Propagation of radiation with partial spatial coherence in a transverse bounded randomly inhomogeneous gain medium

F.A. Starikov

Russian Federal Nuclear Center, Sarov, Nizhnii Novgorod Region

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Propagation of laser radiation with partial spatial coherence in a laterally bounded optically inhomogeneous extended gain medium is studied by use of quasi-optical equation for a transverse correlation function of field amplitude. The approach used allows for the radiation diffraction, refraction on a regular profile of dielectric constant, regular gain, scattering by fluctuations of dielectric constant and gain, scattering by random axial (hose-type) displacements. It is shown that fluctuations of the gain coefficient lead to additional regular gain of spontaneous emission, which dominates over the scattering by the fluctuations. Under the condition of small-scale hose-type fluctuations and low amplification within their longitudinal correlation length, fluctuations of the gain coefficient can hardly affect the intensity of amplified spontaneous emission, whereas their effect on coherence can be stronger.

1. Introduction

Propagation of radiation in a laterally bounded medium has some salient features as compared with the case of statistically homogeneous medium (for instance, in free propagation in the atmosphere) depending on boundary conditions at the side surface. For instance, in the case of an extended freely spreading amplifying plasma filament, radiation that passes through the side surface of the filament cannot take part in scattering and refraction processes and come back to the plasma channel. However, it can significantly contribute to output distribution of the radiation intensity. With the increase of spontaneous (spatially incoherent) radiation in a randomly inhomogeneous medium, a series of processes compete that often are antipodal. On the one hand, spatial selection of radiation in the amplifying channel leads to improvement of coherence and sharpening of directional pattern of radiation while, on the other hand, scattering of radiation deteriorates the beam's quality. In addition, this is complicated by the presence of regular refraction and its singularities, amplification inhomogeneity, the presence of non-resonant absorption, etc. This problem has been thoroughly considered in Ref. 1.

Traditionally, the quasi-optical parabolic equation for the complex amplitude of the radiation field is used to describe propagation of partially coherent laser radiation beams with relatively narrow angular width in optically inhomogeneous gain media. The method of classical geometrical optics is inapplicable here and wave effects must be taken into account, when the diffraction length of radiation on the cross size of optical inhomogeneities is comparable with the propagation path or exceeds it. Realistic situations can be described by the numerical solution of parabolic equation with arbitrary distributions of optical parameters.[2–5] However, fluctuations of the field of the source's radiation make it necessary to perform a lot of numerical calculations of the parabolic equation with averaging of the output radiation parameters over time or ensemble[3–5] what takes considerable time. If one takes into account random parameters of the medium, the method of parabolic equation becomes even more complicated.

To avoid the problem of averaging over ensemble of realizations and find the mean energy and coherence properties of radiation in most simple way, one can pass from the parabolic equation to the equation for the transverse correlation function (TCF) of the complex field amplitude.[6–8] When the transverse length of coherence is much less than the beam diameter, the TCF-method enables one to obtain the numerical results much faster as compared with the method of statistical testing for the parabolic equation.[5] The TCF-method was used in studies of the dynamics of spontaneous radiation in a gain randomly inhomogeneous medium (see, for instance, Refs. 7, 9–15, and the review in Ref. 1). It is important that the equation for TCF can be solved not only numerically but, in contrast to the stochastic parabolic equation, enables one to obtain analytical solutions and estimates in some cases. This helps better understanding of the radiation dynamics.

Main results on the dynamics of spontaneous radiation in an active medium with fluctuations of dielectric constant, which are similar to usual turbulent fluctuations in the atmosphere, were obtained in Refs. 7, and 9–15. Effects of axial inhomogeneity of the medium, i.e., its random transverse displacements, on the amplified coherent radiation were considered in Ref. 16. In this paper, within the frames of the TCF-method, usual and axial fluctuations of the dielectric

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constant and gain coefficient are taken into account simultaneously, and their influence on the amplified spontaneous radiation is compared.

2. The equation for the TCF and its analysis

Dynamics of radiation in an optically inhomogeneous active medium in the quasi-stationary case is described by the parabolic equation for slowly varying complex amplitude of the radiation field E:

$$\left[\frac{\partial}{\partial z}+\frac{i}{2k}\frac{\partial}{\partial \mathbf{r}^2}+\frac{ik}{2}\left[\varepsilon(\mathbf{r},z)-1\right]-\frac{\alpha(\mathbf{r},z)}{2}\right]E(\mathbf{r},z)=0, \quad (1)$$

where $\mathbf{r} = \mathbf{i}x + \mathbf{j}y$ is the transverse radius vector, z is the longitudinal coordinate, k is the wave number, α is the gain coefficient for a weak signal (an important case with nonlinear α is considered in detail in Refs. 1 and 15), ε is the dielectric constant ($\varepsilon \approx 1$).

Equation (1) cannot be solved in its general form. In practice, the direct numerical solution is widely used. The width of the transverse grid is $\Delta x \sim \lambda/\theta$, where θ is the maximum angle of radiation divergence, used in calculations. If a part of optical inhomogeneities is significant, the number of calculation points is large, and the integration step over z $\Delta z \sim k\Delta x^2$ is small. For instance, in the short-wave region, modern calculation tools enable one to consider only the case of a plane medium.[3,4] Besides, the obtained solutions of the parabolic equation should be averaged over an ensemble of field realizations and field parameters what takes unreasonably long time for calculations.

To avoid the problem of averaging over the ensemble of realizations, we obtain the equation for the second moment of field amplitude, i.e., for the transverse correlation function (TCF) $B = \langle E(\mathbf{r}_1, z) \times E^*(\mathbf{r}_2, z)\rangle$ from the stochastic equation (1). The brackets denote statistical averaging. In averaging, we take into account the presence of two types of random optical inhomogeneities. The first type corresponds to "usual" fluctuations of the dielectric constant and gain, which are characterized by spatial spectrum, dispersion, and correlation lengths. These are described similarly to, for instance, fluctuations of the turbulent atmosphere. The second type of fluctuations is connected with random deviations of the medium (e.g., a plasma filament) as a whole from the straight-line shape. They can be called axial or "hose-like"[16] fluctuations. Axial fluctuations can be naturally simulated as the random transverse displacements of regular profiles of the gain coefficient and the refractive index. So, we represent ε and α in the form

$$\varepsilon(\mathbf{r},z) = \bar{\varepsilon}(\mathbf{r}+\tilde{\mathbf{r}},z) + \tilde{\varepsilon}(\mathbf{r},z)\ ;$$

$$\alpha(\mathbf{r},z) = \bar{\alpha}(\mathbf{r}+\tilde{\mathbf{r}},z) + \tilde{\alpha}(\mathbf{r},z)\ ,$$

where $\bar{\varepsilon}$ and $\bar{\alpha}$ are the regular components; $\tilde{\varepsilon}$ and $\tilde{\alpha}$ are fluctuation components; $\tilde{\mathbf{r}}$ is the random transverse displacement of the regular profiles $\bar{\varepsilon}$ and $\bar{\alpha}$. The fluctuations $\tilde{\varepsilon}$, $\tilde{\alpha}$, $\tilde{\mathbf{r}}$ are statistically independent and have zero mean values. Assuming the axial fluctuations to have small transverse scale under conditions that $\nabla_\perp \bar{\varepsilon} \gg L_\perp \nabla_\perp^2 \bar{\varepsilon}$ and $\nabla_\perp \bar{\alpha} \gg L_\perp \nabla_\perp^2 \bar{\alpha}$, we obtain

$$\varepsilon(\mathbf{r},z) \cong \bar{\varepsilon}(\mathbf{r},z) + \tilde{\mathbf{r}}(z)\nabla_\perp \bar{\varepsilon}(\mathbf{r},z) + \tilde{\varepsilon}(\mathbf{r},z)\ ;$$

$$\alpha(\mathbf{r},z) \cong \bar{\alpha}(\mathbf{r},z) + \tilde{\mathbf{r}}(z)\nabla_\perp \bar{\alpha}(\mathbf{r},z) + \tilde{\alpha}(\mathbf{r},z)\ ,$$

where $\nabla_\perp$ is the transverse gradient, $L_\perp$ is the root-mean-square transverse displacement of the medium. For simplicity, the correlation functions $\tilde{\varepsilon}$, $\tilde{\alpha}$, and $\tilde{\mathbf{r}}$ are supposed to be Gaussian:

$$\langle\tilde{\alpha}(\mathbf{r}_1,z)\,\tilde{\alpha}(\mathbf{r}_2,z')\rangle = \sigma_\alpha^2 \exp\left[-r'^2/2l_{\alpha\perp}^2 - (z-z')^2/2l_{\alpha\parallel}^2\right];$$

$$\langle\tilde{\varepsilon}(\mathbf{r}_1,z)\,\tilde{\varepsilon}(\mathbf{r}_2,z')\rangle = \sigma_\varepsilon^2 \exp\left[-r'^2/2l_{\varepsilon\perp}^2 - (z-z')^2/2l_{\varepsilon\parallel}^2\right], \quad (2)$$

$$\langle\tilde{\mathbf{r}}(z)\,\tilde{\mathbf{r}}(z')\rangle = L_\perp^2(z) \exp\left[-(z-z')^2/2L_\parallel^2\right],$$

where σ_ε^2 and σ_α^2, $l_{\varepsilon\perp}$ and $l_{\alpha\perp}$, $l_{\varepsilon\parallel}$ and $l_{\alpha\parallel}$ are the dispersion, transverse, and longitudinal lengths of the correlation $\tilde{\varepsilon}$ and $\tilde{\alpha}$, respectively, $L_\parallel$ is the longitudinal length of the correlation $\tilde{\mathbf{r}}$. If the longitudinal length of coherence of the amplified radiation in the medium significantly exceeds $l_{\varepsilon\parallel}$, $l_{\alpha\parallel}$, and $L_\parallel$, one can use the Markovian approximation.[17] Within the framework of this approach, correlations (3) are approximated by delta functions along z:

$$\langle\tilde{\varepsilon}(\mathbf{r}_1,z)\,\tilde{\varepsilon}(\mathbf{r}_2,z')\rangle = A_\varepsilon(\mathbf{r},\mathbf{r}';z)\,\delta(z-z');$$

$$\langle\tilde{\alpha}(\mathbf{r}_1,z)\,\tilde{\alpha}(\mathbf{r}_2,z')\rangle = A_\alpha(\mathbf{r},\mathbf{r}';z)\,\delta(z-z'); \quad (3)$$

$$\langle\tilde{\mathbf{r}}(z)\,\tilde{\mathbf{r}}(z')\rangle = A_h(z)\,\delta(z-z').$$

By setting equal the integrals over z', with the infinite limits, taken from the correlation functions (2) to those taken from Eqs. (3), we obtain

$$A_\varepsilon(\mathbf{r},\mathbf{r}';z) = (2\pi)^{1/2}\,\sigma_\varepsilon^2(\mathbf{r},z)\,l_{\varepsilon\parallel}(\mathbf{r},z)\exp\left[-r'^2/2l_{\varepsilon\perp}^2(\mathbf{r},z)\right], \quad (4)$$

$$A_\alpha(\mathbf{r},\mathbf{r}';z) = (2\pi)^{1/2}\,\sigma_\alpha^2(\mathbf{r},z)\,l_{\alpha\parallel}(\mathbf{r},z)\exp\left[-r'^2/2l_{\alpha\perp}^2(\mathbf{r},z)\right], \quad (5)$$

$$A_h(z) = (2\pi)^{1/2}\,L_\perp^2(z)\,L_\parallel(z)\ . \quad (6)$$

With the account of Eqs. (3), the equation for TCF takes the form

$$\left[\frac{\partial}{\partial z}+\frac{i}{k}\frac{\partial^2}{\partial \mathbf{r}\,\partial \mathbf{r}'}+\frac{ik}{2}\mathbf{r}'\,\nabla_\perp\,\bar{\varepsilon}(\mathbf{r};z)-\bar{\alpha}(\mathbf{r};z)+\right.$$

$$+ \frac{\pi k^2}{4} H(\mathbf{r}, \mathbf{r}'; z)\Big] B(\mathbf{r}, \mathbf{r}'; z) = 0, \quad (7)$$

where $\mathbf{r} = (\mathbf{r}_1 + \mathbf{r}_2)/2$, $\mathbf{r}' = \mathbf{r}_1 - \mathbf{r}_2$. The transverse differential operator in Eq. (7), as an approximation of a two-dimensional plane medium, has the form

$$\frac{\partial^2}{\partial \mathbf{r}\,\partial \mathbf{r}'} = \frac{\partial^2}{\partial x\,\partial x'},$$

and, in the case of a three-dimensional axially symmetric medium takes a more complicated form

$$\frac{\partial^2}{\partial \mathbf{r}\partial \mathbf{r}'} = \cos\varphi\left(\frac{\partial^2}{\partial r\partial r'} - \frac{1}{rr'}\frac{\partial^2}{\partial \varphi^2}\right) - \sin\varphi\frac{\partial}{\partial \varphi}\left(\frac{1}{r}\frac{\partial}{\partial r'} + \frac{1}{r'}\frac{\partial}{\partial r}\right),$$

where φ is the angle between $\mathbf{r}$ and $\mathbf{r}'$. The influence of random parameters of the medium is defined by the function $H(\mathbf{r}, \mathbf{r}'; z) = H_\varepsilon + H_\alpha + H_h$, where

$$H_\varepsilon(\mathbf{r}, \mathbf{r}'; z) = \frac{A_\varepsilon(\mathbf{r}_1, 0; z) + A_\varepsilon(\mathbf{r}_2, 0; z) - 2A_\varepsilon(\mathbf{r}, \mathbf{r}'; z)}{2\pi}, \quad (8)$$

$$H_\alpha(\mathbf{r}, \mathbf{r}'; z) = \frac{A_\alpha(\mathbf{r}_1, 0; z) + A_\alpha(\mathbf{r}_2, 0; z) + 2A_\alpha(\mathbf{r}, \mathbf{r}'; z)}{2\pi k^2}, \quad (9)$$

$$H_h(\mathbf{r}, \mathbf{r}'; z) = \frac{A_h(z)}{2\pi}\left[\nabla_\perp \bar{\varepsilon}(\mathbf{r}_1, z) - \nabla_\perp \bar{\varepsilon}(\mathbf{r}_2, z) + \frac{i}{k}\nabla_\perp \bar{\alpha}(\mathbf{r}_1, z) + \frac{i}{k}\nabla_\perp \bar{\alpha}(\mathbf{r}_2, z)\right]^2 \cong \frac{A_h(z)}{2\pi}\left[r'\nabla_\perp^2 \bar{\varepsilon}(\mathbf{r}, z) + \frac{2i}{k}\nabla_\perp \bar{\alpha}(\mathbf{r}, z)\right]^2 \quad (10)$$

are responsible for the effect of $\tilde{\varepsilon}$, $\tilde{\alpha}$ and $\tilde{\mathbf{r}}$, respectively. Relations (8)–(10) hold when the average density of the radiation flux $B(\mathbf{r}, 0; z)$ weakly varies at a distance of the order of $l_{\varepsilon\perp}$, $l_{\alpha\perp}$, $L_\perp$ and the transverse length of radiation coherence. The restriction on the propagation path, which is connected with the Markovian approximation and takes place in an infinite randomly inhomogeneous medium,[17] is absent in the case of a transversally bounded medium.[12] The salient features of the numerical method for integrating Eq. (7) have been discussed in Refs. 6 and 18.

Below we suppose that variation of σ_ε and σ_α is small at the distances of the same order of magnitude as the correlation lengths. To perform qualitative analysis of the effect of fluctuations $\tilde{\mathbf{r}}$, let us represent the profiles $\bar{\varepsilon}$ and $\bar{\alpha}$ in the quadratic form, as in Ref. 16: $\bar{\varepsilon} = 1 - \Delta\varepsilon(1 - \mathbf{r}^2/a^2)$, $\bar{\alpha} = \alpha_0[1 - \mathbf{r}^2/a^2]$, where a is the half-width of the active medium, z_r is the refraction length, $\Delta\varepsilon = (a/z_r)^2$ is the step-wise variation of the regular component of the dielectric constant in the medium. Beyond the medium (for $|\mathbf{r}| > a$) $\bar{\varepsilon} = 1$ and $\bar{\alpha} = 0$. Then, taking into account Eqs. (8)–(10), we obtain from Eqs. (4)–(6) that

$$H_\varepsilon(\mathbf{r}, \mathbf{r}'; z) = \sqrt{2/\pi}\,\sigma_\varepsilon^2\, l_{\varepsilon\|}\{1 - \exp[-r'^2/(2l_{\varepsilon\perp}^2)]\}, \quad (11)$$

$$H_\alpha(\mathbf{r}, \mathbf{r}'; z) = -\sqrt{2/\pi}\,\frac{\sigma_\alpha^2\, l_{\alpha\|}}{k^2}\{1 - \exp[-r'^2/(2l_{\alpha\perp}^2)]\}, \quad (12)$$

$$H_h(\mathbf{r}, \mathbf{r}'; z) = \frac{4L_\| L_\perp^2}{\sqrt{2\pi}}\left(\frac{\mathbf{r}'}{z_r^2} - i\frac{2\alpha_0\,\mathbf{r}}{ka^2}\right)^2. \quad (13)$$

Let us write Eq. (7) for an active medium with the allowance for Eqs. (11)–(13):

$$\Bigg[\frac{\partial}{\partial z} + \frac{i}{k}\frac{\partial^2}{\partial \mathbf{r}\,\partial \mathbf{r}'} - \alpha_0 - 6\frac{(kal_{\alpha\perp})^2}{z_\alpha^3} + \left(1 - \sqrt{8\pi}\alpha_0 L_\| \frac{L_\perp^2}{a^2}\right)\left(\frac{ik}{z_r^2}\mathbf{r}\mathbf{r}' + \frac{\alpha_0}{a^2}\mathbf{r}^2\right) + 3\frac{(kal_{\varepsilon\perp})^2}{z_\varepsilon^3}\left\{1 - \exp\left(-\frac{\mathbf{r}'^2}{2l_{\varepsilon\perp}^2}\right)\right\} + 3\frac{(kal_{\alpha\perp})^2}{z_\alpha^3}\left\{1 - \exp\left(-\frac{\mathbf{r}'^2}{2l_{\alpha\perp}^2}\right)\right\} + \frac{3(ka)^2}{2z_h^3}\mathbf{r}'^2\Bigg] B(\mathbf{r}, \mathbf{r}'; z) = 0, \quad (14)$$

where

$$z_\varepsilon = [6(2/\pi)^{1/2}(al_{\varepsilon\perp})^2/\sigma_\varepsilon^2\, l_{\varepsilon\|}]^{1/3};$$

$$z_\alpha = [6(2/\pi)^{1/2}(kal_{\alpha\perp})^2/\sigma_\alpha^2\, l_{\alpha\|}]^{1/3}$$

are characteristic lengths of spontaneous radiation scattering at “strong“ (more exactly, strong small-scale or arbitrary large-scale) fluctuations $\tilde{\varepsilon}$ and $\tilde{\alpha}$, respectively (see Refs. 9 and 12). The value

$$z_h = [6(2/\pi)^{1/2}(az_r^2)^2/(4L_\| L_\perp^2]^{1/3}$$

has the meaning of the scattering length of spontaneous radiation on fluctuations $\tilde{\mathbf{r}}$. The term of Eq. (14) with $\mathbf{r}^2$ leads to beam narrowing with the increase in z due to inhomogeneity of the profile $\bar{\alpha}$ and the beam broadening due to influence of $\tilde{\mathbf{r}}$. The summand with $i\mathbf{r}\mathbf{r}'$ leads to reqular radiation refraction on $\bar{\varepsilon}$ profile due to $\tilde{\mathbf{r}}$ influence. The last three summands in Eq. (14) with the corresponding scattering length define the radiation scattering on the fluctuations $\tilde{\varepsilon}$, $\tilde{\alpha}$ and $\tilde{\mathbf{r}}$.

The influence of fluctuations $\tilde{\varepsilon}$ on widely diverged radiation is significant if z_ε is of the same order of magnitude or less than the medium length.[9] Figure 1 presents the results of numerical integration of equation (7) for $\tilde{\alpha} = \tilde{\mathbf{r}} = 0$ and in the absence of regular diffraction. The divergence of input partially coherent radiation considerably exceeds the characteristic diffraction and geometric angles $\lambda/2a$ and $2a/z$, i.e., radiation does not, in fact, differ from spontaneous one, from the standpoint of influence on the output radiation parameters. Figure 1 shows the axial brightness of amplified radiation $q_0(z) = \iint B(\mathbf{r}, \mathbf{r}'; z)\, d\mathbf{r}\, d\mathbf{r}'$, normalized by the brightness of input radiation q_s at a homogeneous amplification in two- and three-dimensional axially symmetric media for $ka = 1.5 \cdot 10^4$, $\Delta\varepsilon = 0$, $\bar{\alpha}a = 2 \cdot 10^{-2}$, $l_{\varepsilon\perp}/a = 2 \cdot 10^{-2}$, $kl_{\varepsilon\parallel} = 3 \cdot 10^2$ and different σ_ε. Beyond the active medium (for $|\mathbf{r}| > a$), we have $\alpha = 0$, $\varepsilon = 1$.

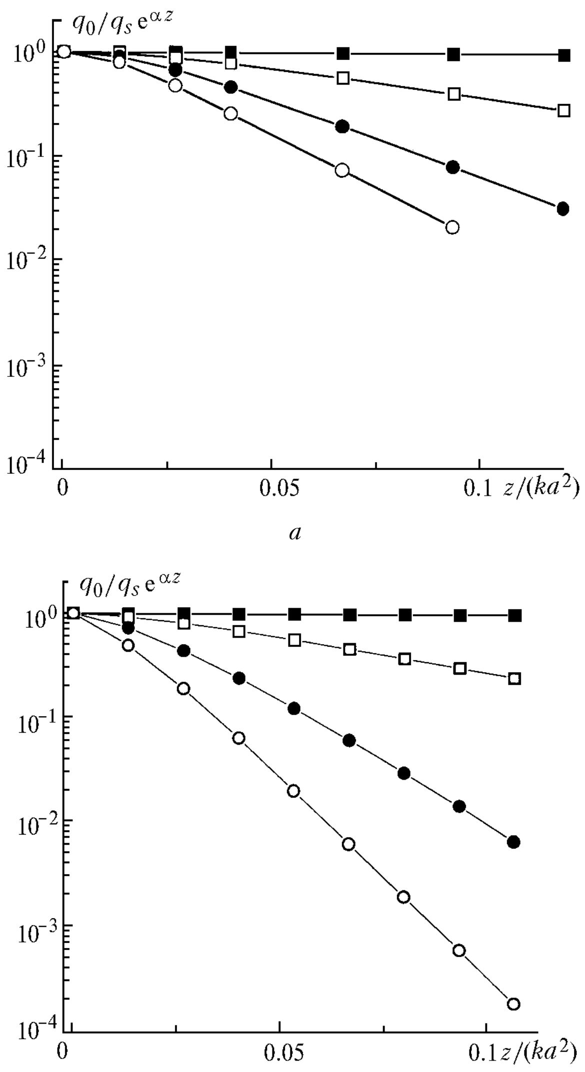

Fig. 1. Plots of axial brightness of radiation $q_0(z)$ in the two-dimensional (a) and three-dimensional (b) cases for $ka = 1.5 \cdot 10^4$, $\Delta\varepsilon = 0$, $\bar{\alpha}a = 2 \cdot 10^{-2}$, $l_{\varepsilon\perp}/a = 2 \cdot 10^{-2}$, $kl_{\varepsilon\parallel} = 3 \cdot 10^2$, and $\sigma_\varepsilon = 0$ ( ), $2 \cdot 10^{-5}$ (~), $4 \cdot 10^{-5}$ (M), $6 \cdot 10^{-5}$ (O).

As seen from Fig. 1, the axial brightness grows exponentially with increase of the medium length in the absence of fluctuations $\tilde{\varepsilon}$, i.e., $q_0(z) = q_s \exp(\bar{\alpha}z)$. The presence of $\tilde{\varepsilon}$ preserves exponential growth of axial brightness for $z \ge z_\varepsilon$, but the exponent decreases, namely, $q_0(z) \sim \exp(\bar{\alpha}z - \iota_\varepsilon z)$. Thus, the scattering of amplified spontaneous radiation on the fluctuations $\tilde{\varepsilon}$ is similar to linear absorption with the intensity coefficient ι_ε which can be effected by the medium geometry. Therefore, the presence of $\tilde{\varepsilon}$ lowers the actual gain coefficient of the medium. Application of the analytical solution of Eq. (1) (Ref. 9), which was obtained for an infinite medium, yields different law of axial brightness variation with the growing length of the active medium: $q_0(z) \sim \exp(\bar{\alpha}z)\,(z_\varepsilon/z)^{3(n-1)/2}$, where n is the dimensionality of space ($n = 2$ for a plane medium, $n = 3$ for a three-dimensional medium). Thus, radiation scattering on fluctuations $\tilde{\varepsilon}$ in a laterally bounded active medium significantly differs from scattering in an infinite medium.

Figure 2 plots the value $\iota_\varepsilon(\sigma_\varepsilon)$ determined by the slope of the linear part of the curves $q_0(z)$ in Fig. 1 for two- and three-dimensional media. The view of the function $\iota_\varepsilon(\sigma_\varepsilon)$ depends on the parameter $d_\varepsilon = kal_{\varepsilon\perp}/z_\varepsilon$, i.e., on the ratio between the diffraction length of a beam having the half-width a and correlation radius $l_{\varepsilon\perp}$ to the characteristic scattering length. For weak fluctuations $\tilde{\varepsilon}$, when $d_\varepsilon^3 \ll 1$, we obtain $\iota_\varepsilon \approx 3d_\varepsilon^2/z_\varepsilon \sim \sigma_\varepsilon^2$, and ι_ε does not depend on $l_{\varepsilon\perp}$ and geometry of the medium. A qualitatively close result was obtained for the scattering coefficient of a coherent beam in an infinite randomly inhomogeneous active medium.[19] For strong fluctuations $\tilde{\varepsilon}$, when $d_\varepsilon^3 \ge 1$, we have $\iota_\varepsilon \approx 1/z_\varepsilon$, for a plane medium, i.e., $\iota_\varepsilon \sim \sigma_\varepsilon^{2/3}$. For a three-dimensional axially-symmetric medium, $\iota_\varepsilon \approx 2/z_\varepsilon$, i.e., the losses increase twice.

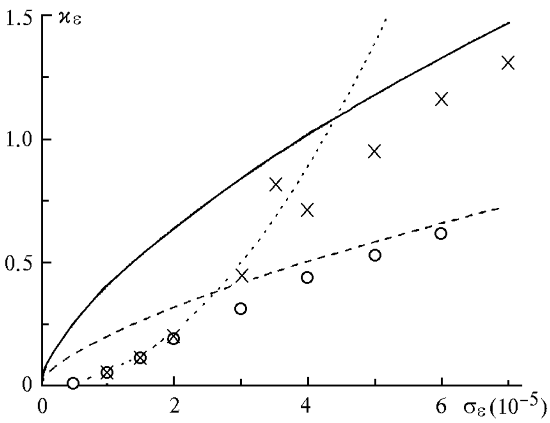

Fig. 2. Numerical dependences ι_ε in the two-dimensional (o) and three-dimensional (×) cases and the values $3d_\varepsilon^2/z_\varepsilon$ (···), $1/z_\varepsilon$ (- - -), and $2/z_\varepsilon$ (——) as functions of σ_ε.

Both in the absence of regular diffraction or its weakness ($z_\varepsilon < z_r$) and strong regular diffraction ($z_\varepsilon > z_r$), scattering of amplified spontaneous radiation by $\tilde{\varepsilon}$ deteriorates angular divergence, coherence, and coherent power of radiation. Coherence grows with the increasing length of medium and tends to a constant value depending on the level of regular diffraction.[9–14]

Under strong defocusing refraction, radiation scattering by $\tilde{\varepsilon}$ is not an analog of linear absorption as it took place at $z_\varepsilon < z_r$. The sharp drop of the degree of coherence is accompanied by a weak decrease in the radiation intensity and broadening of the beam. Fluctuations $\tilde{\varepsilon}$ show a hidden, in a certain sense, effect upon amplified spontaneous radiation at a strong regular diffraction.[10,12,14,20]

As seen from Eq. (14), the mechanism of scattering on fluctuations $\tilde{\alpha}$ is quite similar to the case of $\tilde{\varepsilon}$. But it is worth saying that scattering on $\tilde{\varepsilon}$ at the same scale and depth of fluctuations $\tilde{\varepsilon}$ and $\tilde{\alpha}$ is much more significant due to direct effect on the radiation phase. The part of $\tilde{\alpha}$ can be compared with that of $\tilde{\varepsilon}$ only at a sufficiently high value $\sigma_\alpha \sim k\sigma_\varepsilon$. However, fluctuations $\tilde{\alpha}$ lead not only to scattering but also to the inverse effect, namely, to additional amplification against the background $\bar{\alpha}$ because $H_\alpha \neq 0$ for $\mathbf{r}' = 0$. According to Eq. (14), an addition to regular amplification is $(\pi/2)^{1/2}\sigma_\alpha^2\, l_{\alpha\|} = 6d_\alpha^2/z_\alpha$, where $d_\alpha = kal_{\alpha\perp}/z_\alpha$. The addition significantly changes the effect of radiation scattering on $\tilde{\alpha}$. Indeed, for $\tilde{\varepsilon} = 0$ and homogeneous $\bar{\alpha}$, axial brightness varies as $q_0(z) \sim \exp[(\bar{\alpha} + 6d_\alpha^2/z_\alpha - \iota_\alpha)z]$. For weak fluctuations $\tilde{\alpha}$, when $d_\alpha^3 \ll 1$, scattering on $\tilde{\alpha}$ leads to additional losses with the intensity coefficient $\iota_\varepsilon \approx 3d_\alpha^2/z_\alpha$. In final analysis, total effect of $\tilde{\alpha}$ leads to an increase of axial brightness of radiation, and $q_0(z) \sim \exp[(\bar{\alpha} + 3d_\alpha^2/z_\alpha)z]$.

Under strong fluctuations $\tilde{\alpha}$, when $d_\alpha^3 \geq 1$, we have $\iota_\alpha \approx (n-1)/z_\alpha$; then $q_0(z) \sim \exp[(\bar{\alpha} + 6d_\alpha^2/z_\alpha - (n-1)/z_\alpha)z]$. It follows that the total effect of $\tilde{\alpha}$, like in the above case, leads to an increase in the axial brightness of amplified spontaneous radiation as $d_\alpha^3 \geq 1$. Therefore, in presence of $\tilde{\varepsilon}$ and $\tilde{\alpha}$, the negative effect on brightness due to radiation scattering on $\tilde{\varepsilon}$ is somewhat weaker by the effect of $\tilde{\alpha}$.

As seen from Eq. (14), the relative contribution of the axial fluctuations $\tilde{\mathbf{r}}$ to additional regular refraction and regular broadening of a beam is defined by the scale of the value $(8\pi)^{1/2}\alpha_0 L_\|(L_\perp/a)^2$ as compared with unity. This value depends on amplification at the length $L_\|$ and on the ratio $L_\perp/a$ which is small because the axial fluctuations are taken to have small transverse scale. As seen from Eqs. (10) and (13), the contribution of fluctuations $\tilde{\mathbf{r}}$ to radiation scattering takes place only in the presence of regular refraction. It is similar in its form to that of strong fluctuations $\tilde{\varepsilon}$ when the exponents in Eq. (14) can be expanded into a series. The values $l_{\varepsilon\|}$, $l_{\varepsilon\perp}$, σ_ε, z_ε in the case of $\tilde{\varepsilon}$ correspond to the values $L_\|$, $L_\perp$, $\sigma_h = 2(L_\perp/z_r)^2$, z_h in the case of $\tilde{\mathbf{r}}$, respectively. However, from the relation $z_h/z_r \approx (z_r/L_\|)^{1/3}(a/L_\perp)^{2/3}$ it follows that radiation scattering on the fluctuations $\tilde{\mathbf{r}}$ (when they are small-scale) is much less as compared with the regular refraction on the profile $\bar{\varepsilon}$. Thus, $z_h > z_r$ and fluctuations $\tilde{\mathbf{r}}$ have a weak effect on the intensity of output radiation but can significantly affect its coherence. Using Eq. (12), we obtain that the length of coherence of output radiation, which grows exponentially with the increase of z when fluctuations $\tilde{\mathbf{r}}$ are absent, is limited by the value

$$L_c \approx 2^{1/2}\, L_\perp \exp[\chi/3d_h^2 - C/2],$$

for $d_h < \chi/2$, where $d_h = kaL_\perp/z_h$; $\chi = -\ln \delta(z_h/z_r)$; C is the Euler constant, δ is the level of the step-wise change in the coherence degree by which L_c is determined.

Conclusion

By use of the quasi-optical equation for TCF of the complex field amplitude, dynamics of spontaneous radiation with low spatial coherence in a randomly inhomogeneous laterally bounded extended gain medium is studied. The model allows for the diffraction, regular refraction and amplification, scattering on usual fluctuations of the dielectric constant and amplification, scattering by random transversally small-scale axial deviations of an active medium. The equation has been considered analytically and solved numerically for two- and three-dimensional axially-symmetric medium. Criteria of medium optical parameters' influence on the amplified spontaneous radiation are obtained. It is shown that fluctuations of the gain coefficient yield an additional contribution to regular amplification of radiation, which dominates over radiation scattering on the fluctuations. This means that fluctuations of the gain coefficient decrease the losses due to radiation scattering on the fluctuations of the dielectric constant. Axial fluctuations of an active medium in the presence of regular refraction lead to radiation scattering. However, the influence of axial fluctuations on the radiation intensity is weak if they are small-scale and amplification at their longitudinal correlation length is small. At the same time, axial fluctuations can have an effect on the coherence length; namely, they restrict its growth with the increase of the medium's length.

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