TRANSMISSION OF SIGNALS FROM AN ISOTROPIC SOURCE OF OPTICAL RADIATION THROUGH A CLOUD LAYER

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The problem of the impulse response of a system consisting of a point isotropic source of optical radiation, located between a cloud layer andthe underlyingsurface of the earth, and a remote photodetector was studied.

The problem is solved analytically for the first time in an approximate model in which the underlying surface is replaced by a diffusely scattering Lambertian surface and the cloud layer is replaced by a thin diffusely scattering screen with a Lambertian scattering function. The dependence on the altitude of the source and the layer, the albedo of the surface, and the zenith angle of observation is obtained. A general expression for any number of scattering screens is derived.

1. The problem of the propagation of narrow-directed beams of optical radiation from stationary and nonstationary sources through a layer of clouds has been studied In great detail in the literature.[1–7] The distortion of the form of the signal for nonstationary sources of such radiation is caused by multiple scattering of photons by ihomogeneities of the medium. The associated broadening of the pulses is equal to fractions of or several microseconds.[1]

The effect of the scattering layers on signals from an isotropic radiating source has not been studied as well. Here the significantly larger effect of the spread in the trajectories of the light rays, emanating from an isotropic radiating source in different directions and converging once again on the director after being scattered in the cloud layer, is superposed on the multiple scattering effects. The pulse broadening in this case is much larger than for sources with a narrow directional pattern. It depends on the altitude of the cloud cover and can reach tens of microseconds.

In this paper we construct a model of the propagation path of signals from an isotropic source of optical radiation to a detector which is located at a large distance from the source. This model is suitable for problems involving the remote sensing of the earth's surface through a dense cloud layer. We also derive on the basis of this model analytical expressions for the impulse response and the transfer function of the path.

The model is constructed based on the optical-geometric scheme shown in Fig. 1 The clouds are represented by a thin, diffusely scattering, screen S, located above the source S and the underlying surface P at distances h_1 and $h \geq h_1$, respectively. The detector D is placed above the screen at an angle Θ to the vertical and at a distance much greater than the altitude of the cloud cover h. The model can be extended to the case when the stratified cloud layer is represented by several separate screens. The screens and the underlying surface, which reflected radiation incident on them, are regarded as secondary, Lambertian sources of radiation — their brightness does not depend on the observation angle Θ and the flux densities are proportional to cosΘ.

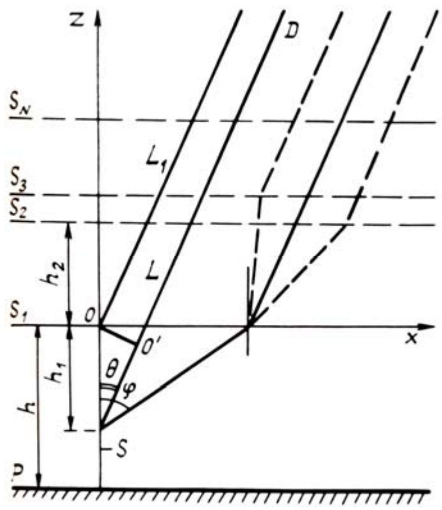

FIG. 1. The optical-geometrical scheme of the system: S is the source; P is the underlying surface; S_1 are the scattering screens.

In constructing the model we employ system methods, which are well know in optics and radio

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electronics, to characterize the signals and transformations. Using these methods we shall regard the propagation path of the signals through a space with scattering layers as a linear stationary (invariant with respect to shifts in time) system, characterized by the impulse response and the transfer function. In the general case we should talk about the point-impulse response $p(\bar{x}, t)$ and the space-time transfer function of the system[11]

$$P(\nu, \omega) = \iiint p(\vec{x}, t) \exp[i(\nu\vec{x} + \omega t)] \, d\vec{x}dt,$$

$$\vec{x} = (x, y). \tag{1}$$

However we shall confine our analysis to the funct ion

$$f(t) = \iint p(\vec{x}, t) \, dx \, dy \tag{2}$$

and

$$F(\omega) = F\{f(t)\} \equiv \int_{-\infty}^{\infty} f(t) \exp(i\omega t) \, dt, \tag{3}$$

which characterize the temporal properties of the system and describe its response to an impulse $\delta(t)$ and a harmonic disturbance $\exp(i\omega t)$ at the input. Here $F\{...\}$ denotes the Fourier transform of the enclosed function and $\delta(t)$ is the Dirac δ function.

The requirement that the detector be far away, which is an important restriction for problems of this class, means that the radiation at the input to the detector can be approximated by a plane-parallel beam.

2. We shall determine the impulse response $f(t)$ of the system represented schematically in Fig. 1. We shall write in the following form the primary pulse generated by the radiation source:

$$W_0(t) = \delta(t). \tag{4}$$

At first we shall neglect the effect of the underlying surface by setting the albedo A of the surface equal to zero.

The problem is divided into three parts:

1) find the Intensity of illumination of the layer by the radiation pulse (4), describing it in the coordinate system of the source by the distribution function $I_0(\bar{x}, t)$; transform $I_0(\bar{x}, t)$ into a form corresponding to the spatiotemporal distribution of the intensity of illumination of the upper boundary of the layer in the coordinate system of the detector; and, 3) calculate the integral

$$f(t) = k \iint I(x, y, t) dxdy, \tag{5}$$

which determines the power of the signal detected by the detector at the time t. The coefficient k in Eq. (5) depends on the properties of the scattering layer, the angle of observation Θ, and the aperture angle of the detector $\Delta\Omega$. For the model, under consideration, with a Lambertian source of secondary radiation and a remote detector

$$k = \Delta \Omega \cos\Theta/\pi. \tag{6}$$

We shall find the distribution $I_0(\bar{x}, t)$ by using the $\cos^3\varphi$ law for the scattering function of the layer for an isotropic point source of incoherent radiation.[12] In accordance with this law the distribution of the radiation intensity on the surface of the screen with $A = 0$ is given by

$$I_0(x, y; h_1) = \frac{h_1}{4\pi} \left(h_1^2 + r^2\right)^{-3/2}, \tag{7}$$

where $r = \sqrt{x^2 + y^2}$ is the distance from the observation point on the surface of the screen to the center of the bright region and

$$\left(1 + r^2/h_1^2\right)^{1/2} = \cos\varphi \tag{8}$$

is the cosine of the angle φ between the normal to the screen and the direction from the source S to the point of the screen with coordinates x, y (Fig. 1). The factor $h_1/4\pi$ on the right side of Eq. (7) was chosen from the condition that $I_0(x, y; h_1)$ be normalized to unit radiation energy in the pulse, since only half of the radiation from the source is scattered by the screen:

$$\iint_{-\infty}^{\infty} I_0(x, y; h_1) dxdy = \frac{h_1}{2} \int_0^{\infty} \frac{rdr}{\left[h_1^2 + r^2\right]^{3/2}} = \frac{1}{2}. \tag{9}$$

Radiation from the source arrives at the point (x, y) with a time delay

$$\tau_0 = \sqrt{h_1^2 + x^2 + y^2/c}. \tag{10}$$

The spatiotemporal distribution of the intensity of illumination of the screen can be written in the form

$$I_0(x, y, t; h_1) = \frac{h_1}{4\pi} \left(h_1^2 + r^2\right)^{-3/2} \times$$

$$\times \delta\left[t - \sqrt{h_1^2 + r^2}/c\right]. \tag{11}$$

In accordance with the thin-screen model chosen the intensity of illumination of the upper boundary of the layer differs from the intensity given in Eq. (11) only by the factor

$$\eta_1 = 1 - R_1 \tag{12}$$

where η_1 is the transmittance and R_1 is the reflectance of the layer. In the coordinate system tied to the detector the distribution of the intensity of the illumination assumes the form

$$I(x,\ y,\ t;\ h_1) = \eta_1 \frac{h_1}{4\pi}\left[h_1^2 + r^2\right]^{-3/2} \times$$
$$\times\, \delta\left[t - \frac{\sqrt{h_1^2 + r^2} - h_1\cos\theta - x\sin\theta}{c}\right]. \tag{13}$$

In writing down this expression we displaced the origin of the time coordinate with respect to the moment the source flashes by an amount L/c, where $L = L + h_1\cos\Theta$ is the distance from the source S to the detector D along the line of sight SD, and c is the velocity of light. The time delay for different trajectories of the signal through the scattering screen is determined in Eq. (13) by the expression

$$\tau = \tau_0 + \frac{L_1 - x\sin\theta}{c} - \frac{L}{c} \tag{14}$$

By substituting Eq. (13) into Eq. (5) we can calculate the response of the system $f_1(t)$ neglecting the underlying surface.

Simple but unwieldy transformations give the expression

$$f_1(t) = k\eta_1 \frac{t_0}{2}\cdot\frac{p}{\left(p^2 + t_0^2\sin^2\theta\right)^{3/2}},$$
$$t \ge 0,\quad t_0 = h_1/c,\quad p = t + t_0\cos\theta. \tag{15}$$

This expression satisfies the normalization condition

$$\int_0^t f_1(t)dt = \frac{k\eta_1}{2}\left[1 - \frac{1}{\sqrt{p^2 + t_0^2\sin^2\theta}}\right] \xrightarrow[t\to\infty]{} \frac{k\eta_1}{2}. \tag{16}$$

Figure 2 shows a family of normalized curves $f_1(t)$, calculated using the formula (15), for angles 8 from 0 to 80°.

3. In order to take into account the effect of the underlying surface on the impulse response of the path it is necessary to know, in addition to the expression (15) found above, the impulse response of the system without the underlying surface $f_1(t)$ to a Lambertian point source located at a distance h from the screen. The response $f_1(t)$ is calculated by the same method as for $f_1(t)$, with the only difference being that the scattering function of the layer of free space for a Lambertian source depends on the angle φ as the fourth power, rather than third power, of the cosine.[12]

The results can be written in the form

$$f_1(t) = k\eta_1\left[\frac{C_1}{\left(p^2 + t_0^2\sin^2\theta\right)^{3/2}} + \frac{C_2}{\left(p^2 + t_0^2\sin^2\theta\right)^{5/2}}\right]$$
$$t \ge 0, \tag{17}$$

where $C_1 = t_0^2(2 + \sin^2\Theta)$, $C_2 = -3t_0^4\sin^2\Theta$.

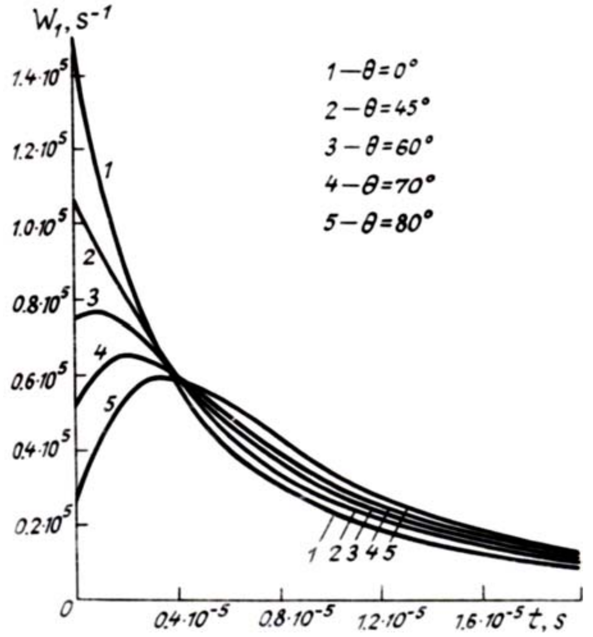

FIG. 2. The normalized impulse response for $A = 0$, $h_1 = 2$ km, and $W_1(t) = f_1(t)\Big/\int_0^\infty f_1(t)dt$.

We shall employ below the normalized functions

$$f_1(t,\ h) = \frac{t_0}{2}\cdot\frac{p}{\left(p^2 + t_0^2\sin^2\theta\right)^{3/2}} \tag{18a}$$

and

$$f_1(t,\ h) = \frac{C_1}{\left(p^2 + t_0^2\sin^2\theta\right)^{3/2}} + \frac{C_2}{\left(p^2 + t_0^2\sin^2\theta\right)^{5/2}} \tag{18b}$$

which differs from $f_1(t)$ and $f_1(t)$ by the normalization factor $k\eta_1$ and by including the parameter h explicitly in the designation of the function. In Eqs. (18)

$$t_0 = h/c,\quad p = t + t_0\cos\theta,\quad t \ge 0. \tag{19}$$

4. The effect of the underlying surface on the impulse response of the system is determined by the contribution of the radiation reflected from the sur-

face to the luminosity of the screen. Neglecting the reflection of waves between the screen and the underlying surface. this contribution is given by the formula

$$I_1(x,\ y,\ t;\ h_1;\ h) = \eta_1 \frac{h - h_1}{4\pi} A \frac{h^2}{\pi} \times$$
$$\times \iint dx' dy' \left[(h - h_1)^2 + x'^2 + y'^2\right]^{-3/2} \times$$
$$\times \left[h^2 + (x - x')^2 + (y - y')^2\right]^{-2} \times$$
$$\times \delta[t - \tau(x,\ y;\ x',\ y')]\ . \qquad (20)$$

where

$$\tau = \frac{\sqrt{(h - h_1)^2 + x'^2 + y'^2}}{c} +$$
$$+ \frac{\sqrt{h^2 + (x - x')^2 + (y - y')^2} - x\sin\theta - h_1\cos\theta}{c}. \qquad (21)$$

It is convenient to rewrite the last expression in the form

$$\tau = \tau_1(x',\ y') + \tau_2(x - x',\ y - y') + \Delta, \qquad (22)$$

where

$$\tau_1 = \frac{\sqrt{(h - h_1)^2 + x'^2 + y'^2}}{c} -$$
$$- \frac{x'\sin\theta + (h - h_1)\cos\theta}{c};$$
$$\tau_2 = \frac{\sqrt{h^2 + (x - x')^2 + (y - y')^2}}{c} -$$
$$- \frac{(x - x')\sin\theta + h\cos\theta}{c}; \qquad (23)$$
$$\Delta = 2(h - h_1)\cos\theta/c.$$

The contribution of the underlying surface to the impulse response of the system can be written, according to Eqs. (5) and (20) in the form

$$f_1(t) = k\eta_1 A f_1(t - \Delta;\ h - h_1) * f_1(t;\ h) \qquad (24)$$

and the impulse response itself can be written in the form

$$f(t) = k\eta_1 \Big[f_1(t;\ h_1) +$$
$$+ A f_1(t - \Delta;\ h - h_1) * f_1(t;\ h)\Big]. \qquad (25)$$

where the simple * denotes the convolution of the functions with respect to the variable t; $f_1(t - \Delta;\ h - h_1)$ denotes the function

$$f_1\left[t - \Delta;\ h - h_1\right] = \begin{cases} \dfrac{t_0}{2} \cdot \dfrac{p}{\left[p^2 + t_0^2\sin^2\theta\right]^{3/2}}, & t > \Delta \\ 0, & t < \Delta \end{cases} \qquad (26)$$

$$p = t - \Delta + t_0\cos\theta,\quad t_0 = (h - h_1)/c.$$

In deriving Eq. (24) we employed the identity[9]

$$\delta(t - \tau_1 - \tau_2 - \Delta) = \delta(t - \tau_1 - \Delta) * \delta(t - \tau_2) \qquad (27)$$

The reduction of the calculation of the integral (5) from the function (20) to the convolution (24) of two impulse functions $f_1(t - \Delta;\ h - h_1)$ and $f_1(t;\ h)$ is made possible by the fact that after substituting the formula (2) the integrand factorizes with respect to the variables $(x,\ y)$ and $(x',\ y')$.

The contribution of reflection of waves between the screen and the underlying surface to the impulse response of the system can be calculated using the same scheme. When these effects are taken into account the impulse response $f(t)$ is obtained by convolving the expression (25) with the function

$$S_1(t) = \delta(t) + \sum_{k=1}^{\infty} \Phi_k(t - k\Delta_1;\ h), \qquad (28)$$

where $\Phi_k(t;\ h)$ is the convolution of k identical functions $\Phi(t;\ h)$ of the form

$$\Phi(t;\ h) = AR_1 f_1(t;\ h) * f_1(t;\ h); \qquad (29)$$

Δ_1 is the time delay of the reflected pulse and is given by the formula

$$\Delta_1 = 2h\cos\theta/c. \qquad (30)$$

The family of curves $f(t)$, constructed from the formula (25) taking into account the reflection of waves between the screen and the underlying surface, is presented in Fig. 3.

The effect of additional cloud layers, approximated by screens, can be taken into account similarly. The impulse response of a system of N scattering screens (Fig. 1), taking into account reflection of waves between the first screen and the underlying surface, can be written in the form

$$f(t) = k\eta S(t) * S_1(t) * S_2(t), \qquad (31)$$

here

$$S(t) = f_n(t;\ h_1) + A f_1(t - \Delta;\ h - h_1) * f_1(t;\ h); \qquad (32)$$

$S_1(t)$ is given by Eq. (28); $S_2(t)$ is given by the formula

$$S_2(t) = f_1(t;\ h_2) \bullet \ldots \bullet f_1(t;\ h_N), \tag{33}$$

where h_2 is the distance between the first and the ith additional screens ($i = 2, 3, \ldots, N$); and, η is the resulting transmittance of N layers

$$\eta = \prod_1^N \eta_1. \tag{34}$$

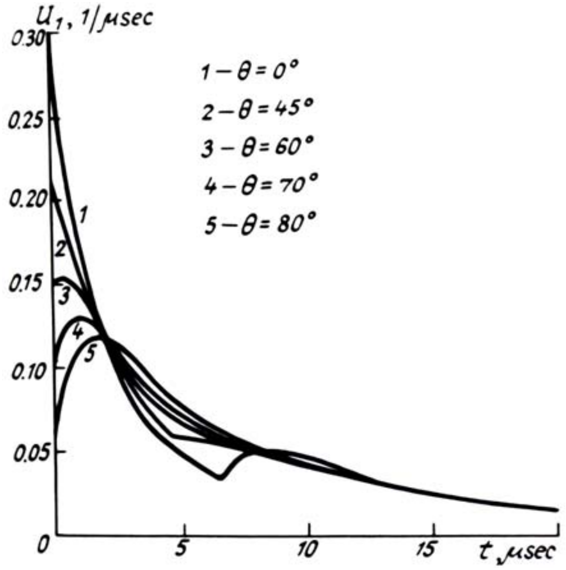

FIG. 3. The normalized impulse response for $A = 0.35$, $h = 3$ km, *and* $h_1 = 1$ km.

5. The transfer function F(ω) is determined as the Fourier transform of the impulse response $f(t)$ (see the formula (3)). The transfer into the frequency domain simplifies the analytical description of the system, and makes it possible simpler operation of multiplying the Fourier transform,[9]

$$F\Big\{ f_1(t) \bullet f_2(t) \Big\} = F\Big\{ f_1(t) \Big\} \cdot F\Big\{ f_2(t) \Big\}$$

and to use the formula for a geometric progression to sum the series:

$$1 + r + r^2 + \ldots + r^n = \frac{1 - r}{1 - r} \xrightarrow[n\to\infty]{} \frac{1}{1 - r},\ r < 1. \tag{35}$$

Using Eqs. (34) and (35) we obtain from Eq. (31) the following expression for the transfer function of the path:

$$F(\omega) = \frac{k\eta \prod\limits_{i=2}^{N} F_1(\omega;\ h_1)}{1 - R_1 A\left[F_1(\omega;\ h)\exp\left(i\omega \frac{h}{c}\cos\theta\right)\right]^2} \times$$

$$\times \left\{F_1(\omega;\ h_1) + A\, F_1(\omega;\ h - h_1)\, F_1(\omega;\ h) \times \right.$$

$$\left. \times \exp\left[i2\omega\left(\frac{h - h_1}{c}\right)\cos\theta\right]\right\}, \tag{36}$$

where $F_1(\omega;\ h)$; and $F_1(\omega;\ h)$ are the Fourier transform of the functions (18). The form of these functions is presented in Figs. 4 and 5.

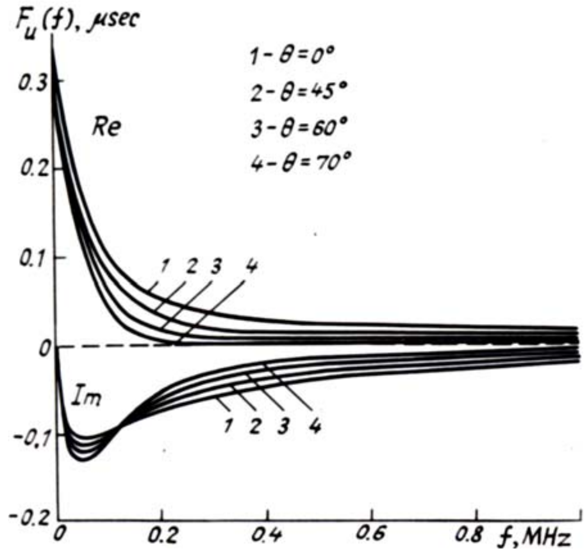

FIG. 4. The transfer function $F_1(f)$. Re *and* Im *denote the real and imaginary parts, respectively, and* $f = \omega/2\pi$.

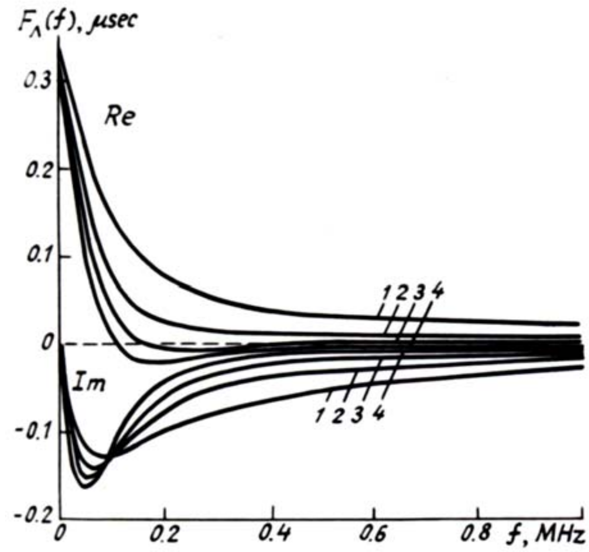

FIG. 5. The transfer function $F_1(f)$. Re *and* Im *denote the real and imaginary parts, respectively, and* f = ω/2π.

6. The model constructed in this paper for the path is based on replacing the real cloud layer by a thin diffusely scattering screen, together with the upper boundary of the layer, or a series of such screens, representing a layered cloud structure. It is assumed that the brightness of the screens does not depend on the observation angle, and that in each sufficiently small region it is proportional to the intensity of illumination of the screen by the sources. It is also assumed that the time dependences of the input signal are identical everywhere on the screen. The spread of the photons along each ray direction separately, which always exists owing to multiple scattering, is neglected.

These assumption agree poorly with another. The assumption that the screen a Lambertian surface is justified if the optical thickness of the cloud layer simulated by the screen is large. But, for large optical thickness of the layer it is not obvious that the spread of the photon path along separate ray trajectories can be neglected. The applicability of the assumptions made can be checked in each specific case only by a physical experiment under natural conditions or by the methods of computer simulation.

The model can be improved by using known solutions of the problem of radiation transfer through an optically thick medium with anisotropic scattering. The strong elongation of the scattering phase function of water droplets in a cloud layer makes it possible to use the smal1-angle approximation of the theory of radiation transfer. However a different, phenomenological (or engineering) approach to constructing a realistic model of the path can also be used. It consists of introducing into the formulas derived above the possible deviation of the characteristics of the screen from the characteristics of a Lambertian radiator and possible spreading of the pulse on each ray trajectory separately, and then estimating the numerical values of these parameters from the experimental results.

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