# Effective source technique for express-estimation of total power of gas impurity emissions in the atmosphere

**Yu.E. Geints and A.A. Zemlyanov**

*Institute of Atmospheric Optics,*
*Siberian Branch of the Russian Academy of Sciences, Tomsk*

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The paper describes the method of express-estimation of total power of the gas impurity emitted in the atmosphere by a group of continuous point sources. The method is based on the trace measurements of integral pollutant concentration and corresponding replacement of an actual group of sources by a virtual effective surface source, which encompasses the whole emission area. The Gaussian plume model is used to describe the atmospheric dispersion of each independent source. The relative error of the method is estimated depending on the dispersion between the emission power of individual sources, spatial configuration of sources within emitting area, turbulent state of the atmosphere, and effective height of emissions.

## Introduction

One of important problems of ecological and industrial monitoring of the atmosphere chemical composition is the estimation of the net power of industrial emissions from one or several local sources using measurements of the impurity concentration field within the region under study. To the present, this problem can be solved by different ways. Most of them use the model of atmospheric dispersion of gas or aerosol impurity (model of the Lagrangian or Eulerian type) provided the spatial location of emission sources and field of values of meteorological parameters are known. This inverse problem of atmospheric diffusion of impurity, in its turn, can be solved both by back trajectory methods[1,2] or the method proposed by G.I. Marchuk[3] and connected with solution of a special equation conjugated with the equation of turbulent diffusion.[4,5]

At the same time, the laser computer tomography is used for ecological monitoring already during several decades,[6,7] by means of which the two-dimensional field of actual impurity concentration is reconstructed from data array of its measured path concentrations (integral along the optical path) using a special algorithm. Subsequent processing with the use of a particular dispersion model yields the sought source power and its space location.

Without going into details of the above-mentioned methods, we note only that all they require, first, a large number of measurements of the local or integral concentration of a studied matter at different points of the investigated region, and, second, intensive numerical calculations. However, only a rough estimate of overall quantity of impurity, emitted in a unit of time by a group of sources, located within the limits of some region, is sufficient in practice. The estimate, if possible, should be obtained with minimum of field measurements and computation time.

One of the possible ways of solution of the problem[8] consists in the simplifying and reducing the price of the concentration sensors (which detect only the excess above a given threshold) with a simultaneous increase of their quantity. In this case the sensors are spread in a random order at the measurement area (for example, from an aircraft). Then, after obtaining information from the sensors, using a simple dispersion model of the Gaussian plume,[9,10] the source power is retrieved through the regression analysis.

One more method to determine the net power of emissions from several sources, which location is unknown, is proposed in a given paper, which comprises elements characteristic both for tomography of emission and for the technique of randomly located sensors. An idea of the proposed method consists in substitution of real group of point sources by a virtual effective area source with the same total intensity, which is limited in spatial scales, that excludes zone of action of actual sources. Considering the impurity scattering law in the atmosphere to be Gaussian and having the data at least of one measurement of the trace concentration in the transverse direction to the mean wind, it becomes possible to obtain the express-estimation of the total emission power with a sufficient accuracy.

Limitations of this method follow from the conditions of applicability of the model of Gaussian emission plume. It is commonly meant that such modeling of the atmospheric transport gives true results under more or less stable atmospheric conditions, continuous emission sources, simple orography of the area, and distances from the emission up to several tens of kilometers. The accuracy of the proposed method of an effective source is not below 50%, provided our recommendations are obeyed.

## Key relationships

Let $N$ point continuous impurity sources be located in the rectangular region ($L_x \times L_y$). Each point source is at the arbitrary (unknown) spatial

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point $(x_i, y_i)$ at one and the same height $h_c$ and emits $Q_i$ mass of impurity ($i = 1...N$) per unit time. Let the origin of Cartesian coordinates be at a point ($x = 0$, $y = L_y/2$, $z = 0$) and OX axis is oriented windward. The coordinates of location of the emission sources are denoted as $(x_i, y_i)$. The state of the atmosphere is characterized by a certain given vertical profile of the mean wind velocity $u(z)$ and the coefficients of the turbulent diffusion $\sigma_y(x)$, $\sigma_z(x)$. It is required to estimate the net emission power $Q_\Sigma = \sum_{i=1}^{N} Q_i$ from data of path measurements of the concentration $C(x,y,z)$. In the general case, particular coordinates of each source are arbitrary and unknown.

Use the model of the Gaussian emission plume. This model is based on the approximate analytical solution of the impurity turbulent diffusion equation and exploits the statistical mean characteristics of jet and the atmosphere. The model of Gaussian plume is applicable for large periods of diffusion and stationary conditions of emissions.

The impurity concentration from an isolated source, averaged over a large period of time, at each point of space is given by the formula

$$C_i(x,y,z) = \frac{1}{2\pi u(z)} \times$$

$$\times \frac{Q_i}{\sigma_y(x-x_i)\sigma_z(x-x_i)} f^y(x-x_i) f^z(x-x_i), \qquad (1)$$

where the functions of longitudinal and transverse plume dispersion are of the form

$$f^y = \exp\left(-\frac{(y-y_i)^2}{2\sigma_y^2(x-x_i)}\right)$$

and $$f^z = \left[\exp\left(-\frac{(z-H_e)^2}{2\sigma_z^2(x-x_i)}\right) + \exp\left(-\frac{(z+H_e)^2}{2\sigma_z^2(x-x_i)}\right)\right], \qquad (2)$$

$H_e = h_c + \Delta H$ is the effective source height; $\Delta H$ is the height of plume elevation due to the jet moment of inertia and its floating up under the action of the Archimedean forces.[11]

In principle, at the known space position of sources $(x_i, y_i)$ and availability of a series of local measurements of the impurity concentration $C_i$ at some points, it is possible to calculate the power of each source and obtain the estimate of $Q_\Sigma$, using Eqs. (1) and (2). However, if $(x_i, y_i)$ are unknown, it is impossible to make this directly.

Let us assign a certain virtual effective distributed source, limited by the longitudinal $L_x$ and transverse $L_y$ scales (including the operation zone of real sources), to the real group of point sources. Its mean over area density of emission power is $q_e = Q_\Sigma/(L_x L_y)$. Then the expression for the impurity concentration field from such a source is the following

$$C_e(x,y,z) = \frac{q_e}{2\pi u(z)} \int_0^{L_x} \int_{-L_y/2}^{L_y/2} dx'dy' \frac{f^y(x-x')f^z(x-x')}{\sigma_y(x-x')\sigma_z(x-x')} =$$

$$= \frac{q}{2\sqrt{2\pi}u(z)} \int_0^{L_x} \frac{f^z(x-x')}{\sigma_z(x-x')} \left[\operatorname{erf}\left(\frac{L_y+2y}{2\sqrt{2}\sigma_y(x-x')}\right) - \right.$$

$$\left. - \operatorname{erf}\left(\frac{-L_y+2y}{2\sqrt{2}\sigma_y(x-x')}\right)\right] dx'. \qquad (3)$$

Assume that the integral trace concentration of impurity is measured at the point $(x_0, z_0)$, for example, from the magnitude of energy extinction of an optical signal. The measurement path runs in the horizontal plane across the wind and encloses a certain range of values of the variable $y$: $-Y_0/2 \le y \le Y_0/2$.

Mathematically, this corresponds to the integral of Eq. (3) with respect to $y$ (in the corresponding limits), which can be written as

$$C_{eY_0}(x,z) = \int_{-Y_0/2}^{Y_0/2} C_e(x,y,z)\,dy =$$

$$= \frac{q_e}{2\sqrt{2\pi}u(z)} \int_0^{L_x} \frac{f^z(x-x')}{\sigma_z(x-x')} F(L_y, Y_0)\,dx', \qquad (4)$$

where

$$F = \operatorname{erf}\left(\frac{L_y+Y_0}{2\sigma_y}\right)(L_y+Y_0) + \operatorname{erf}\left(-\frac{Y_0-L_y}{2\sigma_y}\right)(Y_0-L_y) +$$

$$+ 2\sqrt{\frac{2}{\pi}}\sigma_y\left(\exp\left[-\frac{(L_y+Y_0)^2}{8\sigma_y^2}\right] - \exp\left[-\frac{(L_y-Y_0)^2}{8\sigma_y^2}\right]\right).$$

The unknown quantity of the net emission power $Q_\Sigma$ is obtained from the following ratio

$$Q_\Sigma = C_{eY_0}(x_0,z_0)\frac{u(z_0)}{K(x_0,z_0)}, \qquad (5)$$

where

$$K(x_0,z_0) = \frac{1}{2\sqrt{2\pi}(L_x L_y)} \int_0^{L_x} \frac{f^z(x_0-x',z_0)}{\sigma_z(x_0-x')} F(L_y,Y_0)\,dx'.$$

In most cases it is necessary to calculate this integral numerically. However, sometimes, when the limits of measurement of trace concentration $Y_0$ are large as compared to linear size of emission area, i.e., $Y_0 \gg L_y$, the integral can be simplified considerably:

$$K(x_0,z_0) \simeq \frac{1}{\sqrt{2\pi}L_x} \int_0^{L_x} \frac{f^z(x_0-x',z_0)}{\sigma_z(x_0-x')}\,dx'.$$

If the condition $x_0 \gg L_x$ is fulfilled, i.e., the measurements are carried out far from the source boundaries, we obtain for $Q_\Sigma$:

$$Q_\Sigma = C_{eY_0}(x_0,z_0)\frac{\sqrt{2\pi}u(z_0)f^z(x_0,z_0)}{\sigma_z(x_0)}. \qquad (6)$$

In the limiting case of one point source we obtain from Eqs. (1)–(3):

$$C^1_{eY_0}(x,z_0) = \int_{-Y_0/2}^{Y_0/2} C(x,y,z_0)\,dy = \frac{Q_1}{\sqrt{2\pi}u(z_0)\sigma_z(x)} f^z(x)$$

and, consequently,

$$Q_1 = C^1_{eY_0}(x_0, z_0)\frac{u(z_0)}{K^1(x_0, z_0)};\quad K^1(x_0, z_0) = \frac{f^z(x_0)}{\sqrt{2\pi}\sigma_z(x)},$$

which is fully similar to Eq. (6).

In the general case, the appropriateness of the above substitution is unobvious. The accuracy of determination of the net power of emission by the formula (5) depends on the specific configuration of sources and magnitudes of their partial powers $Q_i$. And only within the limits of a localized group of emitters, when inequalities $y_i \gg Y_0$, $x_i \gg x_0$, $\forall x_i$, $y$ are true, we come exactly to Eq. (6).

## Estimate of the method error

Now estimate the error introduced to $Q_\Sigma$ by the method of the effective source. To do this, we first obtain an exact expression for the path concentration of impurity from a group of point sources. Integrating Eq. (1) with respect to $y$ and summing, we obtain

$$C_{Y_0}(x,z) = \int_{-Y_0/2}^{Y_0/2} C(x,y,z)\,dy = \frac{1}{2\sqrt{2\pi}u(z)} \times$$

$$\times \sum_{i=1}^{N} \frac{Q_i f^z(x - x_i)}{\sigma_z(x - x_i)}\left[\operatorname{erf}\left(\frac{Y_0 + 2y_i}{2\sqrt{2}\sigma_y}\right) - \operatorname{erf}\left(\frac{-Y_0 + 2y_i}{2\sqrt{2}\sigma_y}\right)\right].$$

We consider that the condition $y_i \ll Y_0$ is fulfilled, which in most cases could be realized in practice. Then

$$C_{Y_0}(x,z) \simeq \frac{Q_\Sigma}{\sqrt{2\pi}u(z)} \sum_{i=1}^{N} \frac{\bar{Q}_i f^z(x - x_i)}{\sigma_z(x - x_i)} \operatorname{erf}\left(\frac{Y_0}{2\sqrt{2}\sigma_y(x - x_i)}\right),$$

$$\bar{Q}_i = \frac{Q_i}{Q_\Sigma}$$

and the estimate of the net emission power can be written as

$$Q_\Sigma = \frac{C_{Y_0}(x_0, z_0)u(z_0)}{R_N(x_0, z_0)}, \qquad (7)$$

where

$$R_N(x_0, z_0) =$$

$$= \frac{1}{\sqrt{2\pi}} \sum_{i=1}^{N} \frac{\bar{Q}_i f^z(x_0 - x_i, z_0)}{\sigma_z(x_0 - x_i)} \operatorname{erf}\left(\frac{Y_0}{2\sqrt{2}\sigma_y(x_0 - x_i)}\right).$$

Based on the conditions of the problem, it is meant that the actually measured value of the trace concentration $C_{Y_0}(x_0, z_0)$ from the group of point sources coincides with the value, which would be obtained as a result of action of the virtual area source $C_{eY_0}(x_0, z_0)$, i.e., the equality $C_{Y_0}(x_0, z_0) = C_{eY_0}(x_0, z_0)$ is true. Then, writing the Eq. (7) to Eq. (5) ratio, we introduce the following parameter of the method error:

$$\eta = 1 - \frac{K(x_0, z_0)}{R_N(x_0, z_0)}. \qquad (8)$$

The deviation of the parameter from zero reflects the accuracy of the method of an effective source; in this case, the positive $\eta$ values correspond to the overestimated retrieved net power of emissions, and the negative values, on the contrary, point to the understated $Q_\Sigma$.

To determine the range of $\eta$ variation, we have simulated the atmospheric dispersion of conditional impurity from group of point sources of different powers, varying the coordinates of path measurement of concentration and classes of turbulent stability of the atmosphere,[9] affecting the form of dependence of the turbulent diffusion coefficients $\sigma_y(x)$, $\sigma_z(x)$ on the longitudinal coordinate.

As an illustration of a computer model, figure 1 shows 2D fields of concentration of the conditional impurity from a group of point sources of different powers.

At the top of Fig. 1, the distribution of relative concentration $\bar{C} = C(x,y)/C_m$ is shown, where $C_m$ is the maximum concentration in the field under consideration for two different classes of atmospheric stability **A** and **F**. Below the normalized concentration profiles across the wind $\bar{C}(x_0)$ are given, calculated along the path with the centre at $(x_0, z_0)$. A rectangular area $L_x \times L_y$ is shown, where the sources are located. The effective height of emissions $H_e$ is 10 m and the concentration field was calculated at the height $z_0 = 2$ m. The transverse profile of the net concentration is also shown, which represents the surface cross section $C(x, y, z = z_0)$ by a plane parallel to the axis $OY$ at the point $x_0$. It is seen that a high degree of the atmospheric turbulence (**A** class of stability according to the international classification) leads to strong mixing of the impurity with air, and, as a result, individual plumes quickly merge into one. In the stable atmosphere (**F** class of stability) this process takes a much longer time.

Figure 2 shows two realizations of $\eta$ values at different values of standard deviation $\sigma_Q$ of the power level of individual sources.

This parameter was determined by the following formula:

$$\sigma_Q = \frac{1}{\bar{Q}}\left[\frac{1}{N}\sum_{i=1}^{N}\left(Q_i - \bar{Q}\right)^2\right]^{1/2},$$

where $\bar{Q} = \frac{1}{N}\sum_{i=1}^{N} Q_i$ is the mean value of emission power of several sources. The position of sources inside the area $L_x \times L_y$ and their partial power are random (normal distribution of probability density) and noncorrelated to one another.

Two limiting cases were considered: the unstable atmosphere (**A** class of stability) and stable atmosphere (**F** class of stability), characterized by different dependences of coefficients of the turbulent dispersion of the plume ($\sigma_y$ and $\sigma_z$) on the distance along the wind drift. Besides, the behavior of $\eta$ was studied depending on path coordinates of measuring the impurity concentration $x_0$.

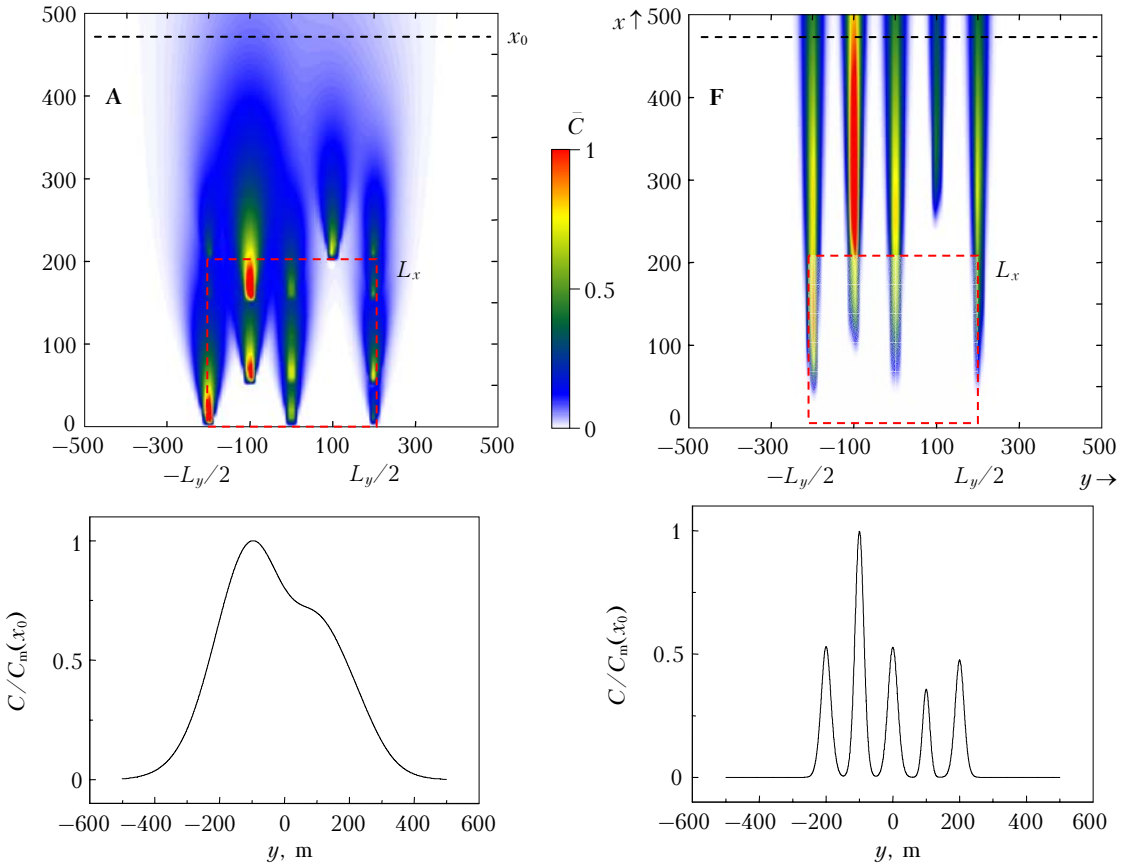


**Fig. 1.** Numerical simulation of the impurity concentration distribution of a group of point sources of different power under conditions of stable and unstable atmosphere.

Figure 2 shows that the increase in the spread of values of source power $\sigma_Q$, as a whole, results in the rise of the reconstruction error; in this case the remoteness of the concentration measuring path (the increase of $x_0$) from the emission sources is accompanied by the decrease of the error. The influence of the class of the atmospheric turbulent stability on $\eta$ manifests itself insignificantly.

A detailed study of $\eta$ behavior has revealed its functional dependence not only on the power dispersion of sources but on their configuration within the limits of the emitting area. It turned out that the largest in modulus deviations of $\eta$ from zero were observed for sources crowding near the closest and farthest boundaries of the emission area: $x = 0$; $x = L_x$. Formally, this corresponds to the fact that the effective source, by which a group of real emitters is replaced, is degenerated from an area source into a linear one, for which the coefficient $K(x_0, z_0)$ is calculated differently:

$$K_\delta(x_0, z_0) = \lim_{L_x \to 0} K(x_0, z_0) =$$

$$= \frac{1}{2\sqrt{2\pi} L_y} \frac{f^z(x_0, z_0)}{\sigma_z(x_0)} F(L_y, Y_0). \qquad (9)$$

Relative location of sources inside an effective emission area is characterized by the parameter of center of gravity $X_c$ of discrete distribution $Q(x_i)$, which is determined as first moment by $x$ of a given function:

$$X_c = \frac{1}{Q_\Sigma} \sum_{i=1}^{N} Q_i x_i.$$

The results of calculation of the function $\eta(X_c)$ at large standard deviation of powers $\sigma_Q = 4.2$ are given in Fig. 3 and show that the error of restitution of net concentration is minimal at a balanced position of partial sources over the emission area ($X_c \simeq L_x/2$). In this case the increase of measurement distance $x_0$ as compared to the linear size of the area $L_x$ decreases this error, since under condition $x_0 \gg L_x$ a transition to the approximation of the linear source (9) takes place, and equations (5) and (7) give one and the same result.

Consider the dependence of the coefficient $K(x_0, z_0)$ on basic parameters of the problem. Figures 4 and 5 show the influence of the ratio $L_x/x_0$ and the effective height of the source $H_e$ emissions on the value of a given coefficient at varying $x_0$. It follows from Figs. 4 and 5 that the largest deviations of $K$ from its limiting ($L_x \to 0$) value $K_\delta$ are observed at close location of the measurement path from the outer boundary of the effective source (large values of $L_x/x_0$). In this case the growth of the

length of the emission area in absolute units (see curves *1*, *2*, *4*, *5* in Fig. 3) leads to the increase of $K$.

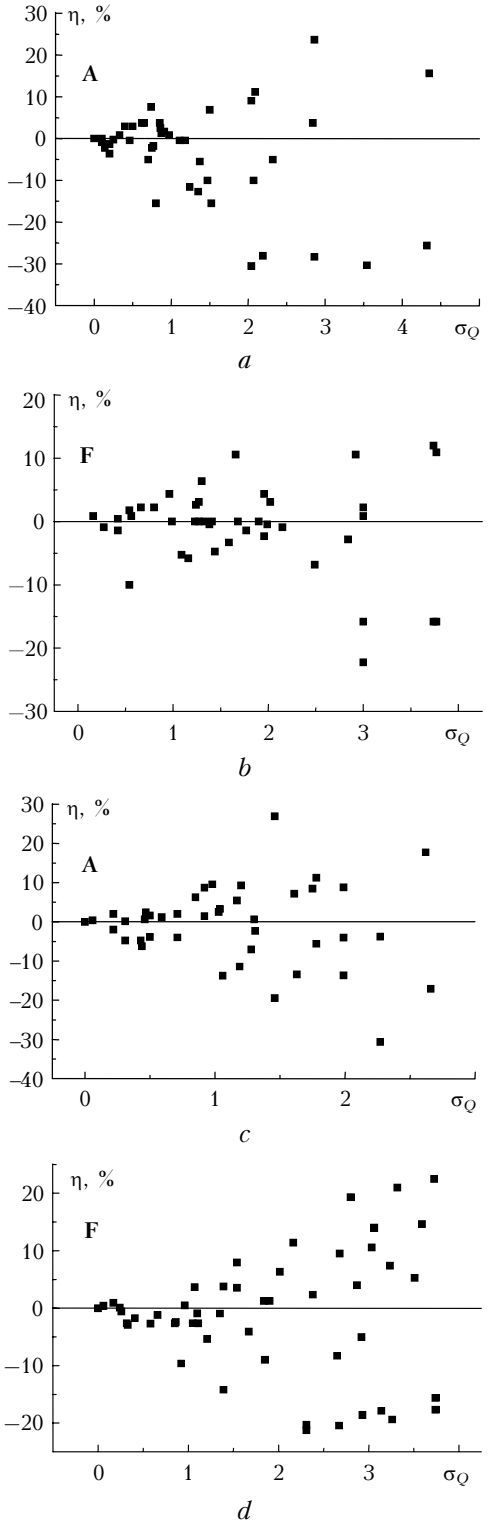


**Fig. 2.** The tendency in η variation at different standard deviations of powers of individual sources of $\sigma_Q$ (normal distribution) and different $x$-coordinate of the measurement path $x_0 = 500$ (*a*, *b*) and 250 m (*c*, *d*); $L_x \times L_y = 200 \times 200$ m; $z_0 = 2$ m; $H_e = 10$ m, $N = 15$.

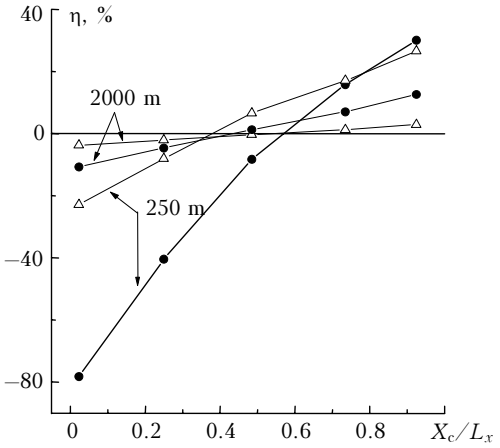


**Fig. 3.** The dependence of η on the $x$-coordinate of the centre of gravity of power distribution of local sources $X_c$ at $\sigma_Q = 4.2$ and different remoteness of the measurement point $x_0$ (is shown by digits) and for classes of stability **A** (•) and **F** (Δ) at the same parameters (see Fig. 2).

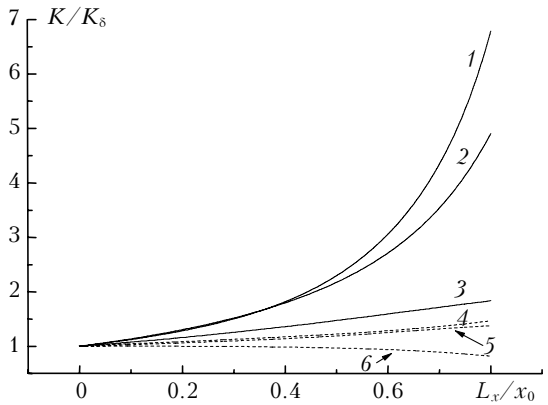


**Fig. 4.** The value of $K/K_\delta$ depending on $x_0 = 2$ (*1*, *4*); 1 (*2*, *5*); 0.25 km (*3*, *6*). The stability class: **A** is defined by solid curves; **F** – by dash lines; $z_0 = 2$ m; $H_e = 10$ m.

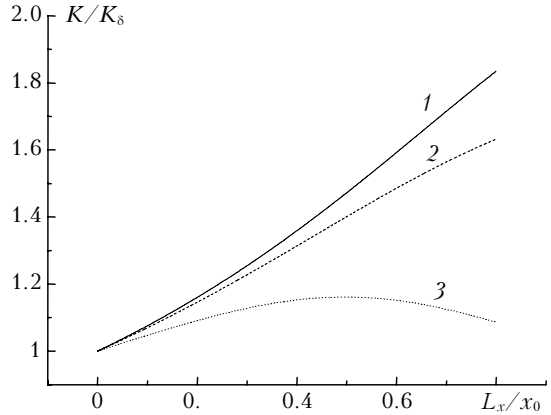


**Fig. 5.** The value of $K/K_\delta$ depending on the effective heights of emission $H_e = 5$ (*1*), 10 (*2*), and 20 m (*3*). $z_0 = 2$ m; $L_x/x_0 = 0.25$ km; the stability class **A**.

In stable atmosphere, $K$ values differ slightly from $K_\delta$ in the chosen variation range of the calculation parameters. The growth of the effective height of emissions $H_e$ (see Fig. 4), and, to be more precise,

the increase of $|H_e - z_0|$, decreases the ratio $K/K_\delta$, because at large difference between heights of the plume axis and the measurement point the contributions from each local source in the limits of emission area have time to mix and to form one Gaussian plume with characteristics close to the linear source.

## Conclusion

Thus, the above-mentioned investigations have shown that when estimating the net emission power of distributed emitters, the method of effective virtual source may be accurate to 50% provided the following conditions are fulfilled:

1) Statistically provided measurements of the path integral concentration should be carried out transversely to wind at distances $x_0 \gg L_x$; the height of measurements $z_0$ does not play a key role.

2) Linear dimensions of the area of an effective source should be selected in such a way as to prevent crowding of actual local sources close to the front $x = L_x$ or back $x = 0$ boundaries of the area.

3) It is desirable (but not obligatory, see Fig. 3) to have equal values of power of real sources inside a chosen area $L_x \times L_y$. The standard deviation of the source powers, obtained as a result of a preliminary evaluation of a particular situation, should not exceed the mean power over an ensemble of sources.

Let us formulate in conclusion an approximate order of actions for calculating the net power of emission using the method of effective source.

1. Preliminary estimation of space configuration and power of individual emission sources at the area under study; determination of the effective emission area $(L_x, L_y)$; selection of the height $z_0$ and the distance $x_0$ for path measurements.

2. Conducting of meteorological measurements; determination of wind velocity at the required level by direct measurements or by calculating from measurements at other levels with the use of the formulae for vertical profile of the wind velocity.[9,12]

3. Determination of turbulent state of the atmosphere (classes of stability) for particular conditions from the measured or visually observed parameters (wind velocity, insolation, cloudiness).[9]

4. Determination of the effective height of emission $H_e$ using expressions for plume elevation, taking into account the rate of outflow, density, and temperature of emissions.[11,13]

5. Determination of coefficients of turbulent diffusion $\sigma_y(x)$, $\sigma_z(x)$ at a given distance from plots or formulae (Packville–Gifford sigma curves) in accordance with a selected model of the atmospheric diffusion.[11,14]

6. Carrying out of the statistically averaged path measurements of the impurity integral concentration $C_{Y_0}(x_0, z_0)$; calculation of $K(x_0, z_0)$ and determination of the net emission power using Eq. (5).

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