ESTIMATE OF THE EXTINCTION OF OPTICAL RADIATION BY CRYSTALS LACKING PLANE-PARALLEL FACES

A.A. Popov and O.V. Shefer

Institute of Atmospheric Optics,
Siberian Branch of the Academy of Sciences of the USSR, Tomsk
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An analytical expression is obtained for the optical radiation extinction efficiency factor in crystals lacking plane-parallel faces. A round plate with chamfered upper face is taken as a general model for such crystals. It is shown that in the IR region the efficiency factor can be assumed to be equal to 2 only for crystals with faces angled at more than 10–12° to each other.

Judging from the character of the extinction of optical radiation the entire range of shapes of crystals in the atmosphere may be conditionally divided into two groups. It stands to reason to include in the first group only those crystals which have at least two plane-parallel faces. All the others can be clustered into the second group. Below we try to justify such a classification.

For crystals with plane-parallel faces the total scattered field radiation within small scattering angles is determined as the coherent sum of the diffracted and scattered fields.[1] Therefore, the extinction efficiencies computed for such particles can differ significantly from 2 (Refs. 1–3). The widest possible variability of that factor (0, 4) is encountered in flat plates.[1–2] An increase of the number of plane-parallel faces narrows this interval. The extinction efficiency of hexagonal columns varies approximately within the range 1–3 (Ref. 3).

When the crystal faces deviate from parallel, the refracted beams deviate from the direction of the primary wave that much more, the larger is the angle between the faces. Now, the more these beams deviate from the initial direction, the more reason there is to consider the total scattered field within the small scattering angle to be given by the diffracted field alone. This, on the other hand, means that the extinction efficiency for such particles becomes equal to 2.

Note that simple analytic expressions can be found for the extinction efficiency for every crystal plane-parallel shape that account for the polarization state of the incident radiation. At the same time it appears impossible to obtain similar expressions for crystals with non-parallel faces. However, there is no need for such expressions: first, there are many more shapes of such crystals and, second, the extinction efficiencies for most of them would automatically be equal to 2. Consequently, it is more interesting to estimate the angles between the faces for those crystals for which this factor is equal to 2 with some preset level of accuracy. The present study is dedicated to that problem.

The problem is formulated as follows: an elliptically polarized plane wave falls upon a round plate with a chamfered upper face, in the direction normal to its lower face (Fig. 1). The shape of such a plate may be uniquely described by the following parameters: a is the radius of the lower face; $d_{\min}$ is its minimum thickness, θ is the angle between the faces. We wish to find the scattering efficiency of such a scatterer. Note that the efficiency factor tends to 2 for a crystal of any shape as its absorption factor κ increases. Assuming $\kappa = 0$ to start with, we immediately arrive at a model of the crystal in which the deviation of its faces from parallel remains the only mechanism affecting the character of the extinction of the incident optical radiation.

Within the physical optics approximation the electric component of an electromagnetic field, scattered by a chamfered round plate into the forward hemisphere, is given by the following relations:

$$\vec{E}_{s1} = (\vec{A}_{d1} - \vec{A}_{r1}) \frac{e^{ikr}}{ikr},$$
$$\vec{E}_{s2} = (\vec{A}_{d2} - \vec{A}_{r2}) \frac{e^{ikr}}{ikr}. \quad (1)$$

The vector amplitudes of the diffracted field, $\vec{A}_{d1}$ and $\vec{A}_{d2}$, have the form

$$\vec{A}_{d1} = A_{d1}(\vec{\theta}_{01}\cos\varphi_1 - \vec{\varphi}_{01}\sin\varphi_1);$$
$$\vec{A}_{d2} = A_{d2}(\vec{\theta}_{01}\sin\varphi_1 + \vec{\varphi}_{01}\cos\varphi_1);$$
$$A_{d1} = \frac{k^2}{4\pi}(1 + \cos\theta_1)E_1F(\theta_1, \varphi_1)\, e^{i\psi_d};$$
$$A_{d2} = \frac{k^2}{4\pi}(1 + \cos\theta_1)E_2F(\theta_1, \varphi_1)\, e^{i\psi_d}. \quad (2)$$

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Here $k = 2\pi/\lambda$ is the wave number; E_1 and E_2 are the complex amplitudes of the incident field; ψ_d is the phase run-on for the diffracted field; $\vec{\theta}_{01}$ and $\vec{\varphi}_{01}$ are the basis vectors of the spherical coordinate system (r, θ_1, φ_1), its angles θ_1 and φ_1 are described in the Cartesian coordinate system $ox_1y_1z_1$: θ_1 is measured from the oz_1 axis, and φ_1, from the ox_1 axis in the ox_1y_1 coordinate plane. The angular function $F(\theta_1, \varphi_1)$ is the Fraunhofer integral

$$F(\theta_1, \varphi_1) = \iint_S \exp\left\{-ikx_1\cos\varphi_1\sin\theta_1 - iky_1\sin\varphi_1\sin\theta_1\right\} dx_1dy_1. \quad (3)$$

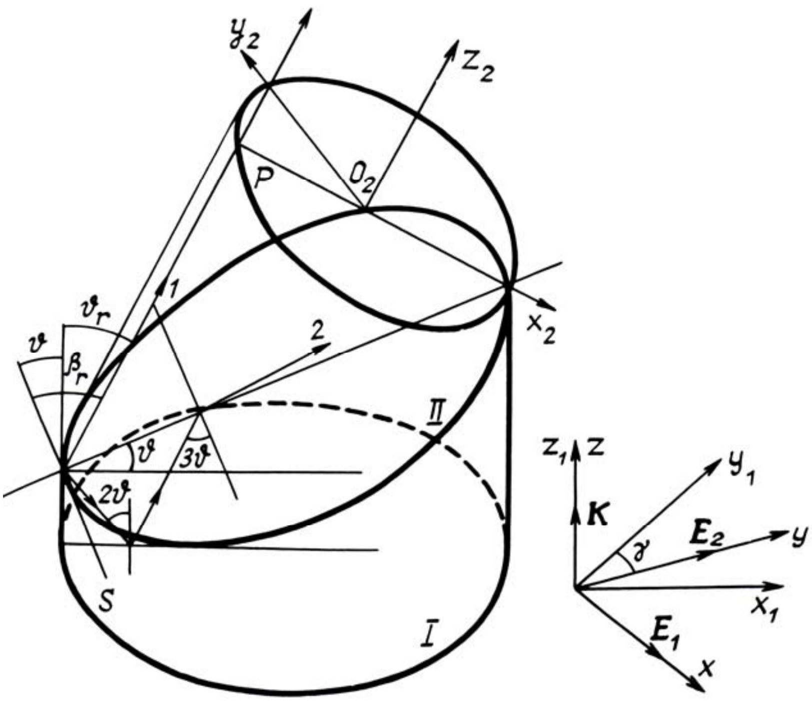

FIG. 1. Scheme of formation of the refracted beam.

Expression (3) is integrated over the surface of geometrical shadow S, which in our case coincides which the area of the lower crystal face.

We define the vector amplitudes $\vec{A}_{r1}$ and $\vec{A}_{r2}$ as the amplitudes of the scattered field in the refracted beam that has passed once through the crystal (Fig. 1). This is the beam that contributes decisively to the diffracted field in the oz_1 direction, which is the direction of interest to us.

Following Ref. 2. we have for the desired amplitudes $\vec{A}_{r1}$ and $\vec{A}_{r2}$

$$\vec{A}_{r1} = A_{r1}(\vec{\theta}_{02}\cos\varphi_2 - \vec{\varphi}_{02}\sin\varphi_2);$$

$$\vec{A}_{r2} = A_{r2}(\vec{\theta}_{02}\sin\varphi_2 - \vec{\varphi}_{02}\cos\varphi_2);$$

$$A_{r1} = \frac{k^2}{4\pi}(1 + \cos\theta_2)a_1G(\theta_2, \varphi_2)\, e^{i\psi_r};$$

$$A_{r2} = \frac{k^2}{4\pi}(1 + \cos\theta_2)a_2G(\theta_2, \varphi_2)\, e^{i\psi_r};$$

$$a_1 = (t_\parallel\cos^2\gamma + t_\perp\sin^2\gamma)E_1 - \sin\gamma\cos\gamma\,(t_\parallel - t_\perp)E_2;$$

$$a_2 = -\sin\gamma\cos\gamma(t_\parallel - t_\perp)E_1 + (t_\parallel\sin^2\gamma + t_\perp\cos^2\gamma)E_2;$$

$$t_\parallel = T_\parallel\tilde{T}_\parallel, \quad t_\perp = T_\perp\tilde{T}_\perp, \quad (4)$$

where ψ_r is the phase run-on for the refraction beam; $\vec{\theta}_{02}$ and $\vec{\varphi}_{02}$ are the basis vectors of the spherical coordinate system (r, θ_2, φ_2), its angles θ_2 and φ_2 are defined in the Cartesian system $o_2x_2y_2z_2$; γ is the an8le between the incident field vector and the normal to the plane of incidence. The angular function $G(\theta_2, \varphi_2)$ is the Fraunhofer integral

$$G(\theta_2, \varphi_2) = \iint_S \exp\left\{-ik_2x_2\cos\varphi_2\sin\theta_2 - iky_2\sin\varphi_2\sin\theta_2\right\} dx_2dy_2. \quad (5)$$

Expression (5) is integrated over the cross section of a beam of elliptical shape. Finally, the

Fresnel coefficients $T_{\parallel}, \tilde{T}_{\parallel}, T_{\perp}$ and $\tilde{T}_{\perp}$ are given by the relations

$$\tilde{T}_{\parallel} = \tilde{T}_{\perp} = \frac{2}{n+1}, \quad T_{\parallel} = \frac{2n\cos\theta}{\cos\theta + n\cos\beta_r},$$

$$T_{\perp} = \frac{2n\cos\theta}{n\cos\theta + \cos\beta_r}, \tag{6}$$

where n is the plate refractive index; β_r is the angle of escape from the plate of the refracted beam, which is related to the angle θ by Snell's law: $\sin\beta_r = n\cos\theta$.

The expression for the extinction includes the amplitudes of the total field scattered in the direction of the initial wave. Therefore we may reduce relations (2)—(5) to the oz_1 direction. To do this we must assign values to the angles θ_1, θ_2, φ_2: $\theta_1 = 0$, $\theta_2 = \theta_r = \beta_r - \theta$; $\varphi_2 = 0$. As a result the angular functions $F(\theta_1, \varphi_1)$, $G(\theta_2, \varphi_2)$ assume the form

$$F(0, \varphi_1) = \iint_S dx_1 dy_1 = S = \pi a^2; \tag{7}$$

$$G(\theta_r, 0) = \iint_P e^{-ikx_2\sin\theta_r} dx_2 dy_2 = \pi b_{min} b_{max} \frac{2J_1(kb_{min}\sin\theta_r)}{kb_{min}\sin\theta_r}, \tag{8}$$

where $b_{min} = a\cos\beta_r/\cos\theta$ and $b_{max} = a$ are the semiminor and seni-major axes of the ellipse P, and $J_1(t)$ is the Bessel function of first order.

Using the formula for the extinction of polarized radiation[4] we obtain (after some obvious transformations) the following relation for the extinction efficiency factor:

$$\Theta = 2 - f\cos\delta T, \tag{9}$$

where

$$f = (1 + \cos\theta_r)\frac{\cos\beta_r}{\cos\theta}\frac{2J_1\left[ka\dfrac{\cos\beta_r\sin\theta_r}{\cos\theta}\right]}{ka\dfrac{\cos\beta_r\sin\theta_r}{\cos\theta}}; \tag{10}$$

$$\delta = \psi_r - \psi_d = kd_{min}(n-1) + ka\left[2n\,\mathrm{tg}\theta - \frac{\sin\beta_r}{\cos\theta} - \mathrm{tg}\theta\right]; \tag{11}$$

$$T = \frac{t_{\parallel} + t_{\perp}}{2} + \frac{t_{\parallel} - t_{\perp}}{2}\left[\frac{I_2}{I_1}\cos 2\gamma - \frac{I_3}{I_1}\sin 2\gamma\right] \tag{12}$$

The variables I_1, I_2, and I_3 in relation (12) are the first three Stokes parameters, related to the complex amplitudes E_1 and E_2 of the incident field by the relations

$$I_1 = |E_1|^2 + |E_2|^2,$$

$$I_2 = |E_1|^2 - |E_2|^2, \quad I_3 = 2\mathrm{Re}\,(E_1, E_2^*).$$

Analyzing the expression for the coefficient T, it is easy to see that electromagnetic waves of differing polarizations must be differently extinguished. However, even for large angles between the plate faces, the values $T_{min} = t_{\perp}$ and $T_{max} = t_{\parallel}$ from each other only slightly. The parameter $(T_{max} - T_{min})/(T_{max} + T_{min})$ does not exceed the value 0.027 for refractive indices $n \le 1.5$ and angles $\theta \le 30°$. Note that all the dependences $\theta(d_{min})$ and $\Theta(\theta)$ shown in Figs. 2–4, were obtained for such a linear polarization of the incident wave: $I_2/I_1 = 1$ for $\gamma = 0$, at which the coefficient T attains its maximum value.

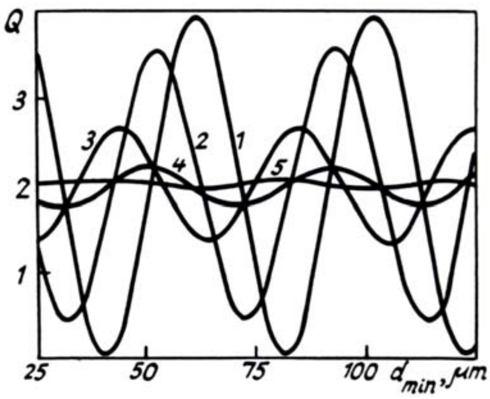

FIG. 2. Dependence of the extinction efficiency on plate thickness: $I_2/I_1 = 1$, $\gamma = 0°$, $n = 1.26$, $a = 250$ μm. $\lambda = 10.6$ μm; 1) $\vartheta = 0°$, 2) $\vartheta = 2°$, 3) $\vartheta = 4°$, 4) $\vartheta = 7°$, 5) $\vartheta = 10°$.

Variations of the plate thickness d_{min} result in oscillations of the extinction efficiency Θ (Fig. 2). At larger angles θ between the crystal faces these oscillation amplitudes quickly diminish, so that at $\theta = 10°$ one may already neglect the dependence of Θ on d_{min}. Apparently, $\Theta = 2$ is an asymptote of all the dependences $\Theta(\theta)$ presented in Figs. 3 and 4. An Interfacial angle of 10–12° may be considered sufficient for disregarding the effect of crystal thickness on the character of the radiation extinction. The crystal shape classification presented above appears more understandable in light of this. Indeed, if a natural crystal has no plane-parallel faces in that classification, it is implied that they meet at angles larger than 10–12°. According to our estimates the extinction efficiency for such crystals is equal to 2.

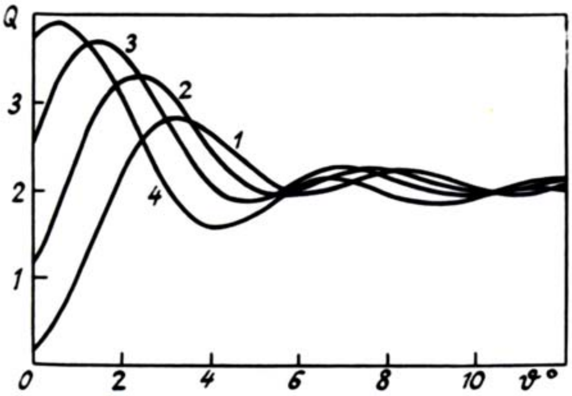

FIG. 3. Dependences of the extinction efficiency on the angle between the faces: $I_2/I_1 = 1$, $\gamma = 0°$, $n = 1{,}26$, $a = 250$ μm*;* *1)* $d_{\min} = 43$ μm, *2)* $d_{\min} = 48$ μm, *3)* $d_{\min} = 53$ μm, *4)* $d_{\min} = 58$ μm.

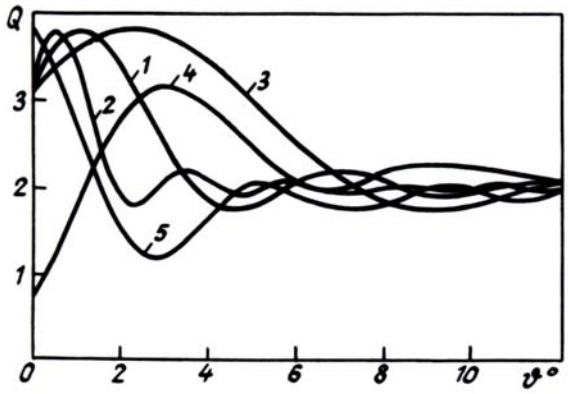

FIG. 4. Dependences of the extinction efficiency on the angle between the faces: $I_2/I_1 = 1$, $\gamma = 0°$, $d_{\min} = 55$ μm, $\lambda = 10.6$ μm,
1) $a = 250$ μm, $n = 1.26$,
2) $a = 125$ μm, $n = 1.26$;
3) $a = 500$ μm, $n = 1.26$;
4) $a = 250$ μm, $n = 1.22$;
5) $a = 250$ μm, $n = 1.30$.

Let us consider another important aspect of the problem, i.e., the total extinction of radiation by the entire ensemble of particles contained in the scattering volume. As a rule, these particles are considered to be chaotically oriented. Such an orientation is associated with the neutral spectral trend of such an integral characteristic of light scattering as the extinction coefficient in the visible spectral range. However, we have shown[5] that such a trend in the visible is also typical of a system of oriented plates. Therefore the absence of a dependence of the extinction coefficient on the wavelength cannot be given as an argument for the chaotic orientation of particles within the scattering volume. To our mind, it would be more logical to assume only those particles to be chaotically oriented, whose maximum and minimum moments of inertia differ only insignificantly from each other. Thus, needle and plate particles should have some preferred orientation. Experiments have been conducted in which such systems of oriented crystals were observed.[6,7] Note that, as a rule, plate crystals are "strictly" oriented. For example, Ref. 7 described in detail experiments with scattering volumes containing ensembles of plates. Each plate was characterized by a small amount of flutter (by no more then 0.56°) around the horizontal. Characteristically, the oscillating terms in the extinction efficiency for the total extinction of optical radiation due to such a set of oriented particles cancel out (partially in the IR range and completely in the visible).[5] In other words, total or partial compensation for fields refracted from oriented crystals of the investigated polydisperse medium take place in the forward direction (the direction of sounding) This means that the estimates obtained for the angles between non-parallel crystals faces (when the addition of the refracted beams to the diffraction part of the scattered field can already be disregarded) are even more reliable for systems of such particles.

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