DETERMINATION OF THE ATMOSPHERIC PARAMETERS BY MEANS OF DIFFERENTIAL GROUND–BASED REFRACTION OBSERVATIONS

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A possibility of reconstructing the near-ground value and gradient of the refractive index from the data on the derivative of the refraction angle is investigated. To obtain these data, it is proposed to use the ground–based measurements of the apparent difference between the vertical and horizontal dimensions of the solar or lunar disks. The efficiency of the proposed technique has been checked.

Recently developed methods for solving the inverse problems of refraction are based on the relation between the refractive index profile and variations in the parameters of optical and radio waves when a ray penetrates into the atmosphere. The method of sensing proposed in Ref. 1 in which radiation passed once through the atmosphere and the source and the receiver were extraterrestrial objects while the ray penetrated into the atmosphere scanning the height dependence of the refractive index with the zone was of great success. In the approximation of the spherically symmetric model of the atmosphere the problem of reconstructing the height dependence of the refractive index is reduced to the inversion of the Abel transform. This effective technique based on the use of the transmitters placed onboard automatic spacecrafts made it possible to investigate the atmospheres of Mars, Venus, and Sun.[2] For the global investigation of the Earth's atmosphere this approach would call for the use of the two artificial earth satellites.[3] This is associated with certain difficulties. The observations of such permanent natural light sources as moon and sun in the optical and microwave ranges are more effective. The technique for solving the inverse problem of refraction by means of spaceborne photographing and subsequent analysis of the recorded distortions in the shape of the solar disk was developed in Refs. 4–7. On the whole, sensing with radiation which passed once through the atmosphere ensures the high accuracy while disadvantages are both the high expense and the narrow range of applicability.

In the present paper it is suggested to use the ground–based observations of the distortions in the shape of solar or lunar disk when their position in the sky varies. The measurements of the relative difference between the vertical and horizontal dimensions of disk determine the derivative of the refraction angle and eliminate the need for an *a priori* information about the expected position of the radiation source. The ground–based observations are accessible but in this case the problem of reconstructing the atmospheric parameters cannot be reduced to the simple Abel transform. It became a more complicated and ill–posed problem.[8-11] Therefore, in this paper the simple technique is proposed to reconstruct two such important parameters as near–ground value and gradient of the refractive index. Then the inverse–problem equation is derived which differs from the well–known equations[8–11] because of using the measurements of the derivative of the refraction angle as the input data.

Let us assume that the radiation source (sun or moon) has the shape of a regular circle. Thus the flatness of the solar disk does not exceed 0.005% (Ref. 12), that is quite an exceptable assumption. If δ is the angular diameter of this circle, then for observations made through the atmosphere the vertical diameter decreases by the value Δ which is equal to

$$\Delta = \xi\left(\varphi - \frac{\delta}{2}\right) - \xi\left(\varphi + \frac{\delta}{2}\right) ,$$

where $\xi(\varphi \pm \delta/2)$ is the angle of astronomical refraction and φ is the elevation angle of the disk center above the horizon. Owing to triviality of the horizontal refraction, the horizontal size of diameter is practically equal to δ. The ratio of the difference between the horizontal and vertical diameters of the ellipse to its horizontal diameter is equal to $\gamma = \Delta/\delta$, and as a consequence of the condition $\delta \ll 1$, can be estimated as

$$\gamma(\varphi) = d\xi(\varphi)/d\varphi = -\dot{\xi}(\varphi) . \quad (1)$$

In accordance with Ref. (13), the refraction angle ξ for $\varphi \geq 5^{\circ}$ can be represented as

$$\xi(\varphi) = N_0 \cot\text{an}\,\varphi \left[1 - H/(a \sin^2\varphi)\right] , \quad (2)$$

where $N_0 = (n_0 - 1)$ is the near–ground value of the normalized refractive index, H is the effective height of the troposphere, and a is the Earth's radius. The near–ground gradient of the refractive index $G = dN/dh$ is connected with H by the relation $G = N_0/H$. The first term in Eq. (2) describes the refraction in the approximation of a plane–layered model of the atmosphere and predominates for the vertical viewing angles. The second term is responsible for the spherical symmetry of the atmosphere. Taking Eqs. (1) and (2) into account one can write

$$\zeta(\varphi) = -\hat{\xi}(\varphi))\sin^2\varphi = N_0\left(1 - \frac{H}{a}(1 + 3\text{cotan}^2\varphi)\right) . \quad (3)$$

The function $\xi(\varphi)$ for different H is depicted in Fig. 1 on a double logarithmic scale. Figures *6*, *7*, *8*, *9*, and *10* indicate the height H expressed in kilometers. The calculation was carried out for $N_0 = 250$ N–units. It can be seen that the dependence of ξ on H increases with decreasing of φ and vanishes for $\varphi \geq 25$ when the curves merge. For large φ $\xi(\varphi) = N_0 =$ const.

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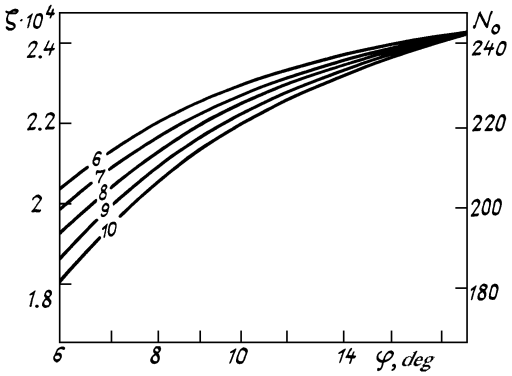

FIG. 1. Differential parameter of refraction as a function of the observation angle for different effective thickness of the troposphere.

In general the measured values of $\xi(\varphi)$ can be used for reconstructing N_0 and H by means of the least–squares technique and those values of N_0 and H will be chosen which allow Eq. (3) to approximate the experimental values of $\xi(\varphi_i)$ more accurately ($i = 1, 2, \dots, M$; $M \geq 2$). The system of standard equations involved in this technique is linear and can be easily solved.

If there are only two measured values ($M = 2$) then one can use the figure as a nomogram to determine N_0 and H (and, consequently, G). Let us consider the sequence of operation in this case. First of all it is necessary to mark two reference values of $\xi(\varphi_1)$ and $\xi(\varphi_2)$ by corresponding points and draw a line segment among them. By a parallel dropping this line segment one should find such its position in which the segment ends hit one and the same curve from the family of the depicted ones. The value of the line segment and the curve are fixed. The value of H corresponding to this curve is the value of the atmosphere altitude to be found. The value of N_0 is read to the right of the coordinate axis after adding to the reference value 250 N–units of a segment the length of which is equal to the found parallel displacement.

To test the efficiency of the method, the table of astronomical refractive angles was used calculated numerically in Ref. 14 for the standard model of the atmosphere. As a result, it was found that $\xi(5°) = 2.0 \cdot 10^{-4}$, $\xi(10°) = 2.5 \cdot 10^{-4}$, and $\xi(20°) = 2.7 \cdot 10^{-4}$. The values of the atmosphere parameters found with the help of the above–described technique turned to be equal for $N_0 = 280$ N–units and $H = 8.3$ km ($G = -34$ N–units/km). Exact values of N_0 and H which were used for calculating the table were equal to 278.24 N–units and 8.4345 km, respectively. The accuracy of reconstruction is comparable with the corresponding accuracy of measurements of the meteorological parameters. The decrease of the error in reconstructing the parameters is possible by means of conducting more accurate measurements and the initial data sampling size of $M \gg 2$.

Differential measurements of $\xi(\varphi)$ can be used for solving the inverse problem of reconstructing the height dependence of the atmospheric refractive index $N(h)$. For this purpose it is necessary to take instead of Eq. (2) more general relation

$$\xi(\varphi) = [N_0 - \nu(\varphi)] \cot\varphi , \quad (4)$$

where

$$\gamma(\varphi) = s \int_0^\infty N(u) u \left[u^2 + s^2\right]^{-3/2} du , \quad (5)$$

$$s = a(1 + N_0) \sin\varphi ,$$

and the integration variable u is related to the height h of the point above the earth's surface by the relation

$$u^2 = (a + h)^2 n^2(a + h) - a^2 n^2(a) .$$

Here the height dependence of the refractive index is given by

$$n(a + h) = 1 + N .$$

The simple differentiation of Eq. (4) results in the following equations:

$$\zeta(\varphi) = N_0 - s \sin^2\varphi \int_0^\infty f(u, s) N(u) u \left[u^2 + s^2\right]^{-3/2} du , \quad (6)$$

$$f(u, s) = \left[u^2 + s^2(1 + 3\cot^2\varphi)\right] / \left[u^2 + s^2\right] .$$

Numerical estimates for $\varphi > 3°$ showed that the value $f(u, s) \approx 1 + 3\cot^2\varphi$ and then according to Eq. (3) we have

$$\zeta(\varphi) = N_0 - (1 + 3\cot^2\varphi) \sin^2\varphi \cdot \nu(\varphi) . \quad (7)$$

In accordance with Eq. (7) one can find $\nu(\varphi)$ from the measured values of $\xi(\varphi_i)$. The methods of solving integral equation (5) were studied in Refs. 10 and 11. If the condition $\varphi > 3°$ is not satisfied, then integral equation (6) must be used for solving the inverse problem of reconstructing the height dependence of the refractive index.

The proposed method for measuring the atmospheric parameters from the data of ground–based observations of the distortion in the shape of solar or lunar disk can be widely employed for monitoring the atmospheric conditions in real time. To obtain the initial differential data, one can use the measurements of the shifts in the relative positions of stars in the sky caused by refraction.

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