Determining parameters of the spectrum of turbulent medium refractive index fluctuations

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A method is proposed for determining the parameters of the spectrum of turbulent medium refractive index fluctuations. Experimental data for laser beams of different diameter and wavelengths are used for this purpose. The relation between the variance of the beam centroid displacements and the correlation function of intensity fluctuations in the observation plane is employed. The method consists in estimating the correlation function of intensity fluctuations and beam wandering in the observation plane and solving the set of equations written based on analytical relationship between the above parameters.

Introduction

The radiation passed though a path in a turbulent medium bears information about the medium. This information can be obtained by analyzing fluctuations of the radiation parameters. Such an approach to studying turbulent media has some advantages over contact methods. Actually, optical radiation does not change characteristics of the medium as well as causes no redistribution of the energy among turbulent vortices if the power density of a light beam is low, whereas any contact sensor introduces distortions into the turbulent medium. Optical methods of measuring the parameters of turbulent atmosphere originate from Refs. 1 and 2 and have a lot of modifications.[3]

Thus, for example, the inner scale of atmospheric turbulence is estimated from the measured variance of intensity fluctuations with the use of data of independent measurements of C_n^2. Another method of estimating the inner scale of turbulence is based on measurements of the spatial correlation function of the amplitude logarithm of a plane optical wave. The inner scale is estimated from the ratios of the variance of fluctuations in narrow spectral intervals. The structure characteristic C_n^2 of the refractive index is determined from measurements of the variance of optical radiation intensity under conditions of weak fluctuations. The main source of errors is the possible influence of the inner scale. This influence can be excluded using specialized methods. There also exist numerous techniques not mentioned here.

The main disadvantage of such methods is that the turbulence parameters are determined separately, although measured values depend on the whole set of parameters of a turbulent medium.

In this paper, we propose a method for determining simultaneously the whole set of parameters of the spectrum of the dielectric constant fluctuations of a turbulent medium. The accuracy of such estimation is, at least, no worse, because this method employs more accurate relationship between the measured parameter σ_c^2 and the turbulence spectrum than the relationships mentioned in Ref. 3. In addition, this relationship includes dependence on the three parameters of the spectrum (C_n^2, l_0, L_0) and allows for the properties of the beam (correlation function of intensity distribution in the observation plane).

Assumptions and equations used

To determine the parameters of the spectrum we propose to make use of the experimental data for laser beams of different diameters and wavelengths. The method employs the relationship between the variance of beam centroid wandering and the correlation function of intensity fluctuations in the observation plane. The method consists in estimating the correlation function of intensity fluctuations and beam wandering in the observation plane from the experimental data and solving the set of equations written on the basis of analytical relationship between the above two parameters.

In the approximation of δ-correlated random inhomogeneities of the refractive index and the Markovian random process, the variance of random wandering of the centroid of a beam propagating along the Z-axis has the following form[4]:

$$\sigma_c^2 = \frac{\pi}{2P_0^2}\int\limits_0^L d\xi (L-\xi)^2 \int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty} \Phi_\varepsilon(\mathbf{p},\xi)\,\mathbf{p}^2 F(\mathbf{p},\xi)\,d^2\mathbf{p}, \quad (1)$$

where

$$F(\mathbf{p},\xi) = \int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty} d^2\mathbf{R}_1 \int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty} d^2\mathbf{R}_2 \exp\{i\mathbf{p}(\mathbf{R}_1-\mathbf{R}_2)\} B(\mathbf{R}_1,\mathbf{R}_2,\xi);$$

$B(\mathbf{R}_1, \mathbf{R}_2, \xi) = \langle I(\xi, \mathbf{R}_1) I(\xi, \mathbf{R}_2)\rangle$ is the correlation function of the intensity distribution in the observation

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plane ξ; $\Phi_\varepsilon(\mathbf{p}, \xi)$ is the spectrum of the dielectric constant fluctuations; P_0^2 is the beam power; L is the distance to the observation plane normal to the beam propagation direction.

Equation (1) allows determining the parameters of the spectrum of fluctuations $\Phi_\varepsilon(\mathbf{p}, \xi)$ from the known variance σ_c^2 and the function $F(\mathbf{p}, \xi)$.

The variance of the beam centroid wanderings σ_c^2 can be easily estimated from the experimental intensity distributions in the observation plane. To estimate the function $F(\mathbf{p}, \xi)$, we can use the approximation

$$F(\mathbf{p}, \xi) \approx F(\mathbf{p}, 0) \text{ or } F(\mathbf{p}, \xi) \approx F(\mathbf{p}, L), \quad (2)$$

i.e., the function $F(\mathbf{p}, \xi)$ is estimated in the initial plane $\xi = 0$ or the observation plane $\xi = L$. For the localized turbulent zone, this approximation is quite well acceptable.[4]

From Eq. (1), the equation follows for estimating the parameters of the $\Phi_\varepsilon(\mathbf{p}, \xi)$ spectrum:

$$\acute{\sigma}_c^2 = \frac{\pi}{2P_0^2} \int_0^L d\xi (L-\xi)^2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \Phi_\varepsilon(\mathbf{p}, \xi)\, \mathbf{p}^2 \acute{F}(\mathbf{p}) d^2\mathbf{p}. \quad (3)$$

Here $\acute{\sigma}_c^2$ is the estimate of σ_c^2, and $\acute{F}(\mathbf{p})$ is the estimate of $F(\mathbf{p}, \xi)$.

The number of these equations is equal to the number of experiments with different laser beams. It is clear that selection of beam parameters is important, because medium inhomogeneities have different effects on different beams. Actually, the effect of small-scale inhomogeneities on the wandering of large-diameter beams is weaker than on the small-diameter beams. For every experiment, the estimate $\acute{\sigma} i_c^2$ of the variance σi_c^2 should be obtained from statistical processing, as well as the estimate $\acute{F}i(\mathbf{p})$ of the function $Fi(\mathbf{p})$. The estimates of the parameters of the $\Phi_\varepsilon(\mathbf{p}, \xi)$ spectrum can be obtained from minimization of the discrepancy

$$\text{New}(l_0, L_0, C_n^2) = \sum_{i=1}^{N} \left\{ \sigma i_c^2 - \frac{\pi}{2P_0^2} \times \int_0^L d\xi (L-\xi)^2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} d^2\mathbf{p}\, \Phi_\varepsilon(\mathbf{p}, \xi) \mathbf{p}^2 \acute{F}i(\mathbf{p}) \right\}^2, \quad (4)$$

where L_0 and l_0 are the outer and the inner scales of turbulence; N is the number of measurements of the mean characteristics corresponding to different beam diameters and wavelengths.

The discrepancy is minimized numerically, and there is no need to seek the exact minimum, because both σi_c^2 and $Fi(\mathbf{p})$ are estimated in the experiment with some error. Therefore, all the values of the parameters l_0, L_0, and C_n^2 that give the discrepancy corresponding to the measurement errors of these parameters can be considered as solutions. Therefore, the experimental error should necessarily be estimated.

Numerical analysis of the method

To analyze the proposed method, we have conducted numerical experiments on determining the parameters of the spectrum for the case of a homogeneous and isotropic turbulent medium with the Karman spectrum

$$\Phi_\varepsilon(\mathbf{p}, \xi) = \Phi_\varepsilon(\mathbf{p}) = = 4AC_n^2 \exp(-l_0/5.92)/(p^2 + 1/L_0^2)^{11/6}, \quad (5)$$

where

$$A = \frac{1}{(2\pi)^2} \Gamma(8/3) \sin(\pi/3).$$

In the numerical experiment, we took the following values of the parameters $C_n^2 = 5.3 \cdot 10^{-10}$ m$^{-2/3}$, the inner scale $l_0 = 0.2$ cm, and the outer scale $L_0 = 2.3$ cm. The laser beams with the wavelength $\lambda = 1.06$ μm had the diameter of 0.05, 0.5, and 1.5 cm with a round aperture and homogeneous intensity distribution. For these beams and the selected parameters of the turbulent medium, we have calculated the variance of centroid wanderings by Eq. (1). Then, for different errors, we sought the parameters of the spectrum by minimizing the discrepancy (4) in the 3D domain of the parameters variation:

l_0 from 0.1 to 1 cm, L_0 from 1 to 16 cm,
C_n^2 from $1.8 \cdot 10^{-11}$ to $6.8 \cdot 10^{-11}$ m$^{-2/3}$.

The estimation error was characterized by the relative standard deviation in percent:

$$\Delta\sqrt{\acute{\sigma}_c^2} = \frac{\left|\sqrt{\acute{\sigma}_c^2} - \sqrt{\sigma_c^2}\right|}{\sqrt{\sigma_c^2}} 100\%.$$

At a 5% error, this method gives, under the conditions chosen, a compact domain of possible solutions.

The beam size affects the size of the domain of acceptable solutions (see Fig. 1).

The solution corresponding to the error $\Delta\sqrt{\acute{\sigma}_c^2} = 5\%$ is plotted in Fig. 1. Figure 1*a* shows the domain of solutions from measured wanderings of the beam 0.1 cm in diameter, Fig. 1*b* shows the same but for the beam diameter of 1 cm, and Fig. 1*c* – for the beam diameter of 3 cm. Joint consideration of measurements for all the beams gives the solution shown in Fig. 1*d*.

Figure 2 demonstrates the effect of the measurement accuracy on the efficiency of reconstructing the parameters of the spectrum. This effect is, certainly, different for different conditions. Its value depends on both the studied spectrum of the medium and the parameters of the laser beams used. Figure 2 shows how the domain of solutions depends on the error of estimation of the beam wanderings.

The efficiency of the method depends significantly on the medium parameters (Fig. 3). It is more difficult to achieve small errors in determination of the parameters the wider becomes the range of the inhomogeneity scales.

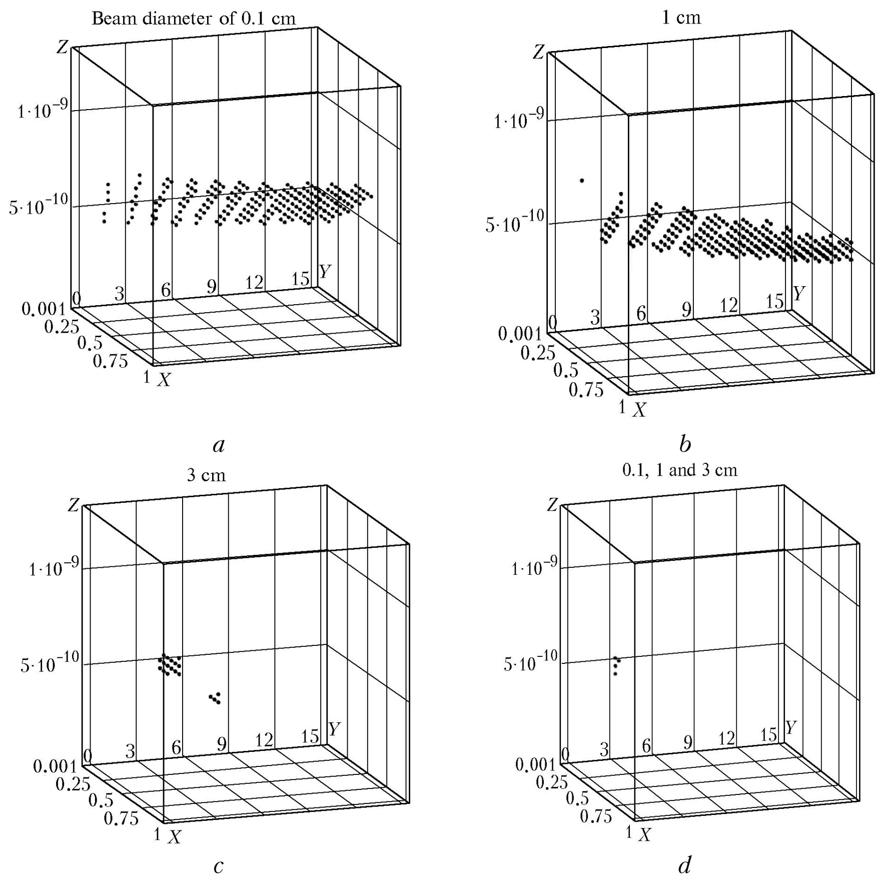


Fig. 1. Domain of acceptable solutions as a function of the beam diameter at l_0 = 0.2 cm, L_0 = 2.3 cm, and $C_n^2 = 5.3 \cdot 10^{-10}$ m$^{-2/3}$, error of 5%. The inner scale of turbulence l_0, in cm, is plotted along the X-axis; the outer scale L_0, in cm, is plotted along the Y-axis, and C_n^2, in m$^{-2/3}$, is plotted along the Z-axis.

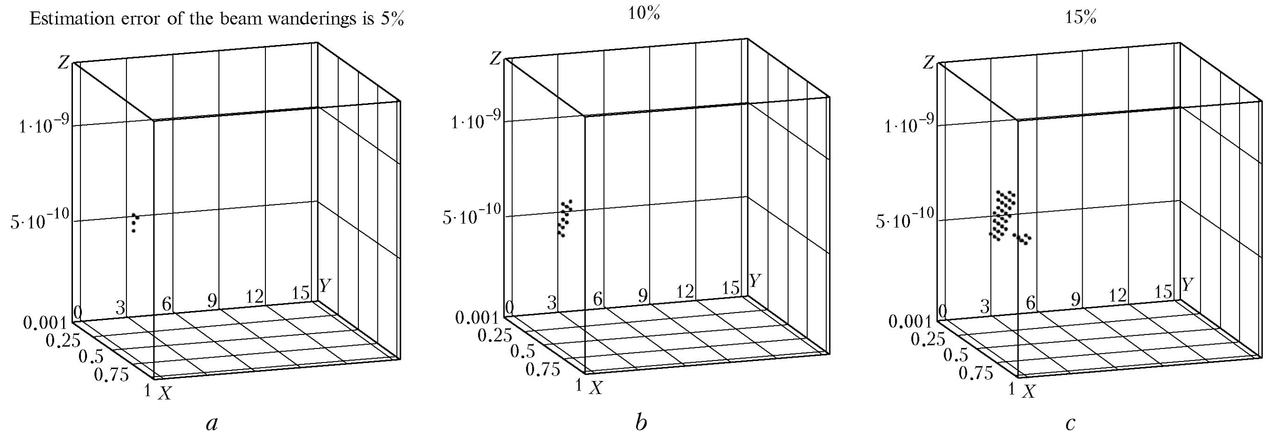


Fig. 2. Domain of solutions as a function of the estimation error of the beam wanderings for beams of 0.1, 1, and 3 cm diameter at l_0 = 0.2 cm, L_0 = 2.3 cm, $C_n^2 = 5.3 \cdot 10^{-10}$ m$^{-2/3}$. The inner scale of turbulence l_0, in cm, is plotted along the X-axis; the outer scale L_0, in cm, is plotted along the Y-axis, and C_n^2, in m$^{-2/3}$, is plotted along the Z-axis.

Actually, to improve the accuracy, one should take laser beams with widely different diameters. However, technically, this is a more complicated problem, and it requires higher accuracy of calculations. For a turbulent medium with different parameters ($C_n^2 = 5.3 \cdot 10^{-10}$ m$^{-2/3}$, the inner scale l_0 = 0.2 cm, and the outer scale L_0 = 100 cm) this technique, at the relative error of 1%, gives, for the beam diameter of 1, 3, and 5 cm, the domains of solutions shown in Fig. 3. The errors of estimation determined by the size of this domain (Fig. 3) are significantly larger than the errors caused only by the medium parameters (Fig. 1). This paper only introduces the proposed method and demonstrates its performance.

The capabilities of the method will be examined in our further investigations.

The measurement technique was tested in the field physical experiments conducted at the Research Institute for Complex Testing of Optoelectronic Devices and Systems (Sosnovyi Bor, Leningrad Region). This experiment was described in Ref. 5. In the experiment, we dealt with the air flow from a commercial air-heater with the exit nozzle diameter of 50 cm. It was the first experiment in a series of experiments on studying turbulence in the exhaust jets of aircraft engines. It was supported by the Defence Evaluation and Research Agency Farnborough (UK). We used the results of this experiment to demonstrate the performance of the proposed method.

Laser beams (with the wavelength $\lambda = 1.06$ μm and diameters of 0.5 and 1.5 cm with the round aperture and homogeneous initial intensity distribution) passed through a 50-cm thick air zone with strong turbulence. The experiment has been conducted when this technique was not yet developed. Therefore, neither beam diameter nor the measurement error was kept constant.

The intensity distribution was measured in the near ($L = 750$ cm) and far zones. Beam centroid wanderings were estimated from 1200 intensity distributions measured for each beam.

The 3D domain, in which the parameters were sought, was the following:

l_0 from 0.08 to 2.5 cm, L_0 from 0.3 to 12 cm,

C_n^2 from $4.0\cdot10^{-10}$ to $43.0\cdot10^{-10}$ m$^{-2/3}$.

The domain of solutions is shown in Fig. 4. Figure 4*a* shows the domain obtained from measured wanderings of the beam of 1 cm diameter, Fig. 4*b* shows similar data but for the beam of 3 cm diameter. The joint consideration of the measurements for both beams gives the solution shown in Fig. 4*c*.

The measurement error has not allowed unambiguous determination of the solution, but the domain of solutions is rather small and the sought parameters are related to each other in a certain way. Thus, for example, at $L_0 = 1.3$ cm the parameter l_0 is estimated as ranging from 0.09 to 0.9 cm at C_n^2 from $6.5\cdot10^{-10}$ to $7.2\cdot10^{-10}$ m$^{-2/3}$; at $L_0 = 1.1$ cm the parameter l_0 is estimated as ranging from 0.09 to 0.9 cm at C_n^2 from $9.7\cdot10^{-10}$ to $10.1\cdot10^{-10}$ m$^{-2/3}$.

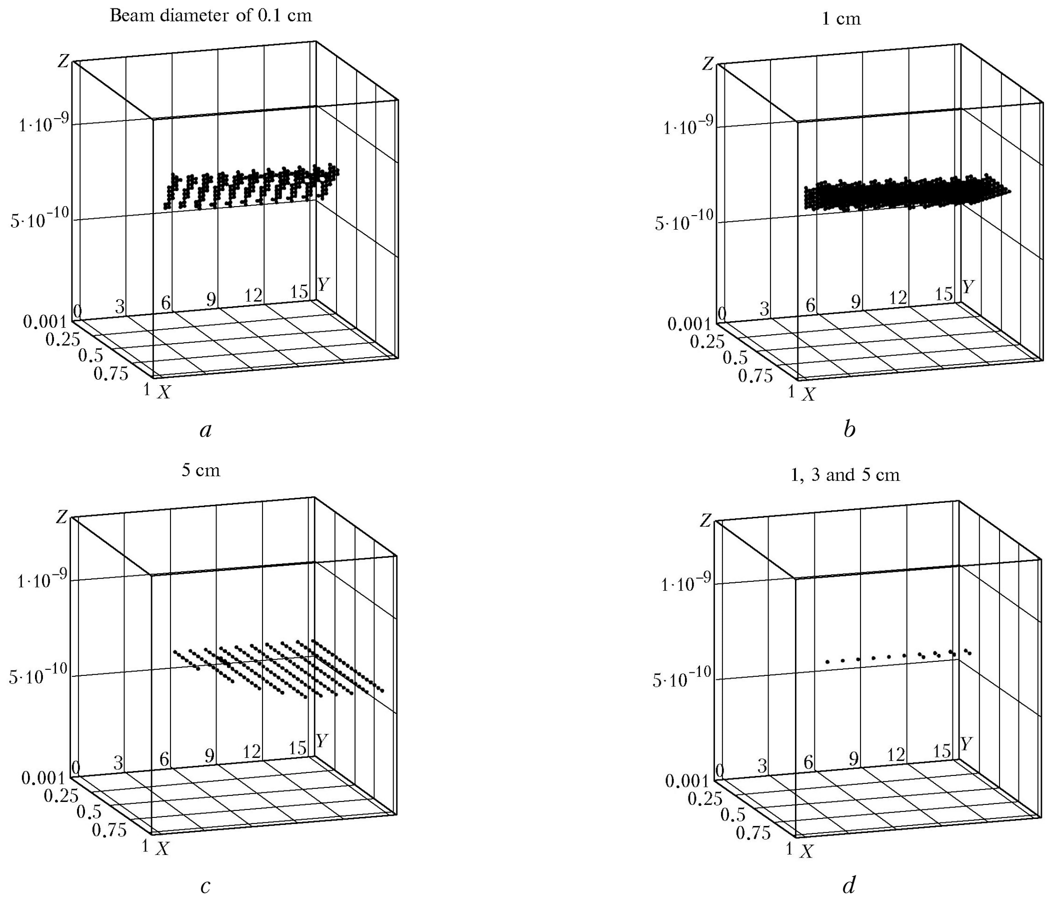
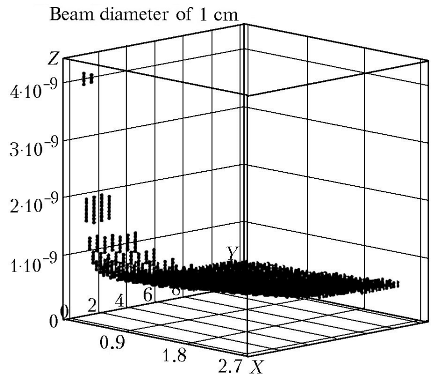


Fig. 3. Domain of solutions as a function of the beam size at the medium parameters: $C_n^2 = 5.3\cdot10^{-10}$ m$^{-2/3}$, the inner scale $l_0 = 0.2$ cm, the outer scale $L_0 = 100$ cm, $\Delta\sqrt{\sigma_c^2} = 1\%$. The inner scale of turbulence l_0, in cm, is plotted along the X-axis; the outer scale L_0, in cm, is plotted along the Y-axis, and C_n^2, in m$^{-2/3}$, is plotted along the Z-axis.



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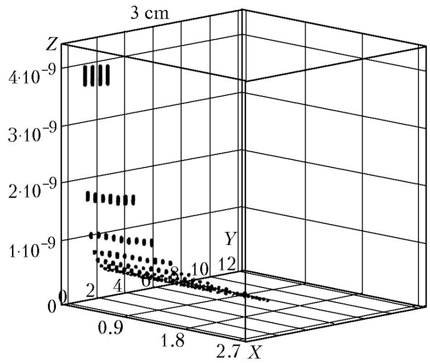

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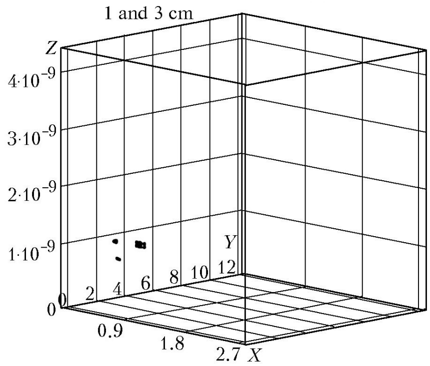


c

Fig. 4. Domain of solutions for the field experiment. The inner scale of turbulence l_0, in cm, is plotted along the X-axis; the outer scale L_0, in cm, is plotted along the Y-axis, and C_n^2, in $m^{-2/3}$, is plotted along the Z-axis.

To check the correctness of the determined parameters, we have compared the mean experimental intensity distribution in the far zone with the mean distribution calculated for the 50-cm thick turbulent zone and with the parameters from the domain of solutions. The results of this comparison are shown in Fig. 5.

The intensity distribution in the far zone in the case of a homogeneous beam limited by a circle of the radius a_0 was calculated by the equation

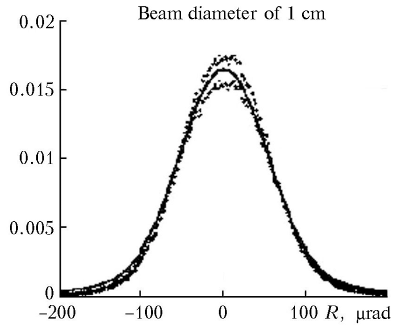
$$<I(\mathbf{R}, L)> = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \gamma_0(\mathbf{p}, 0, L) \exp[-\frac{1}{2} Ds(\mathbf{p}, 0, L) + i\mathbf{Rp}]d^2p, \quad (6)$$

where $\gamma_0(\mathbf{p}, 0, L) = \gamma(\mathbf{p}, -\frac{\mathbf{p}}{k} L, 0)$ is the spectrum of the coherence function, which would be observed in the plane Z in the absence of turbulence;

$$Ds(\mathbf{p}, \mathbf{r}, L) = 2\pi k^2 \int_0^L Hn[\mathbf{r} - \frac{\mathbf{p}}{k}(L - \zeta), \zeta]d\zeta;$$

$$Hn(\mathbf{r}, z) = \frac{1}{2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} [1 - \cos(\mathbf{rp})]\Phi_\varepsilon(\mathbf{p}, 0, z)d^2p;$$

$k = 2\pi/\lambda$ is the wave number.



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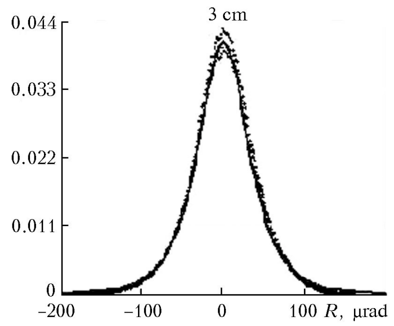

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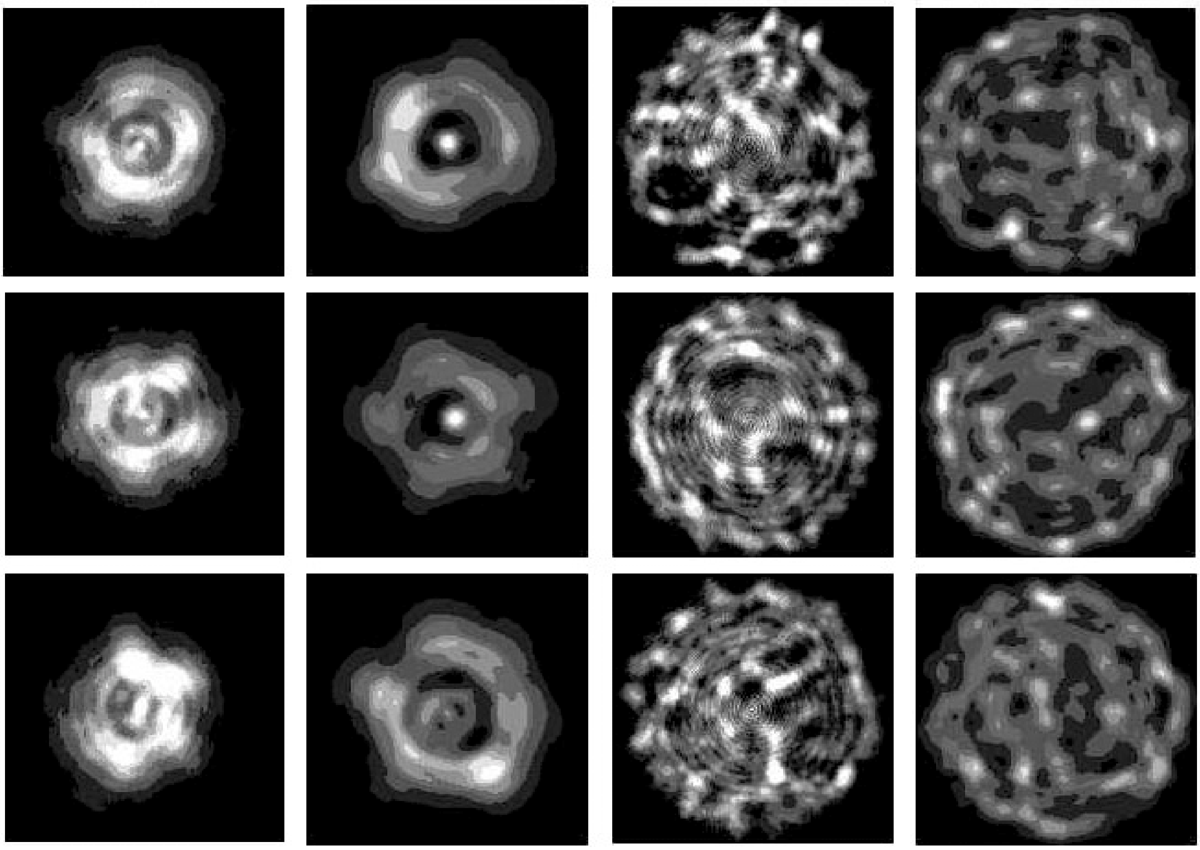
Fig. 5. Comparison of the mean experimental intensity distribution and the distribution calculated by Eq. (6) with the following parameters $C_n^2 = 4.7 \cdot 10^{-10}$ $m^{-2/3}$, the inner scale $l_0 = 0.25$ cm, the outer scale $L_0 = 1.5$ cm: experimental (⋯⋯⋯) and calculated (———) distributions.

Figure 6 compares experimental realizations of the mean intensity distribution with the realizations calculated by numerical simulation for the 50-cm thick turbulent medium with the following parameters: $C_n^2 = 4.7 \cdot 10^{-10}$ $m^{-2/3}$, the inner scale of turbulence $l_0 = 0.25$ cm, the outer scale $L_0 = 1.5$ cm, by the technique described in Ref. 6.

The comparison suggests that for this field experiment the model of a homogeneous isotropic medium is suitable within some accuracy limits (at least for beams with the wavelength $\lambda = 1.06$ μm and the diameter of 0.5 and 1.5 cm), and the proposed method allows one to determine the parameters of this model.

Beam diameter of 1 cm Beam diameter of 3 cm



Experiment Numerical simulation Experiment Numerical simulation

Fig. 6. Comparison of the field and numerical experiments, near zone.

Conclusion

The proposed technique allows determination of the parameters of a turbulent medium from the intensity distributions of laser beams passed through this medium. The technique is suitable for measuring turbulence of media, for which Eq. (1) is valid. The accuracy of the method has been studied numerically for the case that the medium is homogeneous, isotropic, and can be replaced by a phase screen,[4] but the method can be expanded to a more complex parametric case.

The proposed technique allows reliable determination of the parameters of the medium spectrum with a relatively narrow range of inhomogeneities. In the general case, the parameters C_n^2 and l_0 are determined reliably, and the possibility of estimating the outer scale is limited, to a certain degree, by technical problems, because this requires large-aperture lasers to be used.

The success of the practical use of this method depends on the solution of some problems, such as selection of parameters of the sensing beams, accuracy of the wandering magnitude estimation, method of estimation of the functions $F(\mathbf{p})$, and selection of the algorithm for solution of the set of equations.

We succeeded in solving these problems for the case of a localized zone of homogeneous and isotropic turbulence.

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References

1. A.S. Gurvich, Izv. Akad. Nauk SSSR, Ser. Fiz. Atmos. Okeana **4**, No. 2, 160–169 (1968).
2. A.M. Obukhov, Izv. Akad. Nauk SSSR, Ser. Geofiz., No. 2, 155–165 (1953).
3. V.E. Zuev, V.A. Banakh, and V.V. Pokasov, *Optics of the Turbulent Atmosphere* (Gidrometeoizdat, Leningrad, 1988), 270 pp.
4. A.S. Gurvich, A.I. Kon, V.L. Mironov, and S.S. Khmelevtsov, *Laser Radiation in the Turbulent Atmosphere* (Nauka, Moscow, 1976), 276 pp.
5. Yu.N. Evchenko, V.N. Pasunkin, V.S. Sirazetdinov, A.D. Starikov, D.H. Titterton, T.A. Sheremet'eva, and G.N. Filippov, Optich. Zh. **65**, No. 12, 106–112 (1998).
6. T.A. Sheremet'eva and G.N. Filippov, Atmos. Oceanic Opt. **13**, No. 5, 492–496 (2000).