Experimental study of light diffraction on a thin screen with a straight edge

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Received January 22, 1999

The experimentally obtained position of diffraction fringes with respect to the boundary of a geometric shadow in the diffraction pattern of a screen is compared with the position of diffraction fringes calculated based on the Young and Fresnel concepts. The relation between the light flux incident on a thin screen and the edge light flux from an arbitrary area of the deflection zone of the screen is established.

Reference 1 presents the formula

$$h_f = \sqrt{(k_0 + k)\,\lambda\, L\,(L + l)/l}\,, \qquad (1)$$

which characterizes the fringe position in the diffraction pattern from a thin screen with a straight edge formed due to interference of rays from the screen edge with the direct light. In Eq. (1), h_f is the distance between the fringes and the shadow boundary (sh. b.); l and L are, respectively, the distances from the linear light source to the screen and from the screen to the plane of observation of the diffraction pattern; $(k_0 + k)$ is the number of $\lambda/2$ in the propagation difference between interfering rays: $k = 0, 2, 4, \ldots$ correspond to maxima; $k = 1, 3, 5, \ldots$ correspond to minima; $k_0 = 0.69$. The presence of k_0 in Eq. (1) indicates that the edge rays lead the rays, propagating without deflection, by $k_0\,\lambda/2$ at the moment of formation.

To determine k_0, the following equations were used:

$$h_{\max_1} = [2\lambda L\,(L + l)/l - h_{21}^2]/2\,h_{21}; \qquad (2)$$

$$k_0 = h_{\max_1}^2\, l/\lambda L\,(L + l), \qquad (3)$$

where h_{21} is the distance between the first and the second experimental maxima.

According to Tables 1–3 from Ref. 1, the calculated values of h_f (h_{cal}) are in close agreement with the experimental ones, if the point lying at a distance h_{cal} from $\max_1$ is taken as the shadow boundary. At the same time, they are somewhat different for $\max_1$ and $\min_1$ from h_f, found based on the Cornu spiral.

Further investigations[2] have shown that there are deflection zones above the surface of bodies (screens), where light beams deflect to both sides from their initial direction. Just this deflection is the basic cause of formation of the edge wave[3] (boundary diffracted wave[4]). According to experimental data, this deflection increases, as the distance h_z between the initial ray trajectories and the screen edges decreases.

In view of these facts, Eq. (1) determines the distance h_f to projection of incident rays (PIR_1) coming as edge rays into $\max_1$ of the diffraction pattern after deflection in the deflection zone, rather than the distance to the classic shadow boundary (CShB).[5] The distance from fringes to CShB, in its turn, is (Fig. 1)

$$H = \left[\frac{h_z\,(L + l)}{l} + h\right] = \left[\frac{h_z\,(L + l)}{l} + \sqrt{(k_0 + k)\,\frac{\lambda L\,(L + l)}{l}}\,\right], \qquad (4)$$

where the second term determines the distance from the fringes to the corresponding projections of incident rays, which give rise to the edge rays resulting from deflection in the deflection zone and coming into the diffraction fringes.

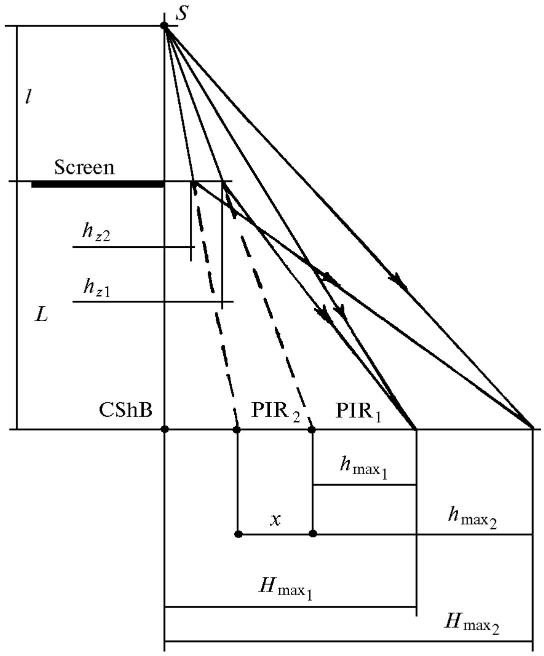

Fig. 1. Diffraction geometry of a light beam from a linear source on a thin screen with a straight edge.

The edge wave propagating into the screen shadow also has an initial shift with respect to the incident wave. The shift is equal, in the absolute value, to the initial shift of the edge wave propagating from the screen, but opposite in sign.[5] In this case, the shift

0235-6880/99/05 395-03 $02.00

between the components of the edge wave propagating towards the screen shadow and on the illuminated side must be equal to $2k_0\lambda/2 = 1.38\,\lambda/2$. At the same time, according to the theory and experiments,[1] it is equal to $\lambda/2$, that is, $k_0 = 0.5$.

The value $k_0 = 0.69$ determined in the above-described way is likely overestimated because of approximated character of Eqs. (2) and (3). They have been derived on the assumption that the edge rays are formed immediately near the screen edge and k_0 is independent on the deflection angles.

With regard for deflection of the edge rays at different distances from the screen edge and possible change of k_0 with increasing order of the diffraction pattern, Eq. (2) takes the form

$$h_{\max_1} = \frac{(k_{02} - k_{01} + 2)\,\lambda L\,(L + l)/l - (h_{21} + x)^2}{2\,(h_{21} + x)}, \quad (5)$$

where x is the distance between the projections PIR_1 and PIR_2 of the initial trajectories of incident beams coming after deflection to the first and the second maxima. As seen, an appearance of x in the formula decreases $h_{\max_1}$ and, consequently, the value of k_0 determined by Eq. (3).

In Ref. 6, the relation between the deflection angles ε of the edge rays in the deflection zone of the thin screen with the straight edge and the distance between their initial trajectories and the screen edge has been found experimentally. It has the following form:

$$h_z = (259.5 - 0.786\,\varepsilon)/\varepsilon \quad (6)$$

(h, μm; ε, min).

This circumstance allows a calculation of diffraction fringe position with respect to CShB by Eq. (4). In this case, ε is calculated as

$$\varepsilon = 3438'\,h/L.$$

Table 1 presents the calculated values of H (H_{cal}), the experimental values H_{exp}, the values H determined based on the Cornu spiral H_C, and the values h_f determined by Eq. (1), where $\Delta H_{exp,cal} = (H_{exp} - H_{cal})$, $\Delta H_{cal,C} = (H_{cal} - H_C)$, $J_{sh.b}$ is the relative intensity of light in the diffraction pattern at CShB.

In the corresponding experiments, a rectangular glass prism set at an angle of 11° with respect to the edge of the right angle towards the adjacent side drift from the light beam axis served as a screen. According to Ref. 7, the prism set at such an angle is equivalent to a thin screen.

A slit of 30-μm wide was used as a light source. The slit was illuminated with a parallel beam of green light at $\lambda = 0.53$ μm, separated from the radiation of a filament lamp by an interference filter.

PMT was used for light recording. The diffraction pattern was scanned with a 20-μm wide slit.

As follows from comparison of the tabulated data, the values of H_{cal} and $J_{sh.b}$ found at $k_0 = 0.5$ practically coincide with the corresponding values of H_C and $J_{C,sh.b}$. To put H_{cal} and H_{exp} into agreement, it is necessary to increase gradually k_0 up to k_0' with increasing diffraction order.

The values of k_0' tabulated in Table 1 were determined by the formula

$$k_0' = \left[(h + \Delta H_{exp,cal})/\sqrt{\lambda L\,(L + l)/l}\,\right]^2 - k. \quad (7)$$

Table 1.

Fringe	k	H_{exp}, mm	H_{cal}, mm	H_C, mm	h_f, mm	$\Delta H_{exp,cal}$, μm	$\Delta H_{cal,C}$, μm	h_z, μm	ε, min	k_0'
$l = \infty$; $L = 99.5$ mm										
$\max_1$	0	0.208	0.208	0.206	0.191	0	2	45.5	5.6	0.5
$\min_1$	1	0.315	0.307	0.306	0.299	8	1	25.9	9.7	0.586
$\max_2$	2	0.395	0.383	0.382	0.377	12	1	19.9	12.6	0.668
$\min_2$	3	0.4675	0.4463	0.445	0.441	21.2	1.3	16.7	14.8	0.853
$\max_3$	4	0.519	0.5018	0.495	0.497	17.2	6.8	14.6	16.8	0.822
$\min_3$	5	0.5715	0.5518	0.544	0.548	19.7	7.8	13.2	18.6	0.91
$\max_4$	6	0.619	0.5976	0.601	0.594	21.4	− 3.4	12	20.2	0.984
$l = \infty$; $L = 279.5$ mm; $J_{sh.b} = 0.247$										
$\max_1$	0	0.349	0.349	0.346	0.32	0	3	76.8	3.35	0.5
$\min_1$	1	0.548	0.515	0.513	0.5	33	2	44	5.8	0.71
$\max_2$	2	0.684	0.642	0.640	0.63	42	2	33.9	7.5	0.85
$\min_2$	3	0.794	0.748	0.746	0.739	46	2	28.5	8.86	0.96
$\max_3$	4	0.892	0.841	0.830	0.834	51	11	25.1	10	1.07
$\min_3$	5	1.004	0.925	0.912	0.918	79	13	22.6	11.1	1.51
$l = 24$; $L = 99.5$ mm; $J_{sh.b} = 0.252$										
$\max_1$	0	0.469	0.469	0.468	0.433	0	1	19.6	12.7	0.5
$\min_1$	1	0.721	0.694	0.694	0.686	27	0	11	22	0.627
$\max_2$	2	0.901	0.867	0.866	0.865	34	1	8.33	28.5	0.71
$\min_2$	3	1.042	1.010	1.009	1.006	31	1	6.92	33.7	0.728
$\max_3$	4	1.162	1.136	1.124	1.126	26	12	6	38.2	0.71
$\min_3$	5	1.290	1.249	1.234	1.249	41	15	5.36	42.2	0.868
$\max_4$	6	1.387	1.353	1.363	1.351	34	− 10	4.87	45.9	0.836

With the revealed character of dependence between h_z and ε, it becomes possible to compare the edge flux coming from some area of the deflection zone with the flux, incident on this area.

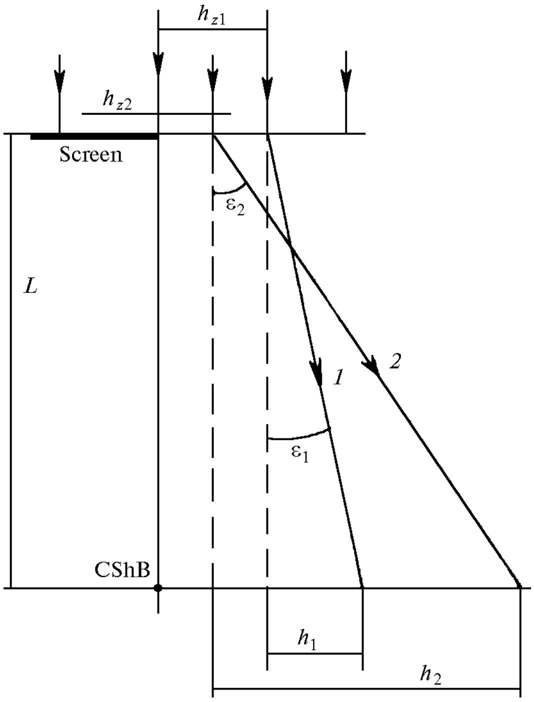

Fig. 2. Diffraction of a plane wave on the thin screen with the straight edge.

Figure 2 shows the geometry of plane wave diffraction on the screen S. In Fig. 2, rays *1* and *2* are deflected in the screen deflection zone at the distances h_{z1} and h_{z2} from the screen at the angles ε_1 and ε_2. Then they fall at the distances h_1 and h_2 from the projections of their initial trajectories onto the plane of the diffraction pattern, situated at the distance L from the screen.

Based on the findings of Ref. 8, the intensity of the edge light in the case of incident plane wave is

$$J_{ed} = 0.02046\ \lambda L J_{in} / h^2, \qquad (8)$$

where J_{in} is the intensity of the incident light.

Hence, the edge light leaving the deflection zone $\Delta h_z = (h_{z1} - h_{z2})$ is described by the following equation:

$$\Phi_{ed} = \int_{h_2}^{h_1} J_{ed}\ \mathrm{d}h = 0.02046\ \lambda L\ J_{in} \left(\frac{1}{h_1} - \frac{1}{h_2}\right) \qquad (9)$$

at the incident light flux $\Phi_{in} = \Delta h_z J_{in}$, where $h_1 = L \tan\varepsilon_1$; $h_2 = L \tan\varepsilon_2$; $h_{z1} = (259.5 - 0.786\ \varepsilon_1)/\varepsilon_1$; $h_{z2} = (259.5 - 0.786\ \varepsilon_2)/\varepsilon_2$.

Table 2 presents the values of Φ_{in}/Φ_{ed} for arbitrary values of ε_1 and ε_2. The data of Table 2 allows the conclusion that this ratio is constant at different diffraction angles and equal to 7.05 on average.

Table 2.

ε_1, min	ε_2, min	h_1, mm	h_2, mm	h_{z1}, μm	Δh_z, μm	Φ_{in}/Φ_{ed}
5	12	0.3	0.7	51.114	30.275	7.33
12	18	0.7	1.04	20.839	7.2084	7.12
30	35	1.74	2.04	7.864	1.2358	6.74
60	70	3.5	4.08	3.539	0.6179	7.01
130	140	7.56	8.14	1.2101	0.1426	7.03

Note. L = 200 mm.

If we take into account the edge fluxes, which propagate from the areas Δh_z of the deflection zone towards the screen shadow area and have the same value,[1] the ratio Φ_{in}/Φ_{ed} is halved, but remains larger than unity.

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