# INTERFERENCE OF MULTIPLY SCATTERED WAVES IN LIDAR RETURNS

**A.G. Borovoi**

*Institute of Atmospheric Optics,
Siberian Branch of the Academy of Sciences of the USSR, Tomsk*
*Received January 8, 1991*

*The effect of interference of the waves scattered by aerosol particles on the magnitude of lidar signals has been studied. It has been shown that in the case of collimated sensing laser beams and coherent detection of the return signals the interference between multiply scattered waves can result in nearly twofold increase of the lidar signal.*

The multiple scattering of sensing radiation in the lidar systems is conventionally described by the basic radiative transfer equation. On the basis of this equation, the radiation can be considered as an ensemble of the "particles carrying electromagnetic radiation", for example, of the photons, in the cases in which the wave nature of radiation is not manifested in an explicit form.

However, it is well–known from the theory of multiple scattering of waves that when the waves are multiply scattered by an ensemble of discrete scatterers in the backward direction, which is typical of lidar measurements, the interference of waves results (starting from the double scattering) in twofold increase of the beam intensity in comparison with the value obtained from the radiative transfer equation. This fact was investigated both theoretically and experimentally by many authors (see, for example, Refs. 1–4). Similar backscattering intensification effects in the turbulent and other randomly nonuniform media were considered in Ref. 5.

The purpose of this article is to analyze whether the above–mentioned effect of twofold increase of the intensity of multiply scattered radiation is manifested in actually measured lidar signals.

It is convenient to consider the effect of twofold increase of the multiply scattered radiation individually using the following simplest scheme close to the actual optical schemes of lidar systems. Let the wave field from a single–point transmitter be scattered by the ensemble of point scatterers and the returned signal be recorded by a quadratic wave detector at the point in which the radiation transmitter is located. Let us take out two arbitrary scatterers *1* and *2* (Fig. 1) in the scattering medium. The singly scattered waves from each of these scatterers, which are usually considered in the problems of sounding, will arrive first, at the observation point. The phases difference between the waves coming from the first and second scatterer is a random value. Therefore, the interference between these waves, on the average, makes a zero contribution to the intensity, and the lidar return is simply formed by summing over the intensities.

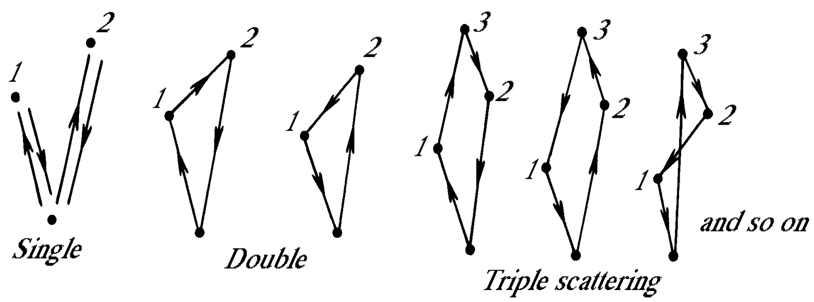


*FIG. 1.*

The situation changes radically starting from the double scattering. As can be seen from the figure, the scatterers *1* and *2* form two doubly scattered waves associated with different sequence of scattering acts, namely, 1) the radiation soural – the scatterer *1* – the scatterer *2* – the observation point and 2) the radiation source – the scatterer *2* – the scatterer *1* – the observation point. The phased difference between these waves at the observation point is always equal to zero. Hence, the wave fields rather than the intensities are summed at the detector. The amplitudes of both waves $A$ are identical. As a result, the detector records the value $I = (2A)^2 = 4A^2$, which doubly exceeds the value obtained by summing over the intensities $I' = A^2 + A^2 = 2A^2$.

Similarly, the triple, quadruple, etc. scattered waves will also arrive at the point of the signal detection from the scatterers *1* and *2*. For instance, Fig. 1 shows the triple scattered field which is formed by the scatterers *1*, *2*, and *3*. The phases difference between these waves at the observation point is, as a rule, a random value, and the recorded signal represents a sum of the intensities of these waves. However, each wave can be associated with one more wave scattered by the same scatterers but with the reverse sequence of scattering acts. These waves have identical amplitudes and zero phase difference. The interference between them results in the twofold increase of the intensity of multiply scattered (starting from the double scattered) wave in comparison with the signal obtained by

0235-6880/91/05 344-03 $02.00 

summing over the intensities on the basis of the radiative transfer equation.

It is easy to understand the reason of the additional flux of energy in the scattered field. The interference here, like many other interference phenomena, results in a simple spatial–redistribution of the energy flux. In the optical scheme chosen, the observation point turned out to be at the center of the central brightest spot. Moving the observation point away from the radiation source, at first we enter a dark interference fringe, then a bright one and so on. It is clear that, if the detector of radiation has the transverse dimension less than the size of the central spot the interference will have an appriciable effect, and in the opposite case, the interference will be negligible.

Let us now consider a more realistic optical model of the lidar. We shall assumed that one and the same telescope is employed as a part of the emitter and the detector, which is typical of practice. Our problem now is to take into account the finite transverse dimension of the telescope (Fig. 2). It is easy to solve this problem, if we introduce a notion of the region, in which spatially coherent scattered waves (CSW) are formed.

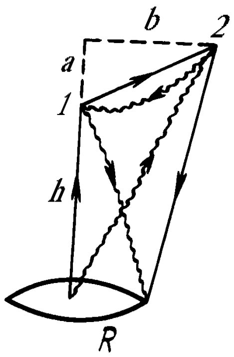


*FIG. 2.*

We shall define the CSW region in the following way. Let us take an arbitrary scatterer at the distance $h$ from the emitter ($h$ is the sensing range) on the optical axis of the lidar with telescope of radius $R$. The CSW region, according to the definition, determines such a vicinity of this scatterer, where the phase difference between two double scattered waves shown in Fig. 1 does not exceed $\pi$ at the distance $R$ from the optical axis, that is, the geometric path difference does not exceed half the wavelength ($\lambda/2$).

The geometric path difference is then equal to

$$s = \sqrt{(h + a)^2 + b^2} - \sqrt{(h + a)^2 + (b - R)^2} + + \sqrt{h^2 + R^2} - h , \quad (1)$$

where $a$ and $b$ are the differences between the coordinates of the scatterers in the vertical and horizontal directions. The values $a/h$ and $b/h$ were assumed to be small along with $R/h$. Expanding then Eq. (1) as a function of three variables $R/h$, $a/h$, and $b/h$ in the Taylor series and taking into account the cubic terms, we obtain

$$s \approx R \left( \frac{b}{h} + \frac{1}{2}\frac{R}{h}\frac{a}{h} - \frac{a}{h}\frac{b}{h} \right) . \quad (2)$$

The third term in Eq. (2) is negligible in comparison with the first term and can be omitted. As a result, denoting the parameters $a$ and $b$ at the boundary by $\alpha$ and $\beta$, we obtain the equation determining the boundaries of the CSW region

$$\frac{\lambda}{2} = R\frac{\beta}{h} + \frac{R}{2}\frac{R}{h}\frac{\alpha}{h} . \quad (3)$$

From Eq. (3) for $a = 0$, we obtain the transverse dimension of the region

$$\beta_{max} \approx h\Theta_0 , \quad (4)$$

where

$$Q_0 \approx \frac{\lambda}{2R} . \quad (5)$$

As can be seen, the transverse dimension of the CSW is determined by the trivial Fraunhofer diffraction of the plane wave by the aperture of the telescope.

The longitudinal dimension of the CSW region, respectively, is equal to

$$\alpha_{max} \approx \lambda\frac{h^2}{R^2} . \quad (6)$$

It is evident that the CSW region is strongly elongated lengthwise

$$\beta_{max}/\alpha_{max} = R/2h \ll 1 .$$

Now, let us discuss the effect of interference on the magnitude of the signals recorded by the lidar. If the lidar is incoherent (direct detection), it is clear that in sensing of the atmospheric aerosol at the altitude $h$, the lidar return is formed as a sum of the photons coming from a certain region surrounding the point *1* in Fig. 2. We shall call this region incoherent detection (ID) region. The ID region is determined by the angular divergence $\theta$ at the exit from the telescope and by the length of the sensing pulse. It is obvious that the correction for the interference in the signal is of the order

$$\varepsilon = v/V , \quad (7)$$

where $V$ is the volume of the ID region and $v$ is the volume of intersection of the ID and CSW regions. The value $\varepsilon$ is always negligible in practice.

The situation will significantly change, if a coherent lidar is employed. Indeed, the valid signal of the coherent lidar is the flux of energy incident on the aperture of the optical system. This signal is caused by the interference between the wave under study and the wave from a local oscillator. If the wavefronts of these waves are not matched, the detector records a zero signal. In sensing of aerosol at the altitude $h$, in order to obtain a perfect matching of the wavefronts, the wave from the local oscillator must be identical to the spherical wave coming from the point *1* in Fig. 2 to the receiving telescope (this wave is then transformed with the optical system). Hence, we obtain an important conclusion that the coherent lidar records only those waves, which come from the above–defined CSW region surrounding the point *1*.

Thus, if we interprete the radiation as an ensemble of the photons, it is possible to state that for the coherent lidars all the multiple scattered photons, whose first and last acts of scattering take place the CSW region (the trajectory shown by solid line in Fig. 3), contribute to the recorded signal with a coefficient close in value to 2.

A more detailed calculation of the coefficient of amplification of the lidar in the case of multiple scattering is outside the scope of exceeds the diffraction angular divergence $\theta_0$, the photons, whose first act of scattering takes place outside the CSW region and the last act of scattering takes place within this region (the trajectory shown by dashed line in Fig. 3), will also contribute to the recorded signal. These photons contribute to the recorded signal with the coefficient 1. Hence, the maximum backscattering intensification effect of the nearly twofold increase of the lidar signal caused by the interference will be observed for the coherent lidars with the divergence angle equal to the diffraction divergence angle $\theta = \theta_0$.

The effect of amplification of the multiple scattered field caused by the interference can be observed only for the coherent lidars. At the same time, the effect depends on the ratio of the angular divergence of the radiating optical system of the lidar to the diffraction angular divergence of the telescope rather than on the absolute dimensions of the telescope.

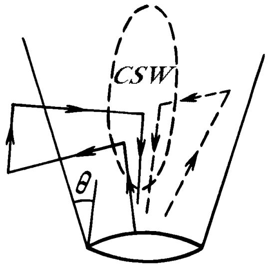


*FIG. 3.*

The author would like to acknowledge K. Flezia for fruitful discussions of this problem.

## REFERENCES

1. K.M. Watson, J. Math. Phys. 10, 688 (1969).
2. D.A. De Wolf, IEEE Trans. **AP–19**, 254 (1970).
3. Yu.N. Barabanenkov, Izv. Vyssh. Uchebn. Zaved., Ser. Radiofiz. **16**, 88 (1973).
4. Yu.N. Barabanenkov, Usp. Fiz. Nauk **117**, 49 (1975).
5. Yu.A. Kravtsov and A.I. Saichev, ibid. **137**, No. 3, 501 (1982).