Non-hydrostatic model of meso- and microclimate

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A non-hydrostatic model is proposed that takes into account water and air compressibility. The numerical algorithm for solution of the problem is based on the method of splitting into the physical processes and geometrical variables. The model can be used in studying and forecasting meso- and microclimate conditions with the account of anthropogenic effects.

The equations of geophysical hydrodynamics, which express the main principles of conservation of energy, momentum, and mass of a stratified continuum, are accepted in this paper as the basis for mathematical simulation of meso- and microclimate.

A non-hydrostatic model taking into account compressibility of water and air is proposed. The model is most universal from the viewpoint of describing the processes with characteristic horizontal scales less than 100 km. The general principle is a unified theoretical approach to studying the atmosphere and hydrosphere.

The system of differential equations of the nonstationary 3D non-linear model includes:

the equation of motion

$$\frac{d\mathbf{v}}{dt} = -\frac{1}{\rho}\,\mathrm{grad}\,p - 2\boldsymbol{\omega}\times\mathbf{v} + \mathbf{g} + D\mathbf{v}, \qquad (1)$$

the equation of continuity

$$\frac{d\rho}{dt} + \rho\,\mathrm{d}i\mathbf{w} = 0, \qquad (2)$$

the equation of heat influx

$$\frac{dT}{dt} - \frac{\alpha T}{c_p\rho}\frac{dp}{dt} = DT + M_T, \qquad (3)$$

the equation of humidity (salinity) transfer

$$\frac{dq}{dt} = Dq + M_q, \qquad (4)$$

the general form of the equation of state

$$\rho = \rho(p, T, q), \qquad (5)$$

where

$$\frac{d}{dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z};$$

$$D\psi = \frac{\partial}{\partial x}k_{x\psi}\frac{\partial\psi}{\partial x} + \frac{\partial}{\partial y}k_{y\psi}\frac{\partial\psi}{\partial y} + \frac{\partial}{\partial z}k_{z\psi}\frac{\partial\psi}{\partial z}.$$

Here ψ is any of the functions of the considered problem; t is time; u and v are the horizontal components of the medium velocity vector $\mathbf{v}$, and w is its vertical component along the axes of the Cartesian coordinate system (x, y, z); x and y are the horizontal coordinates and the axis z is directed upwards; ρ is the medium density; p is the pressure; T is the temperature; ω is the vector of angular velocity of the Earth's rotation (directed parallel to the Earth's axis toward the North Pole); $k_{x\psi}$, $k_{y\psi}$, and $k_{z\psi}$ are the coefficients of turbulent exchange along the horizontal and vertical directions (assumed to be known functions of coordinates and time); $\mathbf{g}$ is the force of gravity; c_p is the specific heat at a constant pressure; $\alpha = -\rho^{-1}\,\partial\rho/\partial T$ is the coefficient of thermal expansion; q is the mass mixing ratio of water vapor in the air (or salinity for water); M_T is the rate of the heat variation due to radiative heat exchange and phase transitions (liberated latent heat); M_q is the power of the substance's sources or sinks.

Equation (5) can be written for air in the following form:

$$p = R\rho T, \qquad (6)$$

where R is the universal gas constant of dry air. For water, the empirical equation of state connecting density, pressure, and salinity is used.

The heat transfer in soil is described by the equation of heat transfer taking into account the fact that the soil consists of several layers with different thermal properties:

$$\frac{\partial T^*}{\partial t} = \frac{\lambda}{c\rho^*}\frac{\partial}{\partial z}\lambda\frac{\partial T^*}{\partial z}, \qquad (7)$$

where T^*, ρ^*, λ, and c^* are the temperature, density, heat transfer coefficient, and specific heat of the soil, respectively.

Transforming Eqs. (2)–(3) by use of the equation of state, we obtain the evolution equations for T and p. To describe mesoscale processes in the atmosphere, these equations take the form

$$\frac{dT}{dt} = (1-\kappa)\,T\,\mathrm{div}\,\mathbf{v} + \kappa DT + \kappa M_T, \qquad (8)$$

$$\frac{\partial p}{\partial t} + \mathrm{div}\,(p\mathbf{v}) = (1-\kappa)\,p\,\mathrm{div}\,\mathbf{v} + R\kappa\rho DT + R\kappa\rho M_T, \qquad (9)$$

where $\kappa = c_p/c_v$, c_v is the specific heat at a constant volume. For water, these equations are more complicated. Below, the system of equations (1), (4), and (6)–(9) is considered.

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The boundary conditions along the horizontal direction are set as fluxes of momentum, heat, humidity, and mass. At the upper and lower boundaries, conditions of the first kind are set.

Since necessary information about hydrometeorological fields is absent, the initial conditions are replaced by the background (large-scale) values or obtained by solving the corresponding stationary problems.

Following the proposed method and using equation of continuity (2), equations (1), (4), (8), and (9) are transformed to the symmetric form:

$$\frac{\partial U}{\partial t} + BU = -\frac{1}{C}\frac{\partial p}{\partial x} + 2(\omega_z V - \omega_y W) + DU; \quad (10)$$

$$\frac{\partial V}{\partial t} + BV = -\frac{1}{C}\frac{\partial p}{\partial y} + 2(\omega_x W - \omega_z U) + DV; \quad (11)$$

$$\frac{\partial W}{\partial t} + BW = -\frac{1}{C}\frac{\partial p}{\partial z} - Cg + 2(\omega_y U - \omega_x V) + DW; \quad (12)$$

$$\frac{\partial T}{\partial t} + BT = (1.5 - \kappa)\, T \operatorname{div} \mathbf{v} + \kappa DT + \kappa M_T; \quad (13)$$

$$\frac{\partial p}{\partial t} + Bp = (0.5 - \kappa)\, p \operatorname{div} \mathbf{v} + R\kappa\rho DT + R\kappa\rho M_T; \quad (14)$$

$$\frac{\partial q}{\partial t} + Bq = 0.5\, q \operatorname{div} \mathbf{v} + Dq + M_q, \quad (15)$$

where

$$B\psi = \frac{1}{2}\,[\mathbf{v} \operatorname{grad} \psi + \operatorname{div}(\psi \mathbf{v})];$$

$$C = \sqrt{\rho}; \quad U = Cu; \quad V = Cv; \quad W = Cv.$$

The equations of the model are integrated in the Cartesian coordinate system by the method of fictitious domains. Introduction of such domains permits one to perform calculations with an arbitrary function describing the terrain and basins' bottoms.

The difficulty of solving the system of equations is caused by the presence of physical processes with different characteristic temporal scales. So, the numerical algorithm for problem (6) and (10)–(15) is constructed using the method of splitting according to the physical processes.[1]

The problem is solved in three stages at each time step: 1) transfer of substances along some trajectories and turbulent exchange; 2) the process of matching hydrometeorological fields ; 3) calculation of radiation and phase heat influxes. This approach permits one, in principle, to use different time steps at every stage.

At the first stage, the evolution equation of the following form is considered for each of the sought functions:

$$\frac{\partial \psi}{\partial t} + L\psi = 0,$$

where $L = \sum_{m=1}^{3} L_m$.

Complicated problems can be reduced to simpler ones in the cases when the initial positively semi-definite operator can be represented as a sum of positively semi-definite simplest operators.

The time approximation is constructed using the method of component-by-component splitting of the geometrical variables[1]: the grid operator $L^h \ge 0$ is decomposed into simpler operators $L_m^h \ge 0$. The operators $L_m^h \ge 0$ are approximated to the second order of accuracy in coordinates.

Let us take a non-uniform grid with the main node points $x_i = i\Delta x$ $(i = 0, 1,\ldots, I+1)$; $y_j = j\Delta y$ $(j = 0, 1,\ldots, J+1)$; $z_k = k\Delta z_k$ $(k = 0, 1,\ldots, K+1)$; $t_n = n\Delta t$ $(n = 0, 1, \ldots)$ and steps Δx, Δy, Δz_k, Δt.

We also use auxiliary points $x_{i+1/2}$, $y_{j+1/2}$, $z_{k+1/2}$ in the middles of the main intervals. Let us denote:

$$\psi^n_{i,j,k} = \psi(x_i, y_j, z_k, t_n); \quad \Delta_k = (\Delta z_{k+1} + \Delta z_k)/2;$$

$$u_{i+1/2,j,k} = (u_{i+1,j,k} + u_{i,j,k})/2;$$

$$v_{i,j+1/2,k} = (v_{i,j+1,k} + v_{i,j,k})/2;$$

$$w_{i,j,k+1/2} = (w_{i,j,k+1} + w_{i,j,k})/2 \quad (k = 1, 2, \ldots, K).$$

The finite-difference analogs of the operators are as follows:

$$(L_1^h \psi)_{i,j,k} = \frac{u_{i+1/2,j,k}\, \psi_{i+1,j,k} - u_{i-1/2,j,k}\, \psi_{i-1,j,k}}{2\Delta x} - \frac{1}{\Delta x^2}\,[k_{x_{i+1/2,j,k}}(\psi_{i+1,j,k} - \psi_{i,j,k}) - k_{x_{i-1/2,j,k}}(\psi_{i,j,k} - \psi_{i-1,j,k})],$$

$$(L_2^h \psi)_{i,j,k} = \frac{v_{i,j+1/2,k}\, \psi_{i,j+1,k} - v_{i,j-1/2,k}\, \psi_{i,j-1,k}}{2\Delta y} - \frac{1}{\Delta y^2}\,[k_{y_{i,j+1/2,k}}(\psi_{i,j+1,k} - \psi_{i,j,k}) - k_{y_{i,j,\,-1/2k}}(\psi_{i,j,k} - \psi_{i,j-1,k})],$$

$$(L_3\, \psi)_{i,j,k} = \frac{w^n_{i,j,k+1/2}\, \psi_{i,j,k+1} - w^n_{i,j,k-1/2}\, \psi_{i,j,k-1}}{2\Delta_k} - k_{z_{i,j,k+1/2}}\frac{\psi_{i,j,k+1} - \psi_{i,j,k}}{\Delta z_{k+1}\, \Delta_k} + k_{z_{i,j,k-1/2}}\frac{\psi_{i,j,k} - \psi_{i,j,k-1}}{\Delta z_k\, \Delta_k}.$$

Using the Crank–Nicolson scheme at each fractional step $[t_n, t_{n+1}]$, we obtain the splitting algorithm

$$\frac{\psi^{n+m/3} - \psi^{n+(m-1)/3}}{\Delta t} + L_m^h\, \frac{\psi^{n+m/3} + \psi^{n+(m-1)/3}}{2} = 0,$$

$$m = 1, 2, 3.$$

To improve the accuracy of calculations, the bicyclic rearrangement of splitting stages was used.

At the second stage, the system of equations has the following form:

$$\frac{\partial U}{\partial t} = -\frac{1}{C}\frac{\partial p}{\partial x} + 2(\omega_z V - \omega_y W),$$

$$\frac{\partial V}{\partial t} = -\frac{1}{C}\frac{\partial p}{\partial y} + 2(\omega_x W - \omega_z U),$$

$$\frac{\partial W}{\partial t} = -\frac{1}{C}\frac{\partial p}{\partial z} - Cg + 2(\omega_y U - \omega_x V),$$

$$\frac{\partial T}{\partial t} = (1.5 - \kappa)\, T \operatorname{div} \mathbf{v},$$

$$\frac{\partial p}{\partial t} = (0.5 - \kappa)\, p \operatorname{div} \mathbf{v},$$

$$\frac{\partial q}{\partial t} = 0.5\, q \operatorname{div} \mathbf{v}.$$

Note that if one uses explicit finite-difference schemes at this stage, the condition of stability imposes a significant restriction upon the time step ($\Delta t = 0.1$ s for the vertical step of 30 m in the boundary layer of the atmosphere). So, to filter sound waves, we use the implicit finite-difference approximation of the first order of accuracy in time, i.e., the scheme of "natural filter":

$$\frac{U^{n+2}_{i,j,k} - U^{n+1}_{i,j,k}}{\Delta t} = -\frac{p^{n+2}_{i+1/2,j,k} - p^{n+2}_{i-1/2,j,k}}{C^n_{i,j,k}\,\Delta x} + 2(\omega_z V^{n+2}_{i,j,k} - \omega_y W^{n+2}_{i,j,k}); \quad (16)$$

$$\frac{V^{n+2}_{i,j,k} - V^{n+1}_{i,j,k}}{\Delta t} = -\frac{p^{n+2}_{i,j+1/2,k} - p^{n+2}_{i,j-1/2,k}}{C^n_{i,j,k}\,\Delta y} + 2(\omega_x W^{n+2}_{i,j,k} - \omega_z U^{n+2}_{i,j,k}); \quad (17)$$

$$\frac{W^{n+2}_{i,j,k} - W^{n+1}_{i,j,k}}{\Delta t} = -\frac{p^{n+2}_{i,j,k+1/2} - p^{n+2}_{i,j,k-1/2}}{C^n_{i,j,k}\,\Delta_k} - \frac{g}{R}\frac{p^{n+2}_{i,j,k}}{C^n_{i,j,k}\,T^n_{i,j,k}} + 2(\omega_y U^{n+2}_{i,j,k} - \omega_x V^{n+2}_{i,j,k}); \quad (18)$$

$$(p^{n+2}_{i,j,k} - p^{n+1}_{i,j,k})/\Delta t = (0.5 - \kappa)\, p^n_{i,j,k}\, d^{n+2}_{i,j,k}\,, \quad (19)$$

$$(T^{n+2}_{i,j,k} - T^{n+1}_{i,j,k})/\Delta t = (1.5 - \kappa)\, T^n_{i,j,k}\, d^{n+2}_{i,j,k}\,,$$

$$(q^{n+2}_{i,j,k} - q^{n+1}_{i,j,k})/\Delta t = 0.5\, q^n_{i,j,k}\, d^{n+2}_{i,j,k}\,,$$

where

$$d^{n+2}_{i,j,k} = \frac{U^{n+2}_{i+1/2,j,k}/C^n_{i+1/2,j,k} - U^{n+2}_{i-1/2,j,k}/C^n_{i-1/2,j,k}}{\Delta x} + \frac{V^{n+2}_{i,j+1/2,k}/C^n_{i,j+1/2,k} - V^{n+2}_{i,j-1/2,k}/C^n_{i,j-1/2,k}}{\Delta y} + \frac{W^{n+2}_{i,j,k+1/2}/C^n_{i,j,k+1/2} - W^{n+2}_{i,j,k-1/2}/C^n_{i,j,k-1/2}}{\Delta_k}.$$

Substituting the velocity components from Eqs. (16)–(18) into Eq. (19), we obtain the equation for pressure. This equation is solved by the component-by-component splitting method over the coordinates. After solving the equation for pressure, one calculates U, V, W, T, and q. The algorithm is realized by use of the non-monotonic Thomas algorithm.[2]

The constructed finite-difference schemes are quite stable; they are of the first order of approximation in time and the second one in coordinates.

The velocities and turbulence characteristics, which are obtained using the hydrothermodynamic model, are used in calculating gas and aerosol pollutant transfer.[3]

To illustrate the capabilities of the model, let us present the results of numerical simulation for the influence of air flow structure upon the pollutant transport in an urban area. The calculations were performed for the following values of the parameters: the vertical and horizontal steps were 2 m; the time step was chosen so that the Courant criterion was satisfied for the highest velocity of a non-disturbed flow, 10 m/s. Figure 1 presents the pollutant concentration isolines in percent of the highest concentration at the emission point over a not very high building. If the source is situated in a rarefaction zone, the pollutant falls into a leeward area behind a high building and spreads in the direction opposite to the nondisturbed flow. Pollutant concentration can be reduced only with a considerable increase of a stack height what is not practical in this particular case.

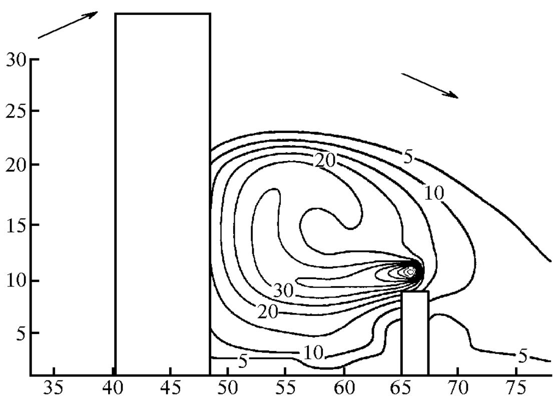

Fig. 1. Influence of the air flow structure upon the pollutant spread in an urban area.

Similar results have been obtained in simulating pollutant spread in a leeward slope when the pollutant plume is kept by a leeward vortex or held down to the Earth by a downward flow if the pollutant source is situated below the vortex zone.

The proposed model can be used in studying and forecasting meso- and microclimate conditions in the presence of anthropogenic factors.

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