ERRORS IN RECONSTRUCTING THE VERTICAL PROFILE OF THE AEROSOL SCATTERING COEFFICIENT FROM THE DATA OF SOUNDING OF THE TWILIGHT EARTH'S ATMOSPHERE

S.V. Loginov

Design and Technology Institute "Optika", Siberian Branch of the Russian Academy of Sciences, Tomsk

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The Tikhonov and Volz–Goody methods are compared on the basis of numerical modeling as applied to reconstructing the aerosol scattering coefficient from the data of spaceborne sounding of the twilight Earth's atmosphere. The Tikhonov method is shown to provide a smaller difference between the initial and reconstructed profiles. It is also shown that the reconstructed profiles are more stable with respect to random errors introduced artificially into the initial data.

The problem of determining the spatial distribution of the aerosol scattering coefficient from satellite measurements of brightness of the Earth's atmosphere near its twilight horizon was considered in a number of papers. The following methods adequately describe the full twilight characterized by the angle of descent of the Sun below the horizon $\delta > 15°$ (Fig. 1): the Bigg differential method[1] improved by Volz and Goody[2] and the effective geometric shadow method developed by Rosenberg.[3] The above–enumerated methods are vivid and easy to calculate. However, in these methods for interpreting the data of sounding of the twilight horizon at angles of descent $\delta < 2°$ with simultaneously varying angle δ and perigee height of the sight line of a device, fast variations of the thickness of an effective scattering layer with the sight ray height must be considered. As a result, a stable solution with respect to measurement errors can be obtained within a shorter altitude range.

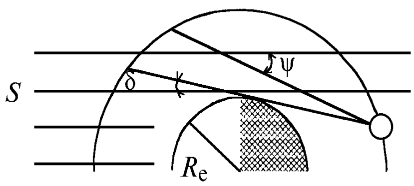

FIG. 1. Scheme of sounding of the twilight Earth's atmosphere. The Sun, the Earth center, and a device lie in the same plane.

In this paper, the errors in reconstructing the scattering coefficient from the data of sounding of the twilight Earth's atmosphere from space by two methods, namely, by the Volz–Goody method and the method based on regularization of a solution, are compared for the example of model data.

Let us write down the radiant intensity of the twilight atmosphere I in the single scattering approximation at zero azimuth angle in the form:[4]

$$I_\lambda(\psi, h) = S_\lambda \int\limits_h^\infty \frac{(\beta^a(x) g^a(\psi, x) + \beta^r(x)\, g^r(\psi))}{\sqrt{(n(x)R(x))^2 - (n(h)\, R(h))^2}}\, e^\tau n(x)\, R(x)\, dx\,, \tag{1}$$

where $\tau = -\tau_1(x, h) - \tau_2(x, h)$; $\tau_{(i)}$ are the optical thicknesses along the rays drawn from the Sun to the point of scattering (τ_1) and from the point of scattering to the detector (τ_2); S_λ is the solar constant; g^a and g^r are the aerosol and Rayleigh scattering phase functions, respectively; ψ is the scattering angle; h is the perigee height of the sight ray; $R(x) = R_e + x$; and, R_e is the Earth's radius (Fig. 1).

Volz and Goody[2] derived the formula that relates the volume scattering coefficient $\sigma(h) = \beta^a(h) + \beta^r(h)$ and the height of the lower boundary of the twilight layer h. They proposed that the optical thickness τ_2 at altitudes above 10 km be primarily determined by the Rayleigh scattering, and the transmission $\Pi(h) = e^{-\tau_1(x,\, h)}$ on the path from the Sun to the scattering point be approximated as follows:

$$\Pi(h) = \begin{cases} 0, & h_0 < h, \\ \dfrac{(h - h_0)}{D}, & h_0 < h < h_0 + D, \\ 1, & h > h_0 + D\,. \end{cases}$$

The parameters h_0 and D were adjusted in model calculations of the twilight horizon brightness. Assuming that

$$\sigma(h) = \sigma(h_0)\, e^{-(h - h_0)/H(h_0)}\,,$$

Eq. (1) can be easy solved for $\sigma(h)$. Volz and Goody obtained that

$$\sigma(h_0) = \frac{I_\lambda}{S_\lambda} \frac{\sin(\varepsilon)\, e^{\tilde\tau}\, D\, G^2\, [1 - e^{D\,G\,q}]^{-1}}{(d\, h_0/ d\, \delta)^2}\,, \tag{2}$$

where

$$G = -\frac{1}{I}\frac{d\, I_0}{d\delta} = \frac{1}{H(h_0)}\frac{d\, h_0}{d\delta}\,,\ \tilde\tau = \frac{\tau^r}{\sin(\varepsilon)}\,,\ q = \left(\frac{d\, h_0}{d\delta}\right)^{-1},$$

$H(h_0)$ is the aerosol vertical scale, δ is the angle of descent of the Sun, ε is the angle of observation, and τ^r is the Rayleigh optical thickness.

In order to calculate the scattering coefficient $\beta^a(h) = \delta(h) - \beta^r(h)$ by Eq. (2), as applied to satellite observations at constant scattering angle, the profile $D(h)$ must be constructed. It is easy constructed by iterations, calculating the brightness of the Rayleigh atmosphere. The process converges in three or four steps.

However, the character of solution (2) leads to a dramatic effect of noise on the reconstructed profile $\beta^a(h)$. One can construct solutions more stable with respect to noise using regularization methods.

Passing to the finite-difference approximation, we can write Eq. (1) in the matrix form:

$$I = \mathbf{A}\,\omega\,, \qquad \omega = g^a\beta^a + g^r\beta^r\,, \qquad (3)$$

or

$$\tilde{I} = \mathbf{B}\beta^a + \xi\,,$$

where

$$\tilde{I} = I - \mathbf{A}(g^r\beta^r),\ \mathbf{B} = \mathbf{A}\,g^a\,,$$

ξ is the measurement error.

By geometric reasoning, some diagonal elements of the matrix $\mathbf{A}$ are equal to zero. So Eq. (3) was solved by the generalized discrepancy method.[5] The matrix $\mathbf{A}$ was preliminary smoothed row–by–row to decrease the discrepancy. The error in reconstructing was determined by the method proposed in Ref. 6:

$$\sigma_i = \left(\sqrt{(\mathbf{F} + \alpha\Omega)^{-1}}\right)_{ii}\,,$$

$$\mathbf{F} = \mathbf{B}^{+}\mathbf{W}\,\mathbf{B},\quad W_{ij} = \frac{\delta_{ij}}{\omega_i\,\omega_j}\,,\quad \begin{matrix} i = 1, \ldots, N, \\ j = 1, \ldots, N, \end{matrix}$$

where Ω is the stabilizer of the second kind; α is the Lagrangian multiplier, ω_j is the rms error in reconstructing, and N is the number of nodes in the reconstructed profile.

The profile $\beta^a(h)$ was reconstructed from the data of modeling the twilight sounding by the Tikhonov and Volz–Goody methods. The value I_λ was calculated from the model profile of the scattering coefficient proposed in Ref. 7. A solution of the inverse problem for the exact values I_λ is shown in Fig. 2. Reconstruction by the Tikhonov method leads to a 30% difference between the calculated and preset profiles $\beta^a(h)$ within the 10–50 km altitude range. Above 50 km the error in reconstructing was more than 100%. The Volz–Goody method satisfactory reconstructs the scattering coefficient within the 10–30 km altitude range. The effect of measurement errors on the reconstruction error was estimated by modeling of the white noise with the amplitude of 5% of the maximum brightness value. Horizontal bars (Fig. 2) show the corridor of reconstruction errors calculated by Eq. (3). As a result, the altitude range of satisfactory reconstruction shortened to 10–35 km for the Tikhonov method and 10–25 km for the Volz–Goody method. This can be explained by the character of solution (2). The Tikhonov algorithm is more stable with respect to noise. However, the time taken for data processing by this method is at least N times greater than the corresponding time taken by the Volz–Goody method.

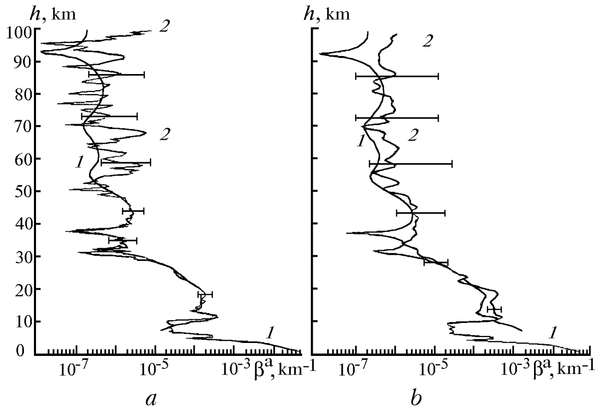

FIG. 2. Initial (1) and reconstructed (2) scattering coefficients $\beta^a(h)$: Tikhonov method (a) and Volz–Goody method (b).

Thus, the Volz–Goody method is suitable for data processing in real time without stringent requirements upon the accuracy. To obtain more accurate results, the Tikhonov method is preferable.

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