

# Fluctuations of natural light outgoing from under the rough sea surface

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Received August 8, 2001

A technique is developed for calculating brightness fluctuations of the solar radiation outgoing from the sea depth through the rough surface. The technique is based on a locally parabolic model of the air/water interface. The dependences of the variance, variation and correlation coefficients of the backscatter on the wind velocity, solar zenith angle, and optical radiation wavelength are studied.

The study of fluctuation characteristics of the natural light outgoing from the sea depth through the rough surface is of special interest in connection with the problems of obtaining and interpretation of shelf images, remote diagnostics of roughness, and real-time estimation of the optical characteristics of the sea water.

In the theoretical aspect, the problem of determining statistical characteristics of the radiation outgoing from the sea is a rather complicated task because of its multifactor character – the backscattered signal (BSS) received by an observation system (OS) is affected by the randomly rough air/water interface, the scattering medium, the limited spatial and angular dimensions of an OS photodetector, and by the geometry of illumination and sighting. The need to take into account correlation effects arising at double passage of the radiation through the same areas of a randomly rough air/water interface leads to particular difficulties. These effects manifest themselves clearly already in studying the first BSS statistical moment.<sup>1,2</sup> The specific features of their manifestation in the higher BSS moments are not sufficiently studied yet. In Refs. 3 and 4, the problem of determination of the spatial spectrum and the spatial correlation function for the brightness of solar radiation outgoing from the sea depth is solved within the framework of the approximation linear with respect to the wave tilts. However, this approximation is valid only under conditions of low scattering at the air/water interface as compared with the scattering in the water medium and therefore it cannot be accepted as absolutely adequate.

This work was aimed at developing the technique for calculating the variance of brightness fluctuations of the radiation outgoing from the sea depth based on a locally parabolic model of the rough air/water interface (without the use of linear approximation) and at the study of the dependence of the variation coefficient and of the spectral correlation function on the characteristics of sounding path within the

framework of the ray-optics approximation in photometric terms. The air/water interface is assumed random and obeying Gaussian statistics.

## 1. Formulation of the problem. Initial equations

The light source (the Sun) with the angular directional pattern  $D_s(\Omega_i)$  illuminates a rough sea surface at an incidence angle  $\theta_s$  (Fig. 1). The radiation penetrating into the water through the surface is scattered and partly absorbed in the water depth. A part of the scattered radiation comes back into the atmosphere and is recorded by a photodetector of the observation system with the receiving directional pattern  $D_r(\mathbf{r}_1, \Omega_i)$ . Hereinafter the vector  $\Omega_i$  denotes the projection of the unit vector  $\Omega_i^0$  onto the plane  $z = \text{const}$ ,  $\mathbf{r}_i$  are the coordinates of points at the plane  $z = \text{const}$ . Nadir sensing of the sea is assumed.

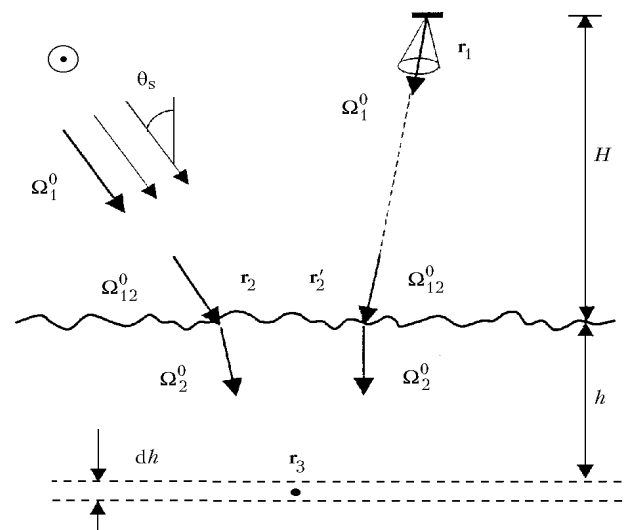


Fig. 1. Observation geometry.

The equation for power of the light signal coming from the sea depth (integral BSS) can be written as follows:

$$P_i = \int_0^\infty P_d(h)dh, \tag{1}$$

where  $P_d dh$  is the power of the backscattered signal from an elementary layer with the thickness  $dh$  from the depth  $h$  (the power of the differential BSS).

The equation for  $P_d$  was derived in Ref. 5 devoted to the study of fluctuations of the lidar BSS as applied to sensing system with an artificial light source:

$$P_d(h) = \frac{B_s \rho_d}{\pi m^2} \iint_{-\infty}^\infty E_s(\mathbf{r}_3, h) E_r(\mathbf{r}_3, h) d\mathbf{r}_3, \tag{2}$$

where

$$\begin{aligned} E_{s,r}(\mathbf{r}_3, h) &= \\ &= \int \dots \int_{-\infty}^\infty D_{s,r}(\mathbf{r}_1, \Omega_1) e_{s,r}(\mathbf{r}_1 \rightarrow \mathbf{r}_3, \Omega_1, h) d\mathbf{r}_1 d\Omega_1, \\ e_{s,r}(\bullet) &= \\ &= \int \dots \int_{-\infty}^\infty G_a(\mathbf{r}_1 \rightarrow \mathbf{r}_2, \Omega_1 \rightarrow \Omega_{12}) G_{in}(\Omega_{12} \rightarrow \Omega_2, \mathbf{r}_2) \times \\ &\quad \times e_w(\mathbf{r}_2 \rightarrow \mathbf{r}_3, \Omega_2, h) d\mathbf{r}_2 d\Omega_{12} d\Omega_2; \end{aligned}$$

$B_s$  is the source brightness,  $G_a$  is the Green's function of the equation of radiative transfer in the atmosphere that determines the brightness of the light field at the point  $\mathbf{r}_2$  in the direction  $\Omega_{12}$  at medium illumination by a point unidirectional source of a unit power located at the point  $\mathbf{r}_1$  and emitting in the direction  $\Omega_1$ ;  $G_{in}$  is the Green's function of the interface surface that describes deformation of the brightness body as the radiation passes through the rough air/water interface;  $e_w$  is the distribution of illumination from a point unidirectional source in the water medium at the depth  $h$ ;  $m$  is the refractive index of water;  $\rho_d = \sigma_\pi/4$ ,  $\sigma_\pi$  is the scattering coefficient of an elementary water volume at the angle of  $180^\circ$ .

Let us modify this equation to meet the conditions of our problem. For the light source (infinitely large in our case) that illuminates the water surface at some angle, we can write

$$\begin{aligned} G_a(\bullet) &= \delta(\mathbf{r}_2 - \mathbf{r}_1 - \Omega_1 H / \gamma_s) \delta(\Omega_1 - \Omega_{12}), \\ G_{in}(\bullet) &= m^2 \delta[\Omega_{12} - m\Omega_2 + A_s \mathbf{q}(\mathbf{r}_2)]; \\ e_w(\bullet) &\equiv e_w(\mathbf{r}_3 - \mathbf{r}_2 - l\Omega_2, l), \tag{3} \\ A_s &= m\gamma_{0s} - \gamma_s, \quad l = h/\gamma_{0s}, \quad \gamma_s = \sqrt{1 - \Omega_s^2}, \\ \gamma_{0s} &= \sqrt{1 - (\Omega_s/m)^2}, \quad \omega_s = \sin\theta_s; \end{aligned}$$

$\mathbf{q}$  is the gradient vector of tilts of the rough water surface.

For the OS photodetector sighting the water surface along the normal to its mean level, similar equations take the form:

$$\begin{aligned} G_a(\bullet) &= \delta(\mathbf{r}_2 - \mathbf{r}_1 - H\Omega_1) \delta(\Omega_1 - \Omega_{12}), \\ G_{in}(\bullet) &= m^2 \delta[\Omega_{12} - m\Omega_2 + A_r \mathbf{q}(\mathbf{r}_2)]; \tag{4} \\ e_w(\bullet) &\equiv e_w(\mathbf{r}_3 - \mathbf{r}_2 - h\Omega_2, h), \quad A_r = m - 1. \end{aligned}$$

It turns out to be practically impossible to analyze statistical characteristics of the integral BSS (1) in the general form with the use of Eqs. (2)–(4) because of the presence of high-order integrals. To overcome (although partly) these difficulties, consider a simplified locally parabolic model of the interface. Assume that the function of interface tilts in some circular region  $M_q(\mathbf{r}_2)$  larger than the OS resolution element is described by the following equation:

$$\mathbf{q}(\mathbf{r}_2) = \mathbf{q}_0 + p\mathbf{r}_2. \tag{5}$$

It describes a spherical lens (with the curvature  $p$ ) located at the sea surface at an angle  $\mathbf{q}_0$  to the horizon. The interface beyond the region  $M_q$  is described by a random function  $\mathbf{q}(\mathbf{r}_2)$ .

With the use of Eqs. (2)–(5) the equations for  $E_{s,r}$  take the following form:

$$E_s(\mathbf{r}_3, h) = E_1(\mathbf{r}_3, h) + E_2(\mathbf{r}_3, h), \tag{6}$$

where

$$\begin{aligned} E_1(\bullet) &= \frac{1}{(2\pi)^2} \iint_{-\infty}^\infty F_s\left(\mathbf{k} \frac{l}{m}\right) F_q[\mathbf{k}(1 + a_s p)] \times \\ &\quad \times F_w(\mathbf{k}, l) \exp[i\mathbf{k}(\mathbf{r}_3 - a_s \mathbf{q}_0)] d\mathbf{k}, \\ E_2(\bullet) &= \frac{1}{(2\pi)^2} \iint_{-\infty}^\infty F_s\left(\mathbf{k} \frac{l}{m}\right) [1 - M_q(\mathbf{r}_2)] \times \\ &\quad \times F_w(\mathbf{k}, l) \exp\{i\mathbf{k}[\mathbf{r}_3 - \mathbf{r}_2 - a_s \mathbf{q}(\mathbf{r}_2)]\} d\mathbf{k} d\mathbf{r}_2, \\ E_r(\mathbf{r}_3, h) &= \\ &= \frac{1}{(2\pi)^2} \iint_{-\infty}^\infty F_r[\mathbf{k}(1 + a_r p), \mathbf{k}H(1 + \alpha + a_r p)] \times \\ &\quad \times F_w(\mathbf{k}, h) \exp[i\mathbf{k}(\mathbf{r}_3 - a_r \mathbf{q}_0)] d\mathbf{k}; \tag{7} \end{aligned}$$

the functions  $F_s$ ,  $F_r$ ,  $F_q$ , and  $F_w$  are the Fourier transforms of the functions  $D_s$ ,  $D_r$ ,  $M_q$ , and  $e_w$ ;

$$a_s = A_s l / m, \quad a_r = A_r h / m, \quad \alpha = h / (mH).$$

Upon substitution of Eqs. (6) and (7) into Eq. (2), we obtain

$$P_d(h) = P_1(h) + P_2(h), \tag{8}$$

where

$$\begin{aligned} P_1(h) &= \frac{B_s \rho_d}{\pi m^2} \frac{1}{(2\pi)^2} \iint_{-\infty}^\infty F_s\left(\mathbf{k} \frac{l}{m}\right) F_r(\mathbf{k} \xi_r, \mathbf{k} H \eta_r) \times \\ &\quad \times F_q(\mathbf{k} \xi_s) F_w(\mathbf{k}, l) F_w(\mathbf{k}, h) \exp[i(a_r - a_s) \mathbf{k} \mathbf{q}_0] d\mathbf{k}, \tag{9} \end{aligned}$$

$$\begin{aligned}
P_2(h) = & \frac{B_s \rho_d}{\pi m^2} \frac{1}{(2\pi)^2} \iint_{-\infty}^{\infty} F_s\left(\mathbf{k} \frac{l}{m}\right) F_r(\mathbf{k} \xi_r, \mathbf{k} H \eta_r) \times \\
& \times [1 - M_q(\mathbf{r})] F_w(\mathbf{k}, l) F_w(\mathbf{k}, h) \exp(i a_r \mathbf{k} \mathbf{q}_0) \times \\
& \times \exp\{-i \mathbf{k}[\mathbf{r} + a_s \mathbf{q}(\mathbf{r})]\} d\mathbf{k} d\mathbf{r}, \quad (10) \\
\xi_{s,r} = & 1 + a_{s,r} p; \quad \eta_r = 1 + \alpha + a_r p.
\end{aligned}$$

## 2. Models of the source, receiver, and scattering medium

For the below consideration, we should concretize the transfer functions of the light source, OS photodetector, and scattering in the water depth. Through approximation of the source and receiver directional patterns by Gaussian functions, we obtain

$$F_s(\omega) = \Delta_s \exp(-\Delta_s \omega^2 / 4\pi), \quad (11)$$

$$F_r(\mathbf{k}, \omega) = \Sigma_r \Delta_r \exp[-(\Sigma_r k^2 + \Delta_r \omega^2) / 4\pi], \quad (12)$$

where  $\Sigma_r$  is the area of the receiving aperture;  $\Delta_{s,r}$  is the solid angle of emission and reception.

As the optical transfer function of the scattering medium, we use the Fourier transform of illumination distribution in a medium with strongly anisotropic scattering that corresponds to the automodel solution of the radiative transfer equation<sup>6</sup>:

$$F_w(\mathbf{k}, h) = \sum_{i=1}^2 C_i(h) \exp[-g_i(h) k^2 / 4\pi], \quad (13)$$

where  $C_1 = \exp(-\varepsilon h)$  is the amplitude of the nonscattered radiation component,  $\varepsilon = \sigma + \kappa$ ;  $C_2 = \exp(-\kappa h) / \cosh \zeta - \exp(-\varepsilon h)$  is the amplitude of the scattered radiation component;  $g_1 = 0$ ;  $g_2 = 4\pi(\zeta - \tanh \zeta) / \{\kappa^2 \Omega_\infty [1 - \cosh \zeta \exp(-\sigma h)]\}$  is the cross section of a narrow light beam at the distance  $h$  from the source,  $\zeta = 0$ ,  $\Omega_\infty \kappa h$ ,  $\Omega_\infty = \sqrt{2\sigma \overline{\gamma^2}} / \kappa$ ,  $\sigma$  is the scattering coefficient of water;  $\kappa$  is the absorption coefficient;  $\overline{\gamma^2}$  is the variance of the scattering phase function.

The region occupied by the lens on the water surface is described by the function

$$M_q(\mathbf{r}) = \exp(-\pi \rho^2 / S_q), \quad (14)$$

where  $S_q = \pi \rho_q^2$ ,  $\rho_q$  is the characteristic correlation length of surface tilts.

## 3. Equations for the BSS moments

Before deriving the equations describing the first and the second statistical moments of the BSS power, let us accept some simplifying assumptions. As is seen from Eqs. (9) and (10), they include two random parameters:  $p$  and  $\mathbf{q}_0$ , as well as the random function  $\mathbf{q}(\mathbf{r})$ . Assume, first, that all these parameters and the

function do not correlate with each other. Second, assume that the function  $\mathbf{q}(\mathbf{r})$  has a  $\delta$ -correlated character (this means that the interface beyond the lens region has small-scale irregularities). Besides, we assume, in what follows, the Gaussian character of the function of probability distribution of tilts (with the variance  $\sigma_q^2$ ) and curvatures (with the variance  $\sigma_p^2$ ).

Let us show that if these conditions are fulfilled, the equations for calculation of the first and second BSS statistical moments are rather simple and, what is also important, physically illustrative.

After substituting Eqs. (9) and (10), with the allowance for Eqs. (11)–(14), into Eq. (8) and statistically averaging over realizations of lens tilts (the integral over tilts  $\mathbf{q}_0$  can be calculated analytically), we obtain the equation for the first BSS statistical moment:

$$\bar{P}_i = \int_0^\infty \bar{P}_d(h) dh, \quad (15)$$

where

$$\bar{P}_d(h) = \bar{P}_1(h) + \bar{P}_2(h),$$

$$\bar{P}_1(h) = A \sum_{i=1}^2 \sum_{j=1}^2 C_i C_j \left\langle \frac{S_q}{Q_{ij}} \right\rangle,$$

$$\bar{P}_2(h) = A \sum_{i=1}^2 \sum_{j=1}^2 C_i C_j \left( 1 - \left\langle \frac{S_q}{T_{ij}} \right\rangle \right);$$

$$A = B_s \Delta_s \Sigma_r \Delta_r \rho_d / (\pi m^2);$$

the angular brackets  $\langle \rangle$  denote averaging over the curvature  $p$ ;

$$Q_{ij} = \Delta_s (l/m)^2 + \xi_r^2 \Sigma_r + \eta_r^2 S_r +$$

$$+ \xi_s^2 S_q + 2\pi \sigma_q^2 (a_s - a_r)^2 + g_i + g_j;$$

$$T_{ij} = \Delta_s (l/m)^2 + \xi_r^2 \Sigma_r + \eta_r^2 S_r + S_q +$$

$$+ 2\pi \sigma_q^2 (a_s^2 + a_r^2) + g_i + g_j;$$

$S_r = \Delta_r H^2$ ; the functions  $C_i$  and  $g_i$  depend on  $h$ , and the functions  $C_j$  and  $g_j$  depend on  $l$ .

It should be noted that the equation for the first BSS moment includes only two integrals: one over the curvature  $p$  and another over the depth  $h$ . These integrals are calculated numerically.

From Eq. (15) we can derive, as a consequence, the equation describing the BSS power under conditions of smooth interface (BSS through the smooth water surface):

$$P_i^0 = A \int_0^\infty \left( \sum_{i=1}^2 \sum_{j=1}^2 C_i C_j dh \right).$$

The equation for the second BSS statistical moment can be obtained in the way similar to Eq. (15). After cumbersome, but simple transformations, this equation takes the form:

$$\bar{P}_i^2 = \int_0^\infty \int_0^\infty M_d(h_1, h_2) dh_1 dh_2, \quad (16)$$

where

$$M_d(h_1, h_2) = M_{11}(h_1, h_2) + M_{12}(h_1, h_2) + M_{21}(h_1, h_2) + M_{22}(h_1, h_2),$$

$$M_{11} = \langle P_1(h_1)P_1(h_2) \rangle, \quad M_{12} = \langle P_1(h_1)P_2(h_2) \rangle;$$

$$M_{21} = \langle P_2(h_1)P_1(h_2) \rangle, \quad M_{22} = \langle P_2(h_1)P_2(h_2) \rangle;$$

$$M_{11} = A^2 \sum_{i=1}^2 \sum_{j=1}^2 \sum_{k=1}^2 \sum_{l=1}^2 C_i C_j C_k C_l \left\langle \frac{S_q^2}{S_{ij} S_{kl}} \{1 + 2\pi\sigma_q^2 \times [(a_{s1} - a_{r1})^2 / S_{ij} + (a_{s2} - a_{r2})^2 / S_{kl}]\}^{-1} \right\rangle;$$

$$S_{ij} = \Delta_s(l_1/m)^2 + \xi_{r1}^2 \Sigma_r + \eta_{r1}^2 S_r + \xi_{s1}^2 S_q + g_i + g_j;$$

$$S_{kl} = \Delta_s(l_2/m)^2 + \xi_{r2}^2 \Sigma_r + \eta_{r2}^2 S_r + \xi_{s2}^2 S_q + g_k + g_l;$$

$$\xi_{r1,r2} = 1 + a_{r1,r2}p, \quad \xi_{s1,s2} = 1 + a_{s1,s2}p,$$

$$\eta_{r1,r2} = 1 + h_{1,2}/(mH) + a_{r1,r2}p, \quad a_{r1,r2} = A_r h_{1,2}/m,$$

$$a_{s1,s2} = A_s l_{1,2}/m, \quad l_{1,2} = h_{1,2}/\gamma_{0s}.$$

Here the functions  $C_i$  and  $g_i$  depend on  $h_1$ , the functions  $C_j$  and  $g_j$  depend on  $l_1$ , the functions  $C_k$  and  $g_k$  depend on  $h_2$ , and the functions  $C_l$  and  $g_l$  depend on  $l_2$ ;

$$M_{12} = A^2 \sum_{i=1}^2 \sum_{j=1}^2 \sum_{k=1}^2 \sum_{l=1}^2 C_i C_j C_k C_l \left\langle \frac{S_q}{S_{ij}} \left( \frac{1}{T_1} - \frac{S_q}{\Sigma_{kl}} \frac{1}{T_2} \right) \right\rangle,$$

$$T_1 = 1 + 2\pi\sigma_q^2 (a_{s1} - a_{r1})^2 / S_{ij},$$

$$T_2 = 1 + 2\pi\sigma_q^2 [(a_{s1} - a_{r1})^2 / S_{ij} + a_{r2}^2 / \Sigma_{kl}],$$

$$\Sigma_{kl} = \Delta_s(l_2/m)^2 + \xi_{r2}^2 \Sigma_r + \eta_{r2}^2 S_r + S_q + 2\pi\sigma_q^2 a_{s2}^2 + g_k + g_l;$$

$$M_{21} = A^2 \sum_{i=1}^2 \sum_{j=1}^2 \sum_{k=1}^2 \sum_{l=1}^2 C_i C_j C_k C_l \left\langle \frac{S_q}{S_{kl}} \left( \frac{1}{T_1} - \frac{S_q}{\Sigma_{ij}} \frac{1}{T_2} \right) \right\rangle,$$

$$T_1' = 1 + 2\pi\sigma_q^2 (a_{s2} - a_{r2})^2 / S_{kl};$$

$$T_2' = 1 + 2\pi\sigma_q^2 [(a_{s2} - a_{r2})^2 / S_{kl} + a_{r1}^2 / \Sigma_{ij}];$$

$$\Sigma_{ij} = \Delta_s(l_1/m)^2 + \xi_{r1}^2 \Sigma_r + \eta_{r1}^2 S_r + S_q + 2\pi\sigma_q^2 a_{s1}^2 + g_i + g_j;$$

$$M_{22} = A^2 \sum_{i=1}^2 \sum_{j=1}^2 \sum_{k=1}^2 \sum_{l=1}^2 C_i C_j C_k C_l \langle (1 - I_1 - I_2 + I_{12}) \rangle,$$

$$I_1 = S_q / (\Sigma_{ij} + 2\pi\sigma_q^2 a_{r1}^2),$$

$$I_2 = S_q / (\Sigma_{kl} + 2\pi\sigma_q^2 a_{r2}^2),$$

$$I_{12} = S_q^2 / (\Sigma_{ij}\Sigma_{kl} + 2\pi\sigma_q^2 a_{r1}^2 \Sigma_{kl} + 2\pi\sigma_q^2 a_{r2}^2 \Sigma_{ij}).$$

As follows from the obtained equations, to determine the second BSS statistical moment, we should calculate the series of triple integrals, in each of which one integral is calculated over the curvature while two others over the depth. It should be noted that the existing personal computers can perform these calculations for rather short time.

In spite of the seeming complexity, Eqs. (15) and (16) are physically simple and illustrative. The keys for their understanding are the parameters  $Q_{ij}$ ,  $T_{ij}$ , and other similar parameters in the equations for  $\bar{P}_i$  and  $\bar{P}_i^2$ , which determine the "concentration" of radiation from the source and receiver at the depth  $h$ . If we simplify successively the considered model fluctuations, first neglecting the effects of scattering in water, then excluding the interface region beyond the lens from the consideration and assuming that  $\mathbf{q}_0 = 0$  inside the lens, and finally taking the radiation to be normal to the interface, then we come to a rather simple one-lens sensing scheme, which was described for the first time and thoroughly studied in Ref. 7.

#### 4. Analysis of numerical results

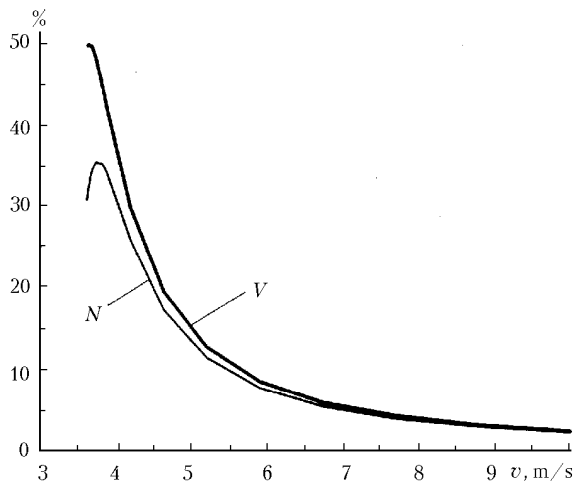
Equations (15) and (16) allow us to study the dependence of the variation and correlation coefficients of the integral BSS on the degree of roughness, the angle of sea surface illumination, and the radiation wavelength. Some of the obtained results are shown in Figs. 2–9. In all the considered cases, the angular pattern of emission was  $0.5^\circ$ , that of reception was  $0.1^\circ$ , the diameter of the receiving aperture was 0.04 m, the OS height was 300 m, and the wind speed was taken 4 m/s, in the cases that it was not a variable.

To calculate the tilt correlation length, which determines the size of the region occupied by the lens, we use the equation from Ref. 5:

$$\rho_q = \sqrt{2\sigma_q^2 / \sigma_p^2},$$

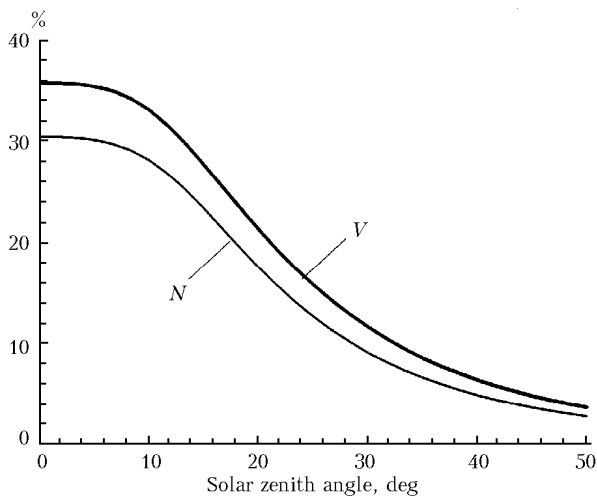
where  $\sigma_q^2 = (3 + 1.92v)10^{-3}$ ,  $\sigma_p^2 = (-4.13 + 1.23v)^2$ ;  $v$  is the wind speed.

Figure 2 depicts the dependences of the so-called<sup>1</sup> BSS amplification coefficient  $N = \bar{P}_i / P_i^0 - 1$  and the BSS variation coefficient  $V = \sigma_i / \bar{P}_i$  (here  $\sigma_i^2 = \bar{P}_i^2 - P_i^0$  is the variance of the BSS fluctuations) on the wind speed over the sea surface  $v$  (the optical parameters  $\sigma = 0.09 \text{ m}^{-1}$  and  $\kappa = 0.0125 \text{ m}^{-1}$  correspond to the ocean water). The general monotonically decreasing character of these dependences is explained by the facts that, first, the area of the lens region of the interface decreases with the increasing wind speed and, second, the angular spread of rays increases at their refraction at the water surface and, as a consequence, illumination at all sensing depths decreases. It should be noted that the monotonically decreasing character of the BSS amplification and variation coefficients keeps for any angles of sea surface illumination by the Sun. In general, it follows from simple physical reasoning that at the transition to the finely rough interface (just this transition occurs with the increase of the wind velocity) the fluctuations, including those connected with the correlation effects, should decrease. A small initial part with the increasing  $N(v)$  is not characteristic, and its origin is explained in Ref. 8.



**Fig. 2.** Dependences of the amplification  $N$  and variation  $V$  coefficients on the wind speed.

Figure 3 depicts the dependences of the amplification and variation coefficients on the solar zenith angle (or, in other words, on the angle between the directions of illumination and sighting of the water surface).

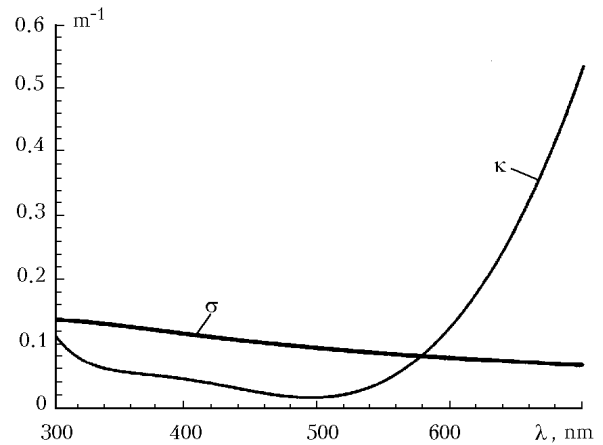


**Fig. 3.** Dependences of the amplification  $N$  and variation  $V$  coefficients on the solar zenith angle.

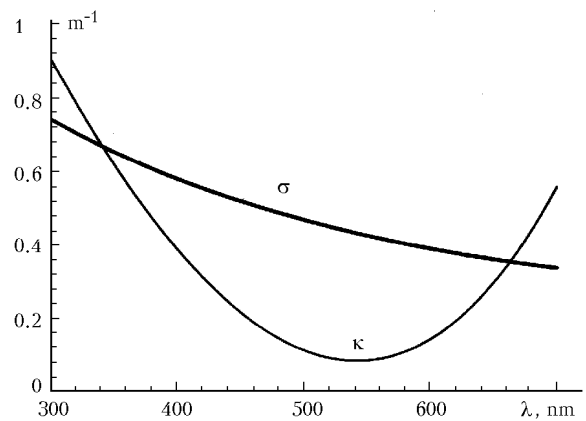
These dependences obtained for the ocean conditions ( $\sigma = 0.09 \text{ m}^{-1}$ ,  $\kappa = 0.0125 \text{ m}^{-1}$ ) are qualitatively very similar, and they both are monotonically decreasing. It should be noted that for the first time the dependence  $N(\theta_s)$  was studied in Ref. 1 based on a more general model of the random air/water interface. The results depicted in Fig. 3 well agree with similar results obtained in Ref. 1.

In studying the BSS fluctuation characteristics, the problem of their dependence on the optical radiation wavelength  $\lambda$  is very important. Since the scattering  $\sigma$  and absorption  $\kappa$  characteristics in our problem depend on  $\lambda$ , the subjects of further

consideration are two characteristic types of water: ocean (clean water) and sea (turbid water). Figures 4 and 5 depict the dependences of  $\sigma$  and  $\kappa$  on  $\lambda$  obtained in Refs. 9 and 10. Below, based on these results, we consider the spectral dependences of such BSS statistical characteristics, as variance, amplification coefficient, and variation coefficient, as well as the coefficient of correlation between signals received at two different wavelengths.



**Fig. 4.** Optical characteristics of the ocean water.

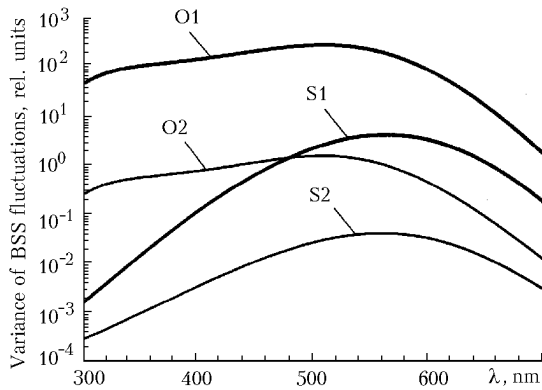


**Fig. 5.** Optical characteristics of the sea water.

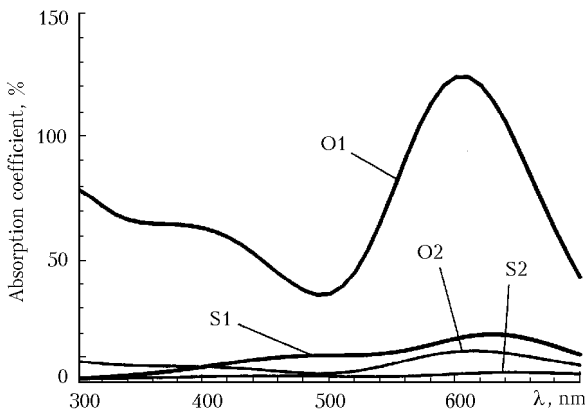
Figure 6 depicts the dependence of the variance of BSS fluctuations  $\sigma_i^2$  on the radiation wavelength for two types of water and two values of the solar zenith angle (0 and 50°). All these dependences are nonmonotonic with the maxima at 510 nm (ocean) and 560 nm (sea).

It should be noted that the minima of the spectral dependences of the absorption coefficient are at  $\lambda = 500 \text{ nm}$  (ocean) and  $540 \text{ nm}$  (sea). Thus, it can be concluded that fluctuations of the integral BSS are maximal in the transparency window of a given type of water.

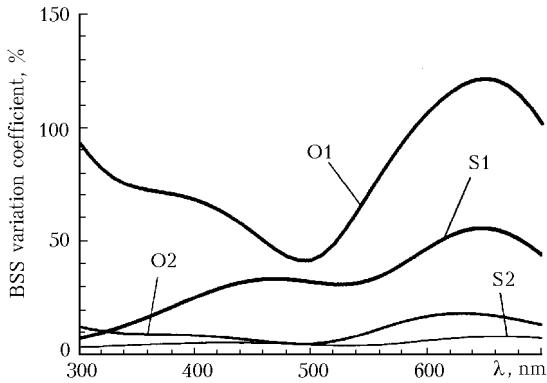
The wavelength dependences of the BSS amplification and variation coefficients depicted in Figs. 7 and 8 are more complicated.



**Fig. 6.** Dependences of the variance of BSS fluctuations on radiation wavelength: ocean,  $\theta_s = 0^\circ$  (O1), ocean,  $\theta_s = 50^\circ$  (O2), sea,  $\theta_s = 0^\circ$  (S1), and sea,  $\theta_s = 50^\circ$  (S2).



**Fig. 7.** Wavelength dependences of the BSS amplification coefficients (designations are the same as in Fig. 6).



**Fig. 8.** Wavelength dependences of the BSS variation coefficients (designations are the same as in Fig. 6).

Comparing the dependences  $N(\lambda)$  obtained at different incidence angles, we can easily see their similarity. The same is true for the dependences  $V(\lambda)$ . It should also be noted that the dependences  $N(\lambda)$  and  $V(\lambda)$  obtained for the same conditions (for example, for the ocean type of water and  $\theta_s = 0^\circ$ ) have similar character. This circumstance is indicative of the same character of manifestation of the correlation effects in the formation of the first and second BSS statistical moments. It is also interesting that all the dependences

shown in Figs. 7 and 8 have local minima, whose positions coincide with the positions of maxima of the corresponding curves for the variance of BSS fluctuations (see Fig. 6), which, in their turn, coincide with the positions of transparency windows of the corresponding types of water. Finally, it is worthy to note the presence of rather pronounced maxima in all the curves, whose positions are determined by roughly the same wavelength of 650 nm. This interesting fact suggests not obvious conclusion that BSS fluctuations are colored in red.

Correlations between BSS received at different wavelengths are of great scientific and practical interest. To study these correlations, we introduce the correlation coefficient (normalized correlation function of BSS at two  $\lambda$ ) as:

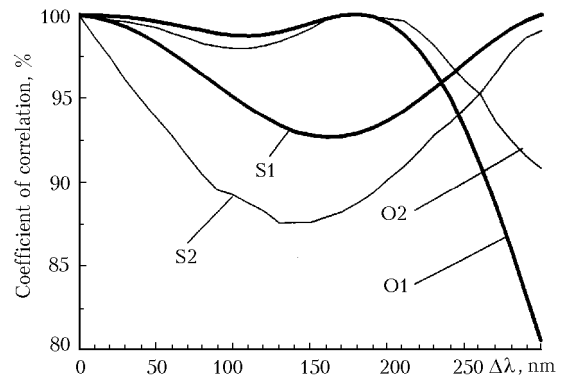
$$R = \sigma_{12}^2 / (\sigma_1 \sigma_2),$$

where

$$\sigma_{12}^2 = \overline{(P)_i(\lambda_1)P_i(\lambda_2)} - \bar{P}_i(\lambda_1) \bar{P}_i(\lambda_2),$$

$$\sigma_{1,2}^2 = \overline{P_i^2(\lambda_{1,2})} - \bar{P}_i^2(\lambda_{1,2}).$$

We study here the correlation between two signals outgoing from the water depth, one of which is received at the wavelength of 400 nm and the other one at the wavelength shifted by  $\Delta\lambda$ . Figure 9 depicts the dependence of the correlation coefficient  $R$  on the wavelength difference  $\Delta\lambda$  for different types of water and different illumination angles. Analyzing this dependence, we can notice high correlation between BSS at different wavelengths. The increase of solar zenith angle does not lead to some significant decorrelation of signals. The local minima of  $R(\Delta\lambda)$  are located so that the wavelength of the second BSS  $\lambda_2 = \lambda_1 + \Delta\lambda$  falls within the transparency window of the corresponding type of water.



**Fig. 9.** Coefficient of correlation between signals at  $\lambda_1$  and  $\lambda_2$  as a function of  $\Delta\lambda = \lambda_2 - \lambda_1$  (designations are the same as in Fig. 6).

High correlation between the signals at different wavelengths is explained by the fact that BSS fluctuations are formed largely by the surface water layer, in which the scattering and absorption effects manifest themselves rather weakly, while the BSS spectral dependence is determined just by these (and

only these) effects. We can assert that for marked manifestation of the decorrelation effect for BSS at different wavelengths, the medium should have strong dispersion of optical characteristics and, second, the photon mean free path in water should be comparable with the characteristic focusing depth of a randomly rough water surface.

### Acknowledgments

This work was partly supported by the Russian Foundation for Basic Research (Project No. 99-05-64798).

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