$ ;  $V_r^* = V_r^*(\xi, \theta)$ ,  $V_\theta^* = V_\theta^*(\xi, \theta)$ ; Pr is Prandtl number; and  $\mathbf{n}_z$  is unit vector along the direction of  $OZ$ -axis.

The form of boundary conditions (4)–(6) indicates that expressions for components of mass velocity  $V_r^e$  and  $V_\theta^e$  are sought in the form of expansions in Legendre and Gegenbauer polynomials.<sup>21</sup>

## 2. Temperature fields outside and inside a particle

In determination of photophoretic force and velocity, we will confine ourselves to corrections of the first order of the smallness parameter  $\varepsilon$ . For their determination, it is necessary to know the temperature fields outside and inside the particle. Successively determining zero- and first-order terms of the expansion, and taking into account the condition of matching the inner and outer expansions, by analogy with Refs. 20 and 21 we obtain:

$$t_e^*(\xi, \theta) = t_{e0}^* + \varepsilon t_{e1}^*, \quad t_e(y, \theta) = t_{e0} + \varepsilon t_{e1},$$

$$t_{i1}(y, \theta) = t_{i0} + \varepsilon t_{i1}, \quad t_{e0}^* = 1, \quad t_{e0}(y) = 1 + \frac{\Gamma_0}{y},$$

$$t_{e1}^*(\xi, \theta) = \frac{\Gamma_0}{\xi} \exp\left\{ \frac{\text{Pr}}{2} \xi(x-1) \right\}, \quad x = \cos \theta,$$

$$t_{i0}(y) = B_0 + \frac{1}{4\pi R \lambda_i T_\infty y} \int_V q_i dV + \int_V \frac{f_0}{y} dy - \frac{1}{y} \int_V f_0 dV,$$

$$t_{e1}(y, \theta) = \frac{\omega}{2y} (\Gamma_1 - y) + \left\{ \frac{\Gamma}{y^2} + \omega \left( \frac{1}{2} - \frac{A_1}{4y^3} + \frac{A_2}{2y} \right) \right\} \cos \theta, \tag{14}$$

$$\omega = \Gamma_0 \text{Pr}, \quad y = r/R,$$

$$t_{i1}(y) = \Gamma_2 + \cos \theta \times$$

$$\times \left( B y + \frac{R J}{3 \lambda_i T_\infty y^2} + \frac{1}{3} \left[ y \int_1^y \frac{f_1}{y^2} dy - \frac{1}{y^2} \int_1^y f_1 dy \right] \right),$$

$$V = \frac{4}{3} \pi R^3,$$

where

$$f_n(y) = -\frac{R^2}{\lambda_i T_\infty} y^2 \frac{2n+1}{2} \int_{-1}^1 q_i(r, \theta) P_n(x) dx;$$

$$J = \frac{1}{V} \int_V q_i z dV$$

is the dipole moment of the density of heat sources; and  $P_n(x)$  are Legendre polynomials.

Integration constants entering into Eq. (14) are determined from the boundary conditions on the particle surface, Eq. (4).

### 3. Determination of photophoretic force and velocity

General solution of the hydrodynamic equations, satisfying the finiteness for  $r \rightarrow \infty$ , has the form<sup>22,23</sup>:

$$P_e(y, \theta) = 1 + \cos\theta \frac{A_2}{y^2}; \quad V_r(y, \theta) = \cos\theta \left[ 1 + \frac{A_1}{y^3} + \frac{A_2}{y} \right];$$

$$V_\theta(y, \theta) = -\sin\theta \left[ 1 - \frac{A_1}{2y^3} + \frac{A_2}{2y} \right]. \quad (15)$$

The resulting force acting on the particle is determined by integration of stress tensor over the surface of aerosol particle; it has the form<sup>22</sup>:

$$F_z = -4\pi R U_\infty A_2, \quad (16)$$

where the coefficient  $A_2$  is determined from boundary conditions on the surface of the aerosol particle (4).

Taking into consideration the form of the coefficient  $A_2$ , the resulting force acting on the solid, large, spherical aerosol particle at small relative temperature drops in its neighborhood, will be a sum of the viscous drag force  $\mathbf{F}_\mu$ , photophoretic force  $\mathbf{F}_{ph}$  proportional to the dipole moment  $J$ , and the force  $\mathbf{F}_{mh}$  caused by motion of the medium (i.e., taking into account the convective terms in the equation of heat conductivity):

$$\mathbf{F} = \mathbf{F}_\mu + \varepsilon \mathbf{F}^{(1)}, \quad \mathbf{F}^{(1)} = \mathbf{F}_{ph} + \mathbf{F}_{mh}, \quad (17)$$

where

$$\mathbf{F}_\mu = 6\pi R \mu_\infty U_\infty f_\mu \mathbf{n}_z, \quad \mathbf{F}_{ph} = -6\pi R \mu_\infty f_{ph} J \mathbf{n}_z,$$

$$\mathbf{F}_{mh} = -6\pi R \mu_\infty f_{mh} \mathbf{n}_z. \quad (18)$$

The coefficients  $f_\mu$ ,  $f_{mh}$ , and  $f_{ph}$  can be estimated from the following formulas:

$$f_\mu = \frac{1 + 2C_m \text{Kn}}{1 + 3C_m \text{Kn}}; \quad \Delta = 1 + 3C_m \text{Kn};$$

$$f_{ph} = K_{TS} \frac{2v_{es}}{3t_{es}} \frac{1}{\lambda_i \delta T_\infty \Delta} \times$$

$$\times \left\{ 1 + \text{Kn} \left[ \beta'_{RT} + 3\beta_{RT}^B - 2\beta_{RT} - \frac{C_V^T}{K_{TS}} (1 + 6C_m \text{Kn}) \right] \right\},$$

$$f_{mh} = K_{TS} \frac{2v_{es}}{3t_{es}} \frac{1}{\lambda_i \delta T_\infty \Delta} \times$$

$$\times \left\{ 1 + \text{Kn} \left( \beta'_{RT} + 3\beta_{RT}^B - 2\beta_{RT} - \frac{C_V^T}{K_{TS}} (1 + 6C_m \text{Kn}) \right) \right\} \times$$

$$\times \left( \frac{\lambda_e}{\lambda_i} + C_T \text{Kn} \right) + \text{Kn} \left( \beta_{RT} - \beta_{RT}^B \right) \left( 1 - 2C_q^T \text{Kn} \frac{\lambda_e}{\lambda_i} \right) \times$$

$$\times \left( 1 + \frac{1 + 2C_m \text{Kn}}{2(1 + 3C_m \text{Kn})} \right).$$

Here subscript  $s$  stands for values of physical quantities taken for the mean relative temperature of particle surface  $t_{es} = t_{e0}|_{y=1}$ , determined from the formula

$$t_{es} = 1 + \frac{1}{4\pi R \lambda_e T_\infty} \int q_i(r, \theta) dV.$$

Equating the resulting force to zero, we obtain general equation for velocity of ordered particle motion, which will be the sum of photophoretic velocity  $\mathbf{U}_{ph}$  and velocity  $\mathbf{U}_{mh}$  caused by the motion of the medium:

$$\mathbf{U}_p = -\varepsilon (\mathbf{U}_{ph} + \mathbf{U}_{mh}), \quad (19)$$

where

$$\mathbf{U}_{ph} = \frac{f_{ph}}{f_\mu} J \mathbf{n}_z, \quad \mathbf{U}_{mh} = \frac{f_{mh}}{f_\mu} J \mathbf{n}_z.$$

### 4. Analysis of the results obtained

From formulas (17) and (18) it is seen that both the magnitude and direction of the force  $\mathbf{F}^{(1)}$  and velocity  $\mathbf{U}_p$  undergo the influence by the magnitude and direction of dipole moment of the density of heat

sources  $J = \frac{1}{V} \int_V q_i z dV$ .

In addition, the  $\mathbf{F}^{(1)}$  and  $\mathbf{U}_p$  quantities depend substantially on the heat conductivity of the particulate matter as well. For  $\lambda_i \rightarrow \infty$ , and for fixed dipole moment,  $\mathbf{F}^{(1)}$  and  $\mathbf{U}_p$  tend to zero. When  $\omega = 0$ , we obtain expression for pure photophoresis of moderately large particle.<sup>14</sup>

To estimate the influence of the medium on photophoresis of moderately large spherical aerosol particle, we will consider the simplest case when the particle absorbs radiation as a blackbody. The absorption takes place in a thin layer of  $\delta R \ll R$  thickness, adjacent to the heated part of the particle surface. Furthermore, the density of heat sources inside the layer of  $\delta R$  thickness is determined from the formula

$$q_i(r, \theta) = \begin{cases} -\frac{I_0}{\delta R} \cos\theta, & \frac{\pi}{2} \leq \theta \leq \pi, \quad R - \delta R \leq r \leq R, \\ 0, & 0 \leq \theta \leq \frac{\pi}{2}, \end{cases}$$

where  $I_0$  is the intensity of incident radiation, related to the temperature of particle surface by:  $T_s = T_\infty + \frac{R}{4\lambda_e} I_0$ .

Taking this into account, we obtain the following formulas:

$$F_{ph}^* = \varepsilon 6\pi R \mu_e f_{ph}^*, \quad U_{ph}^* = \varepsilon h_{ph}^*; \quad (20)$$

$$\begin{aligned}
 f_{ph}^* &= K_{TS} \frac{v_{es}}{3t_{es}\lambda_i\delta T_\infty(1+3C_mKn)} \times \\
 &\times I_0 \left\{ 1 + Kn \left[ \beta'_{RT} + 3\beta_{RT}^B - 2\beta_{RT} - \frac{C_V^T}{K_{TS}}(1+6C_mKn) \right] - \right. \\
 &\quad \left. - \frac{3}{8}Pr \left( 1 + \frac{1+2C_mKn}{2(1+3C_mKn)} \right) \left[ \left( 1 + C_TKn \frac{\lambda_i}{\lambda_e} \right) \times \right. \right. \\
 &\quad \left. \left. \times \left[ 1 + Kn \left( \beta'_{RT} + \beta_{RT}^B - \frac{C_V^T}{K_{TS}}(1+6C_mKn) \right) \right] + \right. \right. \\
 &\quad \left. \left. + Kn \frac{\lambda_i}{\lambda_e} (\beta_{RT} - \beta_{RT}^B) \left( 1 - 2C_q^T Kn \frac{\lambda_e}{\lambda_i} \right) \right] \right\}, \\
 h_{ph}^* &= K_{TS} \frac{v_{es}}{3t_{es}\lambda_i\delta T_\infty(1+2C_mKn)} \times \\
 &\times I_0 \left\{ 1 + Kn \left[ \beta'_{RT} + 3\beta_{RT}^B - 2\beta_{RT} - \frac{C_V^T}{K_{TS}}(1+6C_mKn) \right] - \right. \\
 &\quad \left. - \frac{3}{8}Pr \left( 1 + \frac{1+2C_mKn}{2(1+3C_mKn)} \right) \left[ \left( 1 + C_TKn \frac{\lambda_i}{\lambda_e} \right) \times \right. \right. \\
 &\quad \left. \left. \times \left[ 1 + Kn \left( \beta'_{RT} + \beta_{RT}^B - \frac{C_V^T}{K_{TS}}(1+6C_mKn) \right) \right] + \right. \right. \\
 &\quad \left. \left. + Kn \frac{\lambda_i}{\lambda_e} (\beta_{RT} - \beta_{RT}^B) \left( 1 - 2C_q^T Kn \frac{\lambda_e}{\lambda_i} \right) \right] \right\}.
 \end{aligned}$$

To illustrate the contribution of motion of the medium to the photophoretic rate of solid moderately large spherical aerosol particle, Figure 1 presents the plots of the functions  $h_{ph}^*$  versus the intensity of incident radiation.

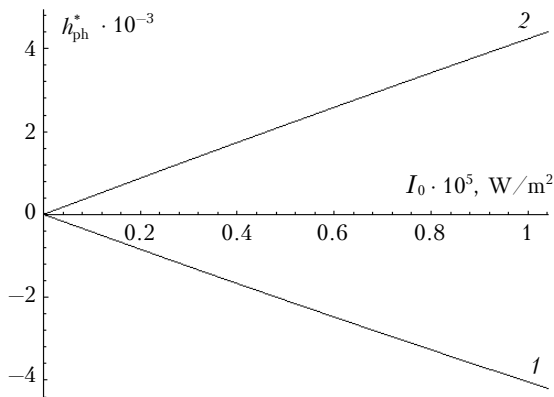


Fig. 1.

The numerical estimates were made for particles of borated graphite ( $\lambda_i = 55 \text{ W}/(\text{m} \cdot \text{deg})$ , curve 1) and soot ( $\lambda_i = 0.029 \text{ W}/(\text{m} \cdot \text{deg})$ , curve 2), suspended in air at  $T_\infty = 273 \text{ K}$ ,  $R = 2 \mu\text{m}$ , and  $P_\infty = 10^5 \text{ Pa}$ .

Figures 2 and 3 present the plots of the function  $h_{ph}^*$  versus Knudsen number ( $0.05 \leq Kn \leq 0.3$ ) at  $R = 10 \mu\text{m}$  (Fig. 2),  $R = 20 \mu\text{m}$  (Fig. 3),  $T_s = 293 \text{ K}$ ,  $313 \text{ K}$  for particles of graphite (curves 2 and 3) and soot (curves 1 and 4), respectively. From this plots it is seen that inclusion of the motion of the medium for particles with high heat conductivity (graphite) has the consequence that the photophoresis rate faster transits to the region of the so-called “negative” photophoresis. This may be one of the reasons for particle levitation in the stratosphere.<sup>10</sup>

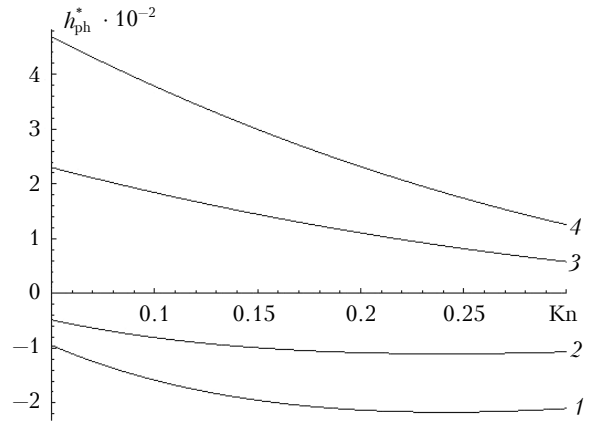


Fig. 2.

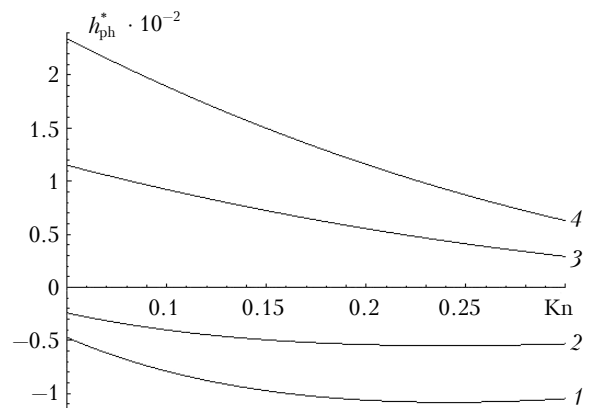


Fig. 3.

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