PROPAGATION OF A LASER BEAM THROUGH A LENS-LIKE MEDIUM ALONG THE ROUND–TRIP PATH

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The spatial structure of a Gaussian laser beam reflected from a specular, corner–tube and Lambertian reflectors of arbitrary size positioned in a lens–like defocusing aberrational–free medium is investigated theoretically. On the basis of the analysis of the mutual coherence function of the second order of reflected sounding radiation, the possibilities are examined of using the round–trip–path modification of the techniques of thermal lens, mirage effect, refocusing, and displacement of an image in order to determine the optical parameters of the investigated medium.

The methods of optical–refraction spectroscopy developed now[1-5] are usually used in their base modification. The base configuration of sensing of a lens–like medium (a refraction channel) assumes that the source and the receiver of optical radiation are positioned at the opposed ends of the measuring path and that sounding radiation passes once through an investigated medium. However, in the particular situations it is possible to use the round–trip–path modification of sensing of a lens–like medium, in which the source and the receiver of optical radiation are positioned at one end of the measuring path while the sounding radiation passes twice through an investigated medium. This may either help to conveniently arrange the instrumentation, or to increase the path length passed by sounding radiation through a lens–like medium, or may be used to study the opaque media, which absorb pumping radiation and reflect sounding radiation. The propagation of the optical radiation along the round–trip path through a lens–like medium was first considered in Ref. 4, but only the cases of specular reflection from the infinite flat and point reflectors were treated there. In this paper the spatial structure of a laser beam reflected from the specular, corner–cube, and Lambertian reflectors of arbitrary size positioned in a lens–like defocusing aberration–free medium is studied theoretically. The possibilities of using the round–trip–path modification of the techniques of thermal lens,[1–3] mirage effect,[1,3] refocusing,[4,5] and displacement of an image[4,5] for the determination of the optical parameters of the investigated medium are studied here.

Let a beam of laser sounding radiation be propagated through an aberration–free defocusing lens–like medium (a refraction channel),[4–6] and the beam optical axis coincides with the $0x$ axis. The beam propagates in the positive direction along the $0x$ axis from the plane $x = 0$ to the reflector positioned in the plane $x = L$. Let this beam be received in the plane $x = 0$ after its reflection and propagation in the backward direction along the $0x$ axis. The formula for the amplitude of the reflected wave $U(0, \boldsymbol{q})$ in this case can be written as follows:

$$U(0, \boldsymbol{q}) = \int d\boldsymbol{q}' d\boldsymbol{q}'' d\boldsymbol{q}''' \tilde{G}(0, \boldsymbol{q}; L, \boldsymbol{q}') \times$$
$$\times\ V(\boldsymbol{q}', \boldsymbol{q}'')\, G(L, \boldsymbol{q}''; 0, \boldsymbol{q}''')\, U_0(\boldsymbol{q}'''), \tag{1}$$

where $V(\boldsymbol{q}', \boldsymbol{q}'')$ is the local reflectance, $G(L, \boldsymbol{q}''; 0, \boldsymbol{q}''')$ is the Green's function of a lens–like medium describing the "forward" propagation of the wave,[6] $\tilde{G}(0, \boldsymbol{q}; L, \boldsymbol{q}')$ is the Green's function of a lens–like medium describing the "backward" propagation of the wave,[6] and $U_0(\boldsymbol{q}''')$ is the initial distribution of a laser radiation field. $\boldsymbol{q} = \{y, z\}$ is the transverse coordinate. In accordance with formula (1), the coherence function of the second order of reflected radiation has the form

$$\Gamma_2(0, \boldsymbol{q}_1, \boldsymbol{q}_2) = \overline{U(0, \boldsymbol{q}_1)U^*(0, \boldsymbol{q}_2)} =$$
$$= \int d\boldsymbol{q}_1' d\boldsymbol{q}_1'' d\boldsymbol{q}_1''' d\boldsymbol{q}_2' d\boldsymbol{q}_2'' d\boldsymbol{q}_2''' \ \overline{V(\boldsymbol{q}_1', \boldsymbol{q}_1'')\, V^*(\boldsymbol{q}_2', \boldsymbol{q}_2'')} \times$$
$$\times\ \tilde{G}(0, \boldsymbol{q}_1; L, \boldsymbol{q}_1')\, \tilde{G}^*(0, \boldsymbol{q}_2; L, \boldsymbol{q}_2')\, \tilde{G}(L, \boldsymbol{q}_1''; 0, \boldsymbol{q}_1''') \times$$
$$\times\ G^*(L, \boldsymbol{q}_2''; 0, \boldsymbol{q}_2''')\, U_0(\boldsymbol{q}_1''')\, U_0^*(\boldsymbol{q}_2'''), \tag{2}$$

where a bar denotes statistical averaging over an ensemble of realizations of fluctuations of the heights of the reflector surface roughness. Let the initial distribution of the field of a laser source be taken to be in the form of a single–mode Gaussian beam

$$U_0(\boldsymbol{q}) = U_0 \exp\left\{-\frac{(\boldsymbol{q}-\boldsymbol{q}_0)^2}{2a_2^0} - \frac{ik}{2R_0}(\boldsymbol{q}-\boldsymbol{q}_0)^2 + ik\varphi\mathbf{n}(\boldsymbol{q}-\boldsymbol{q}_0)\right\}, \tag{3}$$

where U_0 is the amplitude of the optical field at the center of the exit aperture, a_0 is the initial radius of the sounding beam, R_0 is the curvature radius of the wavefront at the center of the radiating aperture, $k = 2\pi/\lambda$, λ is the wavelength of the sounding radiation in the vacuum, $\boldsymbol{q}_0$ is the radial distance which determines the displacement of the center of the radiating aperture with respect to the optical axis of the channel, $\boldsymbol{n}$ is the unit vector of the projection of the normal to the phase front upon the YOZ plane, φ is the angle between the normal to the phase front of a beam and the optical axis of the channel ($\varphi \ll \pi$), and $q = \sqrt{y^2 + z^2}$.

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The Green's functions for "forward" and "backward" propagation of a beam through the aberration–free lens–like medium, whose optical axis coincides with the $0x$ axis, have the form:[6]

$$G(L,\, \mathbf{q};\, 0,\, \mathbf{q}') = \frac{k}{2\pi i\, F_0 U_2\left(\frac{L}{F_0}\right)} \exp\left\{ \frac{ikU_2'\left(\frac{L}{F_0}\right)}{2F_0U_2\left(\frac{L}{F_0}\right)} q^2 - \frac{ik}{F_0U_2\left(\frac{L}{F_0}\right)} \mathbf{q}\mathbf{q}' + \frac{ikU_1\left(\frac{L}{F_0}\right)}{2F_0U_2\left(\frac{L}{F_0}\right)} \mathbf{q}'^2 \right\}, \tag{4}$$

$$\tilde{G}(0,\, \mathbf{q}';\, L,\, \mathbf{q}) = \frac{k}{2\pi i\, F_0 \tilde{U}_2\left(\frac{L}{F_0}\right)} \exp\left\{ \frac{ik\tilde{U}_2'\left(\frac{L}{F_0}\right)}{2F_0\tilde{U}_2\left(\frac{L}{F_0}\right)} q^2 - \frac{ik}{F_0\tilde{U}_2\left(\frac{L}{F_0}\right)} \mathbf{q}\mathbf{q}' + \frac{ik\tilde{U}_1\left(\frac{L}{F_0}\right)}{2F_0\tilde{U}_2\left(\frac{L}{F_0}\right)} \mathbf{q}'^2 \right\}, \tag{5}$$

where $U_1(x)$ and $U_2(x)$ and $\tilde{U}_1(x)$ and $\tilde{U}_2(x)$ are the corresponding partial solutions of the equations

$$U''(x) - \frac{F_0^2}{F^2(x)}\, U(x) = 0$$

and

$$\tilde{U}''(x) - \frac{F_0^2}{F^2(L - x)}\, \tilde{U}(x) = 0$$

with the boundary conditions

$$U_1(0) = U_2'(0) = \tilde{U}_1(0) = \tilde{U}_2'(0) = 1,$$

$$\tilde{U}_1(0) = U_2(0) = \tilde{U}_1'(0) = \tilde{U}_2(0) = 0,$$

while $F(x)$ is the local focal distance of a lens–like medium (a refraction channel)[4,5] $F_0 = F(x = 0)$ is the "initial" focal distance of a lens–like medium. Now we will consider the reflection from a specular, corner–cube, and rough (Lambertian) surfaces. In the first two cases the local reflectance $V(\mathbf{q}',\, \mathbf{q}'')$ is the deterministic function while in the third case – the random function. Specifically for a flat mirror, local reflectance is equal to

$$V(\mathbf{q}',\, \mathbf{q}'') = V_0(\mathbf{q}')\, \delta(\mathbf{q}' - \mathbf{q}''), \tag{6}$$

and similarly for a corner–cube reflector –

$$V(\mathbf{q}',\, \mathbf{q}'') = V_0(\mathbf{q}')\, \delta(\mathbf{q}' + \mathbf{q}''), \tag{7}$$

while for the Lambertian surface the following relation is satisfied:

$$\overline{V(\mathbf{q}_1',\, \mathbf{q}_1'')\, V^*(\mathbf{q}_2',\, \mathbf{q}_2'')} = \frac{1}{k^2}\, V_0(\mathbf{q}_1')\, V_0(\mathbf{q}_2')\delta(\mathbf{q}_1' - \mathbf{q}_1'')\, \delta(\mathbf{q}_2' - \mathbf{q}_2'')\, \delta(\mathbf{q}''_1 - \mathbf{q}_2''), \tag{8}$$

where

$$V_0(\mathbf{q}) = V_0 \exp\left\{ -\frac{(\mathbf{q} - \mathbf{q}_{\mathrm{eff}})^2}{2a_{\mathrm{eff}}^2} \right\};$$

V_0 is the amplitude of the reflectance, a_{eff} is the effective radius of the reflector, and $\mathbf{q}_{\mathrm{eff}}$ is the radial distance determining the displacement of the reflector center with respect to the optical axis of the channel.

By substituting Eqs. (3)–(6) into formula (2), we calculate the mutual coherence function of a Gaussian beam reflected by a flat mirror while calculations according to Eqs. (2)–(5) and (7) yield the mutual coherence function of a Gaussian beam reflected by a corner–cube reflector. The results in both cases can be represented in the following way:

$$\Gamma_2(0,\, \mathbf{q}_1,\, \mathbf{q}_2) = \sqrt{I(0,\, \mathbf{q}_1) I(0,\, \mathbf{q}_2)}\, \exp\left\{ \frac{i\kappa}{2F_0} S(\xi)(\mathbf{q}_1^2 - \mathbf{q}_2^2) \mp \mp (-1)\, \frac{ik}{F_0} \left[S_1(\xi)\, \mathbf{q}_0(\mathbf{q}_1 - \mathbf{q}_2) + S_2(\xi)\, F_0 \varphi \mathbf{n}(\mathbf{q}_1 - \mathbf{q}_2) \mp \mp S_3(\xi)\, \mathbf{q}_{\mathrm{eff}}(\mathbf{q}_1 - \mathbf{q}_2) \right] \right\}, \tag{9}$$

where

$$I(0,\, \mathbf{q}_1) = \frac{U_0^2\, V_0^2\, a_0^2}{a^2(\xi)} \exp\left\{ -\frac{(\mathbf{q}_1 \mp \mathbf{R}_1(\xi))^2}{a^2(\xi)} - \frac{a_0^4}{a_{\mathrm{eff}}^2 a_0^2(\xi)} \left[1 + \frac{a_0^2(\xi)}{a_{\mathrm{eff}}^2} \right] \xi^{-2}\Omega_0^{-2} \tilde{U}_2^2(\xi)(\mathbf{q}_{\mathrm{eff}} \mp \mathbf{R}_2(\xi))^2 - \frac{a_0^4}{a_{\mathrm{eff}}^2 a_0^2(\xi)} \left[\frac{a_0^2(\xi)}{a_0^2}\, \mathbf{q}_l - \frac{a_{\mathrm{eff}}^2(\xi)}{a_{\mathrm{eff}}^2}\, \mathbf{q}_{\mathrm{eff}} \mp \mathbf{R}_2(\xi) \right]^2 \right\}$$

is the intensity of reflected radiation at the point $(0,\, \mathbf{q}_l)$, $l = 1,\, 2$;

$$a(\xi) = a_0 \left\{ [\hat{U}_1(\xi) - \mu \hat{U}_2(\xi)]^2 + \xi^{-2}\Omega_0^{-2} \hat{U}_2^2(\xi) + \xi^{-2}\Omega_0^{-2} \frac{a_0^2}{a_{\mathrm{eff}}^2} \left[2 + \frac{a_0^2(\xi)}{a_{\mathrm{eff}}^2} \right] \tilde{U}_2^2(\xi) \right\}^{1/2}$$

is the radius of the reflected laser beam in the receiver planes;

$$a_0(\xi) = a_0 \{ [U_1(\xi) - \mu U_2(\xi)]^2 + \xi^{-2}\Omega_0^{-2} U_2^2(\xi) \}^{1/2};$$

is the radius of the laser beam in the reflector plane;

$$a_{\mathrm{eff}}(\xi) = a_{\mathrm{eff}} \{ [(U_1(\xi) - \mu U_2(\xi)][\hat{U}_1(\xi) - \mu \hat{U}_2(\xi)] +$$

$+ \xi^{-2}\Omega_0^{-2}U_2(\xi)\hat{U}_2(\xi)\}^{1/2};$

$\mathbf{R}_1(\xi) = \hat{U}_1(\xi)\boldsymbol{q}_0 + \hat{U}_2(\xi)F_0\varphi\mathbf{n}$ is the center shift of the laser beam reflected from the infinite reflector; $\mathbf{R}_2(\xi) = U_1(\xi)\boldsymbol{q}_0 + U_2(\xi)F_0\varphi\mathbf{n}$ is the center shift of the laser beam in the reflector plane,

$$\mathbf{R}_3(\xi) = \{U_2(x) - \mu[U_1(\xi) - \mu U_2(\xi)]\}\, \tilde{U}_2(\xi)\xi^{-2}\Omega_0^{-2}\boldsymbol{q}_0 -$$

$$- [U_1(\xi) - \mu U_2(\xi)]\tilde{U}_2(\xi)F_0\mathbf{n};$$

$S(\xi) = \dfrac{1}{a(\xi)}\dfrac{\mathrm{d}a(\xi)}{\mathrm{d}\xi}$ is the curvature of the wavefront of the reflected laser beam,

$$S_1(\xi) = \frac{a_0^2}{a^2(\xi)}\Bigg\{\mu[\hat{U}_1(\xi) - \mu\hat{U}_2(\xi)] - \xi^{-2}\Omega_0^{-2}\hat{U}_2(\xi) +$$

$$+ \xi^{-2}\Omega_0^{-2}\frac{a_0^2}{a_{\mathrm{eff}}^2}U_1(\xi)\tilde{U}_2(\xi)\Bigg\};$$

$$S_2(\xi) = \frac{a_0^2}{a^2(\xi)}\Bigg\{[\hat{U}_1(\xi) - \mu\hat{U}_2(\xi)] + \xi^{-2}\Omega_0^{2}\frac{a_0^2}{a_{\mathrm{eff}}^2}U_2(\xi)\tilde{U}_2(\xi)\Bigg\};$$

$$S_3(\xi) = \frac{a_0^4}{a_{\mathrm{eff}}^2\, a^2(\xi)}\xi^{-2}\Omega_0^{-2}\left[1 + \frac{a_0^2(\xi)}{a_0^2}\right]\tilde{U}_2(\xi)$$

are the tilts of the wavefront of the reflected laser beam with respect to the optical axis of a refraction channel (these tilts are caused by the displacement of the radiating aperture with respect to the optical axis of a lens–like medium, by the slope of that aperture with respect to the axis, and by the displacement of the reflector center with respect to the axis, respectively);

$$\hat{U}_1(\xi) = U_1(\xi)\tilde{U}_1(\xi) + U_1'(\xi)\tilde{U}_2(\xi);$$

$$\hat{U}_2(\xi) = U_2(\xi)\tilde{U}_1(\xi) + U_2'(\xi)\tilde{U}_2(\xi);$$

$\xi = L/F_0$ is the ratio between the distance from the source–receivers plane ($x = 0$) to the reflector plane ($x = L$) and the initial focal distance of a lens–like medium; $\mu = F_0/R_0$ is the ratio between the initial focal distance of a lens–like medium and the curvature of the wavefront of the initial sounding wave; and, is the Fresnel parameter of the radiating aperture. The minus sign in Eq. (9) corresponds to reflection from a flat mirror and plus sign – from a corner–cube reflector. In addition, the derivative with respect to ξ in the expression for the curvature of the wavefront of the reflected laser beam is taken only of the functions $\tilde{U}_1(x)$ and $\tilde{U}_2(x)$. Let us consider first the limiting cases of the reflector size, namely, the infinite ($a_{\mathrm{eff}} \to \infty$) and the point ($a_{\mathrm{eff}} \to 0$) reflectors.

When a Gaussian beam is reflected from an infinite specular (or corner–cube) reflector ($V_0(\boldsymbol{q}) = V_0$), formula for the mutual coherence function of the second order (9) assumes a simpler form

$$\Gamma_2(0, \mathbf{R}, \boldsymbol{q}) = \frac{U_0^2\, V_0^2\, a_0^2}{\tilde{a}^2(\xi)}\exp\Bigg\{-\frac{(\mathbf{R} \mp \mathbf{R}_1(\xi))^2 + \rho^2/4}{\tilde{a}^2(\xi)} +$$

$$+ \frac{ik}{F_0}\tilde{S}(\xi)\mathbf{R}\boldsymbol{q} \mp (-1)\frac{ik}{F_0}[\tilde{S}_1(\xi)\,\boldsymbol{q}_0\boldsymbol{q} + \tilde{S}_2(\xi)\,F_0\varphi\mathbf{n}\boldsymbol{q}]\Bigg\}, \quad (10)$$

where

$$\tilde{a}(\xi) = a_0\{[\hat{U}_1(\xi) - \mathrm{m}\hat{U}_2(\xi)]^2 + \xi^{-2}\Omega_0^{-2}\hat{U}_2^2(\xi))\}^{1/2};$$

$$\tilde{S}(\xi) = \frac{1}{\tilde{a}(\xi)}\frac{\mathrm{d}a(\xi)}{\mathrm{d}\xi}\Bigg|_{\frac{dU_1(\xi)}{\mathrm{d}\xi} = \frac{dU_2(\xi)}{\mathrm{d}\xi} = 0};$$

$$\tilde{S}_1(\xi) = \frac{a_0^2}{a^2(\xi)}\{\mu[\hat{U}_1(\xi) - \mu\hat{U}_2(\xi)] - \xi^{-2}\Omega_0^{-2}\hat{U}_2(\xi)\};$$

$$\tilde{S}_2(\xi) = \frac{a_0^2}{\tilde{a}^2(\xi)}[\hat{U}_1(\xi) - \mu\hat{U}_2(\xi)];$$

$$\mathbf{R} = (\boldsymbol{q}_1 + \boldsymbol{q}_2)/2\ ;\quad \boldsymbol{q} = \boldsymbol{q}_1 - \boldsymbol{q}_2.$$

Here the functions $\tilde{a}(\xi)$, $\tilde{S}(\xi)$, $\tilde{S}_1(\xi)$, and $\tilde{S}_2(\xi)$ have the same meaning as the functions $a(\xi)$, $S(\xi)$, $S_1(\xi)$, and $S_2(\xi)$ in Eq. (9). The comparison of Eq. (10) for the mutual coherence function of the second order of the Gaussian beam reflected from an infinite specular reflector with the similar formula for the beam which passes the path once (a base scheme)[4,5] reveals their identical structure. The only difference is that the functions $\hat{U}_1(x)$ and $\hat{U}_2(x)$ enter in formula (10) while the functions $U_1(\xi)$ and $U_2(\xi)$ enter in the formula for the coherence function of the laser beam which passes the path once. For a lens–like medium with constant focal distance we have $U_1(\xi) = \tilde{U}_1(\xi) = \mathrm{ch}(\xi)$, $U_2(\xi) = \tilde{U}_2(\xi) = \mathrm{sh}(\xi)$ and $\hat{U}_1(\xi) = \mathrm{ch}(2\xi)$, $\hat{U}_2(\xi) = \mathrm{sh}(2\xi)$, i.e., Eq. (10) is identical to the formula for the mutual coherence function of the second order of the beam which passes a path of length $2L$ only once. A corner–cube reflector reflects the radiation precisely in backward direction but this reflection pattern results only in changing of the direction of the center shift and the tilts of the wavefront of the reflected beam in comparison with the specular reflection. Thus, in the case when the beam is reflected from an infinite reflector, the round–trip–path modification of the methods of thermal lens,[1–3] mirage effect,[1,3] refocusing,[4,5] and displacement of an image has principally the same capabilities as the base scheme. They differ only in the sensitivity of the functions $\hat{U}_1(\xi)$, $\hat{U}_2(\xi)$ and $U_1(\xi)$, $U_2(\xi)$ to the local profile of the focal distance $F(x)$ of the lens–like medium. However, this problem has already been examined in Ref. 4.

When a Gaussian beam is reflected from a point reflector

$$\left(V_0(\boldsymbol{q}) = \frac{2\pi}{k^2}V_0\delta(\boldsymbol{q} - \boldsymbol{q}_{\mathrm{eff}})\right)$$

the mutual coherence function of the second order can be expressed as follows:

$$\Gamma_2(0, \mathbf{R}, \boldsymbol{q}) = \frac{U_0^2\, V_0^2\, a_0^2}{k^2 F_0^2\, \tilde{U}_2^2(\xi)\, a_0^2(\xi)} \exp\left\{ -\frac{(\boldsymbol{q}_{\rm eff} - \mathbf{R}_2(\xi))^2}{a_0^2(\xi)} + \right.$$

$$\left. + \frac{ik}{F_0} \frac{\tilde{U}_2'(\xi)}{\tilde{U}_2(\xi)} \mathbf{R}\boldsymbol{q} - \frac{ik}{F_0} \tilde{U}_2^{-1}(\xi)\, \boldsymbol{q}_{\rm eff}\boldsymbol{q} \right\}. \quad (11)$$

It follows from Eq. (11) that this case is equivalent to the case of propagation of a spherical wave with the amplitude; $\simeq \frac{U_0 V_0 a_0}{k a_0(\xi)} \exp\left\{ -\frac{(\boldsymbol{q}_{\rm eff} - R_2(\xi))^2}{2a_0^2(\xi)} \right\}$ emanating from a point $(L, \boldsymbol{q}_{\rm eff})$. Thus, the round–trip–path modification of the technique of mirage effect[1,3] based on the measurements of the coordinates of the center shift of a sounding beam cannot be used with the point reflector, and the method of thermal lens,[1–3] which consists in recording the intensity of a sounding beam with the use of a point diaphragm, becomes virtually inapplicable. On the contrary, the methods of refocusing and displacement of an image,[4,5] which consist in recording the curvature of the wavefront and tilt $\tilde{U}_2^{-1}(\xi)$ of reflected radiation reveal good capabilities for $\xi \gtrsim 1$ with the use of the point reflector.

Similarly, formula (9) shows that when the sounding beam is reflected from a reflector whose size is comparable with the beam diameter $(a_{\rm eff} \sim a_0(\xi))$, the method of thermal lens based on recording the light intensity and the method of mirage effect consisting in recording the coordinates of the beam center shift cannot be used because the quantities measured and the parameter sought (F_0) are related to each other by the complicated and ambiguous formulas. As for the methods of refocusing and displacement of an image, they remain applicable in this case too. For instance, for a wide $(\Omega_0 \gg 1)$ collimated $(\mu_0 = 0)$ sounding beam we have

$$S(\xi) \simeq \frac{\hat{U}_1(\xi)\hat{U}_1'(\xi) + \Omega_{\rm eff}^{-2}\xi^{-2}U_1^2(\xi)\tilde{U}_2(\xi)\tilde{U}_2'(\xi)}{\hat{U}_1^2(\xi) + \Omega_{\rm eff}^{-2}\xi^{-2}U_1^2(\xi)\tilde{U}_2^2(\xi)};$$

$$S_1(\xi) \simeq 0;$$

$$S_2(\xi) \simeq \frac{\hat{U}_1(\xi)}{\hat{U}_1^2(\xi) + \Omega_{\rm eff}^{-2}\xi^{-2}U_1^2(\xi)\tilde{U}_2^2(\xi)};$$

$$S_3(\xi) \simeq \Omega_{\rm eff}^{-2} \frac{\xi^{-2}U_1^2(\xi)\tilde{U}_2(\xi)}{\hat{U}_1^2(\xi) + \Omega_{\rm eff}^{-2}\xi^{-2}U_1^2(\xi)\tilde{U}_2^2(\xi)},$$

where $\hat{U}_1'(\xi) = U_1(\xi)\tilde{U}_2'(\xi) + U_1'(\xi)\tilde{U}_2(\xi)$, and $\Omega_{\rm eff} = ka_{\rm eff}^2/L$ is the Fresnel number of the reflector. For $\Omega_{\rm eff} \gg 1$ (since $\Omega_{\rm eff} \sim \Omega_0$) we find

$$S(\xi) \simeq \hat{U}_1'(\xi)/\hat{U}_1(\xi), \quad S_1(\xi) \simeq 0,$$

$$S_2(\xi) \simeq 1/\hat{U}_1(\xi), \quad S_3(\xi) \simeq 0.$$

For short paths $(L < F_0)$ this yields a result analogous to that obtained for a base scheme[4–5]

$$S(\xi) \simeq \xi, \quad S_2(\xi) \simeq 1 - \frac{1}{2}\xi^2.$$

For a "quasispherical" wave $(\Omega_0 \ll 1)$ when $\Omega_{\rm eff} \gg 1$ we obtain (since in this situation $\Omega_{\rm eff} \gg \Omega_0$).

$$S(\xi) \simeq \hat{U}_2'(\xi)/\hat{U}_2(\xi);$$

$$S_1(\xi) \simeq 1/\hat{U}_2(\xi), \quad S_2(\xi) \simeq 0; \quad S_3(\xi) \simeq 0.$$

Here $\hat{U}_2'(\xi) = U_2(\xi)\tilde{U}_1'(\xi) + U_2'(\xi)\tilde{U}_2'(\xi)$.

For $\xi \gtrsim 1$ the curvature and tilt of the wavefront of reflected radiation are highly sensitive to the focal distance of the lens–like medium, for both $\Omega_0 \gg 1$ and $\Omega_0 \ll 1$. Note in particular that the curvature of the wavefront of reflected radiation (see Eq. (9)) is independent of the type of the reflector, whether it is specular or corner–cube.

To obtain the mutual coherence function of the second order of a Gaussian beam reflected from a rough (Lambertian) surface, let us substitute Eqs. (3)–(5) and (8) into Eq. (2) and calculate the integrals which enter in them. This makes it possible to derive the following formula:

$$\Gamma_2(0, \mathbf{R}, \boldsymbol{q}) = \frac{U_0^2\, V_0^2\, a_0^2\, a_{\rm eff}^2}{4\pi F_0^2 \tilde{U}_2^2(\xi)\left[a_{\rm eff}^2 + a_0^2(\xi)\right]} \times$$

$$\times \exp\left\{ -\frac{(\boldsymbol{q}_{\rm eff} - \mathbf{R}_2(\xi))^2}{a_{\rm eff}^2 + a_0^2(\xi)} - \frac{\rho^2}{\rho_k^2(\xi)} + \frac{ik}{F_0}\hat{S}(\xi)\, \mathbf{R}\boldsymbol{q} - \right.$$

$$\left. - \frac{ik}{F_0}\left[\hat{S}_1(\xi)\, \boldsymbol{q}_0\boldsymbol{q} + \hat{S}_2(\xi)\, F_0\varphi\mathbf{n}\boldsymbol{q} + \hat{S}_3(\xi)\, \boldsymbol{q}_{\rm eff}\boldsymbol{q}\right] \right\}, \quad (12)$$

where

$$\hat{S}(\xi) = \tilde{U}_2'(\xi)/\tilde{U}_2(\xi);$$

$$\hat{S}_1(\xi) = \frac{a_{\rm eff}^2}{a_{\rm eff}^2 + a_0^2(\xi)} \frac{U_1(\xi)}{\tilde{U}_2(\xi)};$$

$$\hat{S}_2(\xi) = \frac{a_{\rm eff}^2}{a_{\rm eff}^2 + a_0^2(\xi)} \frac{U_2(\xi)}{\tilde{U}_2(\xi)};$$

$$\hat{S}_3(\xi) = \frac{a_0^2}{a_{\rm eff}^2 + a_0^2(\xi)} \tilde{U}_2^{-1}(\xi);$$

$$\rho_k(\xi) = \frac{2F_0\tilde{U}_2(\xi)}{ka_{\rm eff}a_o(\xi)}\sqrt{a_{\rm eff}^2 + a_0^2(\xi)}$$

is the coherence radius of the reflected radiation. The functions $\hat{S}(\xi)$, $\hat{S}_1(\xi)$, $\hat{S}_2(\xi)$, and $\hat{S}_3(\xi)$ in the case of reflection from a rough surface denote the physical variables analogous to the functions $S(\xi)$, $S_1(\xi)$, $S_2(\xi)$, and $S_3(\xi)$ which enter in

Eq. (9) in the case of reflection from a regular surface. When the beam is "completely intercepted" $(a_{\mathrm{eff}} \gg a_0(\xi))$, the intensity of the reflected radiation is approximately equal to

$$\frac{U_0^2\, V_0^2\, a_0^2}{4\pi F_0^2\, \tilde{U}_2^2(\xi)} \exp\left\{ -\frac{(\boldsymbol{q}_{\mathrm{eff}} - \mathbf{R}_2(\xi))^2}{a_{\mathrm{eff}}^2} \right\}, \qquad (13)$$

while

$$\hat{S}_1(\xi) \simeq U_1(\xi) / \tilde{U}_2(\xi)\ , \ \hat{S}_2(\xi) \simeq U_2(\xi)\tilde{U}_2(\xi)\ ,$$

$$\hat{S}_3(\xi) \simeq 0,\ \rho_k(\xi) \simeq 2F_0\tilde{U}_2(\xi) / \left[ka_0(\xi)\right]\ . \qquad (14)$$

The analysis of Eq. (12) for the mutual coherence function of the second order of the Gaussian beam reflected from a rough (Lambertian) surface showed that the techniques of thermal lens and mirage effect are inapplicable in this case: reflection from a randomly uneven surface destroys the spatial structure of a laser beam. At the same time, methods of refocusing and displacement of an image of the sounding beam remain applicable in this situation, as can be seen from Eqs. (12) and (14).

In conclusion it should be noted that the above study of the propagation of a laser beam along the round–trip path through a lens–like medium, when the beam is reflected from a specular, corner–cube, and rough (Lambertian) surfaces, demonstrated the wide possibilities of using the methods of refocusing[4,5] and displacement of an image of a sounding beam[4,5] for the measurement of the optical parameters of an investigated medium, whereas the round–trip–path modification of the methods of thermal lens and mirage effect is applicable only if a beam is reflected from an infinite $(a_{\mathrm{eff}} \gg a_0(\xi))$ specular or corner–cube reflector.

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