COMPUTER SIMULATION OF IMAGE FORMATION FOR AN EXTENDED OBJECT IN THE TURBULENT ATMOSPHERE. PART III. QUALITY ESTIMATION

V.P. Kandidov, M.P. Tamarov, S.S. Chesnokov, and S.A. Shlenov

M.V. Lomonosov State University, Moscow
International Scientific Training Laser Center
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This paper completes the series of papers devoted to direct computer simulation of random realizations of short-exposure images of an extended object in the turbulent atmosphere. We formulate here a criterion of image quality or, on the contrary, degree of degradation. It can be used as the goal function for control over the wave front in adaptive optical systems of image formation.

The development of quality criteria for images of extended objects within the thickness of the Earth atmosphere is of great practical interest. Such criteria can serve for quick estimations of efficiency of adaptive compensation for atmospheric distortions in image-forming systems. Since characteristic time of fluctuation freezing for the refractive index in the atmosphere is usually small, the problem of reducing calculation expenditures is very actual in developing the criteria.

In this paper, we propose a criterion permitting one to estimate quality or, on the contrary, degradation degree of a short-exposure image of an object illuminated incoherently. The image is a random realization of two-dimensional intensity distribution of a light field given in the mesh points (see Part I (Ref. 1) and Part II, pp. 451–455 of this issue). Any *a priori* information about the object is supposed to be absent. In this connection, the central problem is image restoration, i.e., obtaining a "non-distorted image," in comparison with which quality of each realization is estimated. It should be noted that the methods for image restoration from distorted picture and construction of a quantitative quality criterion of image realization are ambiguous; they are discussed in numerous sources (see, for instance, Refs. 2–6). Efficiency of new quality criteria significantly depends on the concrete field of their application, so the approach proposed here can be considered as an alternative one.

1. IMAGE RESTORATION

In the absence of *a priori* information, image restoration must be based on processing of a random realization with allowance for characteristics of the optical system and atmospheric path. The proposed method of processing is oriented to the image-forming model presented in Ref. 1 and in Section 2 of this paper. In this model, distortions of a short-exposure image $\tilde{I}(\mathbf{r}, t)$ include three components:

– diffraction blurring;

– blurring caused by averaged contribution of small-scale atmospheric inhomogeneities;

– image wandering and distortion of image contour connected with large-scale atmospheric fluctuations.

Attempts to construct a restored image free from all the three above components make no sense. Tools of adaptive optics compensate for atmospheric distortions; to exceed diffraction resolution, mathematical processing of the image is usually used. So, in constructing a restored image $I_r(\mathbf{r})$, attempts to obtain diffraction-bounded image $I_0(\mathbf{r})$ of incoherent system from realizations of a distorted short-exposure image $\tilde{I}(\mathbf{r}, t)$ are undertaken.

To exclude contribution of large-scale atmospheric turbulence, we use the averaging procedure for the short-exposure image $\tilde{I}(\mathbf{r}, t)$ over a finite number of realizations $\tilde{I}_\mu(\mathbf{r}) = \tilde{I}(\mathbf{r}, t_\mu)$

$$\langle I(\mathbf{r})\rangle = \frac{1}{M}\sum_{\mu=1}^{M}\tilde{I}_\mu(\mathbf{r}) . \quad (1)$$

With infinite M, this procedure is equivalent to obtaining of a long-exposure image $I_L(\mathbf{r})$ with recording time τ_{exp} exceeding the characteristic time of existence of large-scale inhomogeneities τ_L. With finite M, the averaged image $\langle I(\mathbf{r})\rangle$ is close to the long-exposure image $I_L(\mathbf{r})$:

$$\langle I(\mathbf{r})\rangle \cong I_L(\mathbf{r}) . \quad (2)$$

Let us define the spectra of the long-exposure image $G_L(\Omega)$ and the averaged one $G_{<>}(\Omega)$:

$$G_L(\Omega) = F\{I_L(\mathbf{r})\} ,$$

$$G_{<>}(\Omega) = F\{\langle I(\mathbf{r})\rangle\} . \quad (3)$$

Here Ω is the spatial frequency; F is the Fourier transform operator. According to Eq. (2), the spectrum of the averaged image $G_{<>}(\Omega)$ is close to that of the

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long-exposure one $G_L(\Omega)$

$$G_{< >}(\Omega) \cong G_L(\Omega) . \quad (4)$$

The long-exposure image $I_L(\mathbf{r})$ is distorted by blurring caused by both diffraction and averaged contribution of atmospheric fluctuations of different scales. Its spectrum $G_L(\Omega)$ is equal to the product of the object spectrum $G(\Omega)$ and optical transfer function $H_{0L}(\Omega)$ of an optical incoherent system with long exposure

$$G_L(\Omega) = G(\Omega) \times H_{0L}(\Omega) . \quad (5)$$

According to Ref. 6, the optical transfer function $H_{0L}(\Omega)$ is

$$H_{0L}(\Omega) = H_0(\Omega) \times H_L(\Omega) , \quad (6)$$

where $H_0(\Omega)$ is OTF of a diffraction-bounded incoherent system; OTF $H_L(\Omega)$ represents the contribution of atmospheric inhomogeneities under long exposure. In the case of Kolmogorov fluctuation spectrum of the refractive index, the expression for $H_L(\Omega)$ has the form[7]

$$H_L(\Omega) = \exp \{ - 3.44(\lambda\Omega/r_0)^{5/3}\} , \quad (7)$$

where

$$r_0 = 0.185 [\lambda^2/(C_n^2 z)]^{3/5} \quad (8)$$

is the Fried radius. The image spectrum in a diffraction-bounded system $G_0(\Omega)$ is

$$G_0(\Omega) = G(\Omega) H_0(\Omega) . \quad (9)$$

As follows from Eqs. (5) and (6), the spectrum $G_0(\Omega)$ of the image for a diffraction-bounded system can be reconstructed from a given spectrum $G_L(\Omega)$ of a long-exposure image, if the OTF $H_L(\Omega)$ is known:

$$G_0(\Omega) = G_L(\Omega) \times H_L^{-1}(\Omega) . \quad (10)$$

Hence, the diffraction-bounded image is

$$I_0(\mathbf{r}) = F^{-1} \{G_L(\Omega) \times H_L^{-1}(\Omega)\} . \quad (11)$$

Substituting the spectrum of a long-exposure image $G_L(\Omega)$ with the close spectrum of an averaged one $G_{<>}(\Omega)$ (see Eq. (4)) in Eq. (11), we obtain the restored image

$$I_r(\mathbf{r}) = F^{-1} \{G_{< >}(\Omega) \times H_L^{-1}(\Omega)\} . \quad (12)$$

Finally, using Eqs. (1) and (3), we obtain the restored image $I_r(\mathbf{r})$ in the form

$$I_r(\mathbf{r}) = F^{-1} \left\{ F \left\{ \frac{1}{M} \sum_{\mu=1}^{M} \tilde{I}_\mu(\mathbf{r}) \right\} \times H_L^{-1}(\Omega) \right\} . \quad (13)$$

It should be noted that the restored image $I_r(\mathbf{r})$ does not coincide with the diffraction-bounded one $I_0(\mathbf{r})$. Indeed, the restoration procedure is performed within the frames of the analytical theory,[7] whose validity is not verified experimentally. This theory is developed for the Kolmogorov spectrum of atmospheric turbulence, which is valid only in the inertial interval. In real conditions, the Fried radius r_0 characterizing atmospheric distortions can significantly vary during operation of the adaptive system. To determine the Fried radius, independent measurements of turbulence parameters are necessary. The restored image $I_r(\mathbf{r})$ is calculated from the averaged spectrum $G_{< >}(\Omega)$ which is close but does not coincide with the spectrum $G_L(\Omega)$ of the long-exposure image $I_L(\mathbf{r})$. With increasing sample size M, the deviation of the spectrum $G_{< >}(\Omega)$ from $G_L(\Omega)$ must decrease. However, the time of image restoration also increases with increasing M. On the base of test calculations, it is established that the sample size $M = 5$ is close to optimum in the wide range of problem parameters.

2. QUALITY CRITERION

To calculate the quantitative criterion of quality $\tilde{J}_\mu$ for a random realization of the image, we propose to use the relative value of root-mean-square deviation of intensity distribution in this realization $\tilde{I}_\mu(\mathbf{r})$ from the restored image $I_r(\mathbf{r})$

$$\tilde{J}_\mu = \frac{\int |\tilde{I}_\mu(\mathbf{r}) - I_r(\mathbf{r})|^2 \, d^2\mathbf{r}}{\int I_r(\mathbf{r})^2 \, d^2\mathbf{r}} 100\% . \quad (14)$$

The value $\tilde{J}_\mu$ characterizes the degree of image degradation. From definition of $\tilde{J}_\mu$, it follows that the degradation criterion $\tilde{J}_\mu$ equals zero for the realization $\tilde{I}_\mu(\mathbf{r})$ which is «undistorted[in the sense that it coincides with the restored image $I_r(\mathbf{r})$. On the contrary, degradation $\tilde{J}_\mu$ is 100% for the realization with zero intensity, i.e., black area.

The use of the algorithm developed is illustrated by one of the test objects, namely, the emblem of Moscow State University. One can see sufficiently high quality of the recovered image, in which the long-exposure blurring is eliminated to considerable extent. Under these conditions, the criterion $\tilde{J}_\mu$ is really characterized by the long-exposure blurring of the image, random drift, and distortions of its contour.

Together with the presented definition of the criterion $\tilde{J}_\mu$, its formulation on the base of comparative analysis of image spectra is also possible. However, on our opinion, the criterion $\tilde{J}_\mu$ Eq. (14) more adequately corresponds to natural human recognition of observed distortions.

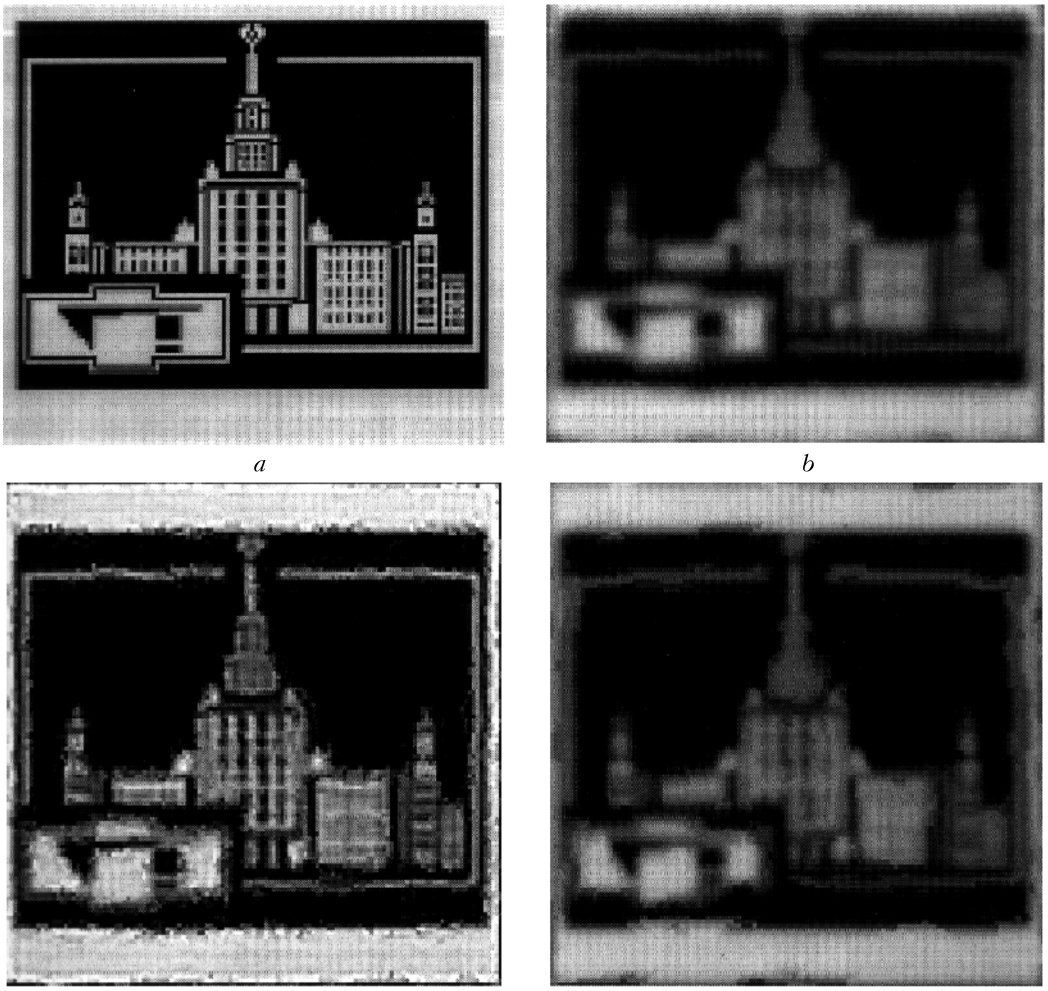

FIG. 1. An example of the use of the algorithm for estimating image degradation: object (a); image averaged over $M = 5$ *realizations (b); restored image (c); realization of a short-exposure image with degradation criterion* $\tilde{J}_\mu = 16.75\%$. *The conditions of the numerical experiment are as follows:* $\lambda = 0.5$ μm; $C_n^2 = 5\cdot10^{-16}$ cm$^{-2/3}$; *object size* $L = 64$ cm; *lens diameter* $d = 10$ cm; *path length* $z = 2$ km; *mesh size* 128×128.

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