STRUCTURE AND INFORMATION CONTENT OF NARROW–BAND NOISE IN LIDAR HETERODYNE DETECTION SYSTEMS

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The structure of narrow–band noise at heterodyne signal detection is analyzed, and its information content is estimated. It is shown that the more the amplitude–frequency response of a narrow–band system differs from the Π–shaped one, the larger is the system information content.

Lidar heterodyne detection systems attract now much attention. Despite of certain complexities, heterodyne systems are undoubtedly preferable against the direct photodetection, in particular, on highly attenuating paths with an intense background noise.

The most important problem of developing heterodyne systems concerns the improvement of their noise protection. One of possible ways to do this is the use of spectral differences of noise and signal.

Because the structure of noise in heterodyne detection is not clearly understood, in this paper we consider the structure of narrow band noise in heterodyne signal detection in order to provide an estimate of its information content.

As known, heterodyne detection systems are essentially narrow–band, and usually satisfy the condition[2]

$$\Delta F \ll F_{\text{inter}}, \tag{1}$$

where ΔF is the width of output radiation spectrum, F_{inter} is the intermediate frequency of a transceiving system.

Systems satisfying condition (1) are referred to as narrow–band systems. Under the influence of a wide–band noise, the signal at the output of such systems is a narrow–band process with the correlation function[2]

$$R(\tau) = \rho(\tau) \cos [\omega_0 \tau + \gamma(t)], \qquad \omega_0 = 2 \pi f_0, \tag{2}$$

while in the case of symmetric amplitude–frequency characteristic (AFC)

$$R(\tau) = \rho(\tau) \cos \omega_0 \tau, \tag{3}$$

where $\rho(\tau)$ is a slowly varying function in contrast to $\cos \omega_0 \tau$, which is determined by the system AFC and depends upon its passband.

According to Ref. 2, a narrow–band process $\xi(t)$ can be represented as a harmonic wave randomly modulated by amplitude and phase

$$\xi(\tau) = A(t) \cos [\omega_0 t + \varphi(t)], \tag{4}$$

where $A(t)$ and $\varphi(t)$ are slowly varying functions in contrast to $\cos \omega_0 \tau$, representing the envelope and random phase of a narrow–band process.

If at the narrow–band system input there is a normal wide–band noise, the system output $\xi(t)$ and the envelope $A(t)$ are also normal with zero mean values,[2] since the output signal is a linear transformation of the normal input process.

Let the envelop $A(t)$ be presented in terms of a Fourier series as

$$A(t) = \sum_{k=1}^{\infty} a_k \cos (k \Omega t + \varphi_k), \tag{5}$$

since $A(t)$ is the normal random process with zero mean, the zeroth term in series (5) is absent.

Taking this into account, the narrow–band process can be presented as

$$\xi(t) = \left(\sum a_k \cos k \Omega t\right) \cos [\omega_0 t - \varphi(t)] =$$

$$= 0.5 \sum a_k \cos [\omega_0 t - k \Omega t + \varphi(t)] +$$

$$+ 0.5 \sum a_k \cos [\omega_0 t + k \Omega t + \varphi_p(t)]. \tag{6}$$

As is obvious from Eq. (6), the narrow–band random process is essentially amplitude–modulated (AM) wave with a suppressed carrier wave. This conclusion is in a good agreement with familiar postulates of information theory and the theory of random processes.

The energy spectrum of narrow–band random process at the output of a linear narrow–band system with the transfer function $K(j\omega)$ is given as

$$S_{\text{out}}(\omega) = S_{\text{in}}(\omega) \, |K(j \omega)|^2, \tag{7}$$

where $S_{\text{out}}(\omega)$ and $S_{\text{in}}(\omega)$ are the spectral power density of noise at the linear narrow–band system output and input, respectively.

In the presence of a wide–band normal noise with a constant spectral density $S_{\text{out}}(\omega) = N_{\text{noise}}$, the equation (7) takes the form:

$$S_{\text{out}}(\omega) = N_{\text{noise}} \, |K(j \omega)|^2. \tag{8}$$

The correlation coefficient of such a process is given as

$$R(\tau) = \frac{K(\tau)}{\sigma^2} = \frac{1}{2\pi \sigma^2} \int_{-\infty}^{\infty} S(\omega) \, e^{j\omega\tau} \, d\tau = \rho(\tau) \cos (\omega_0 \tau + \varphi(\tau)), \tag{9}$$

where σ^2 is the variance of the output process.

The correlation function of the envelope, according to Ref. 2, is

$$\rho_a = \frac{K(t)}{\sigma^2} =$$

$$= \frac{4}{4-\pi}\left[\left(\frac{1}{2}\right)\rho^2(\tau) + \left(\frac{1}{2\cdot 4}\right)\rho^4(\tau) + \left(\frac{1\cdot 3}{2\cdot 3\cdot 4}\right)\rho^6(\tau)\ldots\right] =$$

$$= 0.921\ \rho^2(\tau) + 0.058\ \rho^4(\tau) + 0.0145\ \rho^6(\tau) + \ldots \qquad (10)$$

Since $\rho(\tau) < 1$, equation (10) reduces to

$$\rho_a(\tau) = \rho^2(\tau).$$

Therefore, the correlation coefficient of the envelope is approximately the square of slowly varying factor $\rho(\tau)$

To reveal the envelope changes, let us consider an important practical case of transmitting a noise through an ideal filter with $2\Delta\omega$ bandwidth centered at ω_0; the filter frequency response is assumed uniform within $2\Delta\omega$, with the transmission coefficient K_0. In this case, the energy spectrum at the filter output is uniform in the band from $\omega_0 - \Delta\omega$ to $\omega_0 + \Delta\omega$, with the value $S_{out}(\omega) = S_{out}(\omega_0)$, and the correlation function has the form[1]:

$$K_{out}(\tau) = \frac{S_{out}(\omega_0)}{2\pi}\int\limits_{\omega_0-\Delta\omega}^{\omega_0+\Delta\omega}\cos\omega\tau\, d\omega = \frac{S_{out}(\omega_0)}{2\pi\tau}\,[\sin(\omega_0 + \Delta\omega)\,\tau -$$

$$- \sin(\omega_0 - \Delta\omega)\,\tau] = \frac{S_{out}(\omega_0)}{\pi\tau}\sin\Delta\omega\,\tau\cos\omega_0\,\tau. \qquad (11)$$

From formulas (10) and (11) it follows that the correlation coefficient of the envelope is as follows:

$$R_{e.s.c.} = \frac{S_{out}(\omega)}{\pi^2\,\tau^2}\sin^2\Delta\omega\,\tau\,. \qquad (12)$$

Therefore spectrum of the signal envelope with a suppressed carrier is

$$S_{e.s.c.}(\omega) = 2\int\limits_{-\infty}^{\infty} K_{e.s.c.}(\tau)\, e^{-j\omega\tau}\, d\tau\,, \qquad (13)$$

or, with the spectrum defined for the positive frequency only,

$$S_{e.s.c.}(\omega) = 4\int\limits_{-\infty}^{\infty} K_{e.s.c.}(\tau)\cos\omega\tau\, d\tau. \qquad (14)$$

Calculations by formula (14) have shown that the signal envelope with suppressed carrier has a spectrum shaped as a triangle (see Fig. 1). This finding well agrees with the experimental results,[3] obtained earlier.

As known, a signal at the output of a linear circuit has maximum entropy, at a given energy spectrum, when a wide-band normal noise is applied to the circuit input. This condition is satisfied only in the case when each spectral component is information-bearing. That is, signals with maximum entropy must be carrier-free.

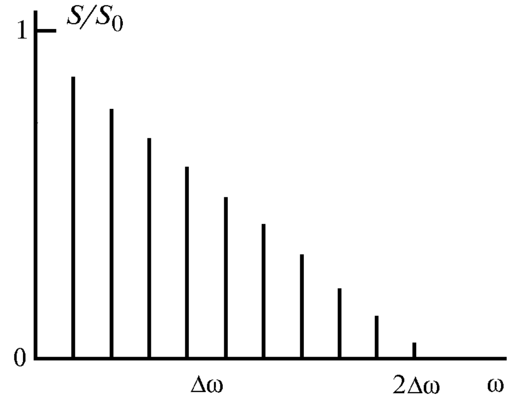

FIG. 1. The spectrum of the narrow-band noise envelope at the output of narrow-band system with Π-shaped AFC.

Let us now consider the information content of a narrow-band signal with a suppressed carrier and compare it to that of amplitude-modulated signal with the envelope of a narrow-band signal with a suppressed carrier. The signals have identical envelopes, hence, they carry the same amount of information. Therefore, having the width of spectrum of both signals known, their ratio gives the change in the signal information content.

In so doing, we first determine the correlation function of amplitude-modulated wave with the carrier for the case when its envelope is identical with the envelope of narrow-band signal with suppressed carrier.

As known,[2] a slowly varying coefficient of correlation function of an amplitude-modulated wave gives the correlation function of envelope.

Therefore, the correlation function of an amplitude-modulated signal with the envelope identical with the envelope of a narrow-band process can be presented as

$$R(\tau) = K_{am}(\tau)/\sigma^2 = (0.921\,\rho^2(\tau) + 0.058\,\rho^4(\tau) +$$

$$+ 0.0145\,\rho^6(\tau) + \ldots)\cos(\omega_0\,\tau + \varphi(\tau)) \qquad (15)$$

or, approximately, as

$$R_{am}(\tau) = \rho^2(\tau)\cos(\omega_0\,\tau + \varphi(\tau)). \qquad (16)$$

The spectrum of a signal having the structure of amplitude-modulated wave is given by the expression

$$S_{am}(\omega) = \sigma\,\frac{1}{2\pi}\int\limits_{-\infty}^{\infty} R_{am}(\omega)\, e^{j\omega\tau}\, d\omega\,. \qquad (17)$$

The information content of the signals can be compared by taking the ratio of required energy bandwidths of both signal structures when the same information is transmitted, namely,

$$P = B_{s.c.}\,/\,B_{am}, \qquad (18)$$

where $B_{s.c.}$ and B_{am} are the energy widths of spectra of amplitude-modulated waves with suppressed and nonsuppressed carriers, respectively.

The theory dictates[2] that the energy width of a spectrum is

$$B = \frac{1}{S_0} \int_0^\infty S(\omega)\, d\omega \,, \qquad (19)$$

where $S_0 = S(\omega_0)$ is the value of the spectral power density at certain characteristic frequency.

It can easily be shown that the systems with the Π–shaped amplitude–frequency response have the coefficient $P = 1$.

Let us evaluate P for an ordinary oscillation circuit. To do this, we use the correlation function for the oscillation circuit output,[2]

$$K(\omega) = \exp(-\alpha\, |\tau|) \cos \omega\, \tau. \qquad (20)$$

So, we use the above formulas to write the expression for the correlation function of an AM signal

$$K_{e.am}(\tau) = \exp(-2\alpha\, |\tau|) \qquad (21)$$

and the correlation function for an AM signal with the suppressed carrier is

$$K_{e.s.c.}(\tau) = \exp(-\alpha\, |\tau|). \qquad (22)$$

Using formulas (21) and (22) for the spectra of envelope at the oscillation circuit output we have

$$S_{s.c.}(\omega) = 2 \int_0^\infty e^{-\alpha\, |\tau|}\, e^{-j\omega\tau}\, d\tau = \frac{2\alpha}{\alpha^2 + \omega^2} - j\, \frac{2\omega}{\alpha^2 + \omega^2} \,, \qquad (23)$$

$$S_{am}(\omega) = 2 \int_0^\infty e^{-2\alpha\, |\tau|}\, e^{-j\omega\tau}\, d\tau = \frac{2\alpha}{4\alpha^2 + \omega^2} - j\, \frac{2\omega}{4\alpha^2 + \omega^2} \,. \qquad (24)$$

Finally, formula (18) for the case of oscillation circuit reduces to

$$P = \frac{1}{S_0} \int_0^\infty S_{s.c.}(\omega)\, d\omega \Big/ \frac{1}{S_0} \int_0^\infty S_{am}(\omega)\, d\omega =$$

$$= \frac{1}{S_0} \int_0^\infty \frac{4a}{2a^2 + w^2}\, d\omega \Big/ \frac{1}{S_0} \int_0^\infty \frac{2\alpha}{\alpha^2 + \omega^2}\, d\omega =$$

$$= \arctan\left(\frac{\omega}{2\alpha}\right) \Big/ \arctan \frac{\omega}{\alpha} = 2. \qquad (25)$$

Thus, the comparison of the information content of AM signals against that of AM signals with suppressed carrier, in the case of identically varying envelopes, shows that P varies from 1 to 2. The maximum gain in information content occurs for narrow–band systems whose amplitude–frequency response differs from Π–shaped one to a maximum extent.

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