Inverse problems in sun photometry for integral aerosol distributions. I. Theory and numerical experiment for submicron range of particle sizes

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Methods and numerical algorithm of aerosol microstructure reconstruction from light scattering characteristics is considered based on integral distribution functions describing dispersion composition of aerosol particles. The efficiency of the proposed methods was investigated in a numerical experiment consisting in conversion of spectral measurements of the extinction coefficients of submicron aerosol particles. The approach developed allows automation of solving inverse problems in processing large data bulks of routine measurements.

Introduction

Methods of sun photometry are widely used for regular monitoring of the optical state of the atmosphere, including its aerosol component. The series of multi-wavelength sun photometers has been developed at the IAO SB RAS, with which regular measurements of the atmospheric transmission are being carried out.[1,2] The developed instrumentation is capable of measuring the brightness of radiation scattered within the solar aureole. The significant amount of experimental data on the optical properties of the atmosphere in a wide wavelength range has been obtained.

The important stage of the researches performed is obtaining the data on microphysical parameters of atmospheric aerosol from the optical characteristics measured. As known, this is related with solving the so-called ill-posed inverse problems and requires the high-level professional skill of a researcher. So, it is very urgent to develop methods for solving the inverse problems of aerosol light scattering suitable for the mass automated processing of the huge arrays of experimental data while using minimum *a priori* information about the solution sought.

The results of investigations on the development of the technique for retrieval of the aerosol microstructure from light scattering characteristics are presented in this paper. It is based on the use of the integral distribution functions for description of the disperse composition of aerosol particles.

Mathematical statement of the problem, basic equations

1.1. Application of the Stieltjes integrals for describing light scattering characteristics of polydisperse aerosol

It is known that particles randomly distributed over space scatter light independently. Therefore, the scattering properties of a system of such particles can be found by summing the relevant optical characteristics of individual particles. For example, let us assume that the medium consists of isotropic spherical particles, whose extinction cross section $\sigma_{ex}(\lambda, r)$ at the wavelength λ depends on their radius r, $0 < r \le R$. Then, dividing the particle size range into the intervals Δr_i that do not overlap, one can represent the volume extinction coefficient of polydisperse ensemble of particles by the following sum

$$\varepsilon(\lambda) = \sum_i \sigma_{ex}(\lambda, \xi_i)\Delta N_i, \qquad (1)$$

where $\xi_i \in \Delta r_i$, ΔN_i is the number of particles per unit volume of the medium per size interval Δr_i. Proceeding to limit in Eq. (1) at tending the length of the maximum interval Δr_i to zero leads to representation of the extinction coefficient $\varepsilon(\lambda)$ in the form of the Riemann–Stieltjes integral

$$\varepsilon(\lambda) = \int_0^R \sigma_{ex}(\lambda, r)\mathrm{d}N(r), \qquad (2)$$

where the function $N(r)$ determines the total number of particles with the radius less than r per unit volume of the medium. By replacing the cross section $\sigma_{ex}(\lambda, r)$ by the extinction efficiency factor $K(\lambda, r) = \sigma_{ex}(\lambda, r)/(\pi r^2)$ one obtains the equivalent representation of the form

$$\varepsilon(\lambda) = \int_0^R K(\lambda, r)\mathrm{d}S(r) \qquad (3)$$

in which the function $S(r)$ is the total geometric cross section of particles with the radii less than r. Thus, the representation of the optical characteristics of a polydisperse ensemble of particles in the form of the Riemann–Stieltjes integral (2) or (3) is quite

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natural generalization of summing the contributions coming from individual particles.

In practice, assuming existence of the derivative of the function $dN(r)/dr = n(r)$ or the derivative of the function $dS(r)/dr = s(r) = \pi r^2 n(r)$ on the interval $[0, R]$ one usually uses the representation of the light scattering characteristics of a polydisperse aerosol by the Riemann integral. However, in reality, aerosol size distributions are discrete, and such a representation is, to a certain degree, a mathematical idealization that can be accepted if a sufficient number of particles have sizes in each narrow interval $[r, r + dr]$, and no localized monodisperse fractions exist in the medium. One often uses smooth functions of simple analytical form or linear combinations of such functions in describing the model distributions $n(r)$.

The fact of quite limited possibilities of describing the aerosol disperse composition by use of differential size distribution functions $n(r)$ or $s(r)$ becomes more obvious in solving the inverse problems. The regularization procedures used in solving such problems lead to narrowing the class of solutions admissible because of the requirements of continuity, smoothness, etc.

One can avoid some of the aforementioned restrictions, by using the integral representation of the particle size distribution in describing the disperse composition of aerosol. For certainty, we shall use, in what follows, the function $S(r)$ that describes the size distribution of the particles' geometric cross sections. Let us note some specific properties of the function $S(r)$ that make it belonging to a certain class of functions Ω. It is a positive function monotonically non-decreasing on the interval $[0, R]$ and continuous from left. Besides, as one always can estimate *a priori*, the total cross section of all particles in a unit volume, let us consider the constant C restricting the functions $S(r) \in \Omega$, $r \in [0, R]$ from above $S(r) \le S(R) \le C$.

It is known that any monotonic function can be represented as a sum of a continuous monotonic function and a saltus function. So one can write the following expansion

$$S(r) = \tilde{S}(r) + \sum_{r_k < r} \sigma_k, \qquad (4)$$

where $\tilde{S}(r)$ is the continuous monotonic function, which has the derivative $d\tilde{S}/dr = s(r)$, and the second term defines the saltus function at the break points r_k. The saltus function in the considered problem determines the presence of monodisperse fractions of aerosol with the particle radii r_k and the total cross section σ_k. Taking into account the expansion (4), the extinction coefficient $\varepsilon(\lambda)$ (3) can be written in the form of the following sum

$$\varepsilon(\lambda) = \sum_i K(\lambda, r_i)\,\sigma_i + \int_0^R K(\lambda, r) s(r) dr\,. \qquad (5)$$

Formula (5) differs from the traditional representation of this optical characteristic of a polydisperse aerosol using the differential distribution function $s(r)$ by the first term, which explicitly defines the total contribution of the monodisperse fractions to this characteristic.

1.2. Statement of inverse problem and the way of solving

Let us consider the inverse problem on seeking the distribution function $S(r)$ from Eq. (3). To solve it, let us first integrate Eq. (3) by parts. This yields the following integral equation relative to the function $S(r)$:

$$K(\lambda, R)S(R) - \int_0^R \frac{\partial K(\lambda, r)}{\partial r} S(r) dr = \varepsilon(\lambda). \qquad (6)$$

If one passes in Eq. (6) from the function $S(r)$ to the function $S_\downarrow(r) = S(R) - S(r)$, then the equation is obtained that, in contrast to Eq. (6), does not contain the function to be sought under the integral sign:

$$\int_0^R \frac{\partial K(\lambda, r)}{\partial r} S_\downarrow(r) dr = \varepsilon(\lambda). \qquad (7)$$

In practice it is preferable to deal with this monotonically decreasing function $S_\downarrow(r)$, because it, in contrast to the function $S(r)$, goes to zero out of the limits of the interval $[0, R]$. Equations (6) and (7) have general structure of the equation of first kind:

$$QS = \varepsilon. \qquad (8)$$

From the physical point of view it is clear that the exact solution of Eq. (8) $S_0(r)$ exists and it belongs to the Ω set. As shown in Ref. 3 the set Ω is compact in the space $L_p[0, R]$, $p > 1$. Because of continuity of the inverse operator Q^{-1} on the set $Q\Omega$, narrowing of the set of admissible solutions to the compact class Ω is sufficient for constructing a stable approximate solution of Eq. (8). The effective numerical algorithms have been developed to date for solving some ill-posed inverse problems on compact sets.[3] To construct a stable approximate solution of Eq. (8) it is sufficient to minimize the discrepancy functional

$$F^2 = \|QS - \varepsilon\|^2 \qquad (9)$$

on the set Ω. Any function $S_\delta(r) \in \Omega$, for which the functional $F^2 \le \delta^2$, where δ^2 characterizes the error in the initial data can be accepted as the approximate solution of Eq. (8). The convergence $S_\delta(r) \to S_0(r)$ takes place in the space $L_p[0, R]$ at $p > 1$.

Let us note other important properties of the approximate solution $S_\delta(r)$ revealed in Ref. 3. If it has been known that $S_0(r)$ is a continuous function,

that corresponds to the absence of the second term in Eq. (4), then $S_\delta(r)$ converges to $S_0(r)$ uniformly. However, the approximate solution $S_\delta(r)$ can be a discontinuous monotonic function. Finally, if $S_0(r)$ has been the piecewise continuous function, then $S_\delta(r) \to S_0(r)$ uniformly within each closed interval, which does not contain the break points of the exact solution $S_0(r)$.

Comparing the described approach to the problem of determination of the aerosol microstructure from the characteristics of light scattering by the methods using regularization algorithms based on minimization of the smoothing functional,[4] one can note the following differences. The use of the method of smoothing functional is oriented mainly to the problems of retrieval of continuous smooth aerosol distributions $s(r)$.

Passing to the integral representation of microstructure of the aerosol size distributions makes it possible to essentially expand the class of correctness at solving the inverse problems. If the distributions $s(r)$ are discontinuous, and are not enough smooth, then it is worth representing the disperse composition of aerosol by the integral distributions $S(r)$. In this case, it is sufficient to use the approximation of the piecewise continuous function $S(r)$ in describing the aforementioned breaks in the distributions $s(r)$. Moreover, if assuming breaks of the function $S(r)$ itself, one can consider the situations when the monodisperse aerosol fractions can exist in the aerosol size distributions that would yield δ-peculiarities in the distributions $s(r)$.

The possibility of estimating the error in the approximate solution based on the data on the errors in the initial data is among other important advantages of the solution of inverse problem for Eq. (8) on the compact Ω (see Ref. 3).

The integral distributions $S(r)$ are less frequently used in describing the disperse composition of aerosol as compared with other ways of setting the aerosol microstructure. Based on the description of the aerosol microstructure in the form (4), one can pass to other parameters that are more widely used in describing the aerosol microstructure. Those are the number density, volume packing factor, moments of different order, including the mean value, half-width, etc. For example, the volume packing factor V and the mean, over the distribution $S(r)$, radius of particles $\bar{r}_s$ are presented by the formulas using the function $S(r)$:

$$V = a\left[RS(R) - \int_0^R S(r)\mathrm{d}r\right]; \qquad (10)$$

$$\bar{r}_s = V/[aS(R)], \quad a = 4/3. \qquad (11)$$

1.3. Finite difference approximation of the Stieltjes integrals

In solving direct and inverse problems stated by the integral equations (6) and (7) relative to the function $S(r)$, the difficulties in calculation can appear because the presence of derivatives of the form $\partial K(.)/\partial r$ of the corresponding Mie efficiency factors in the integrand. Then the necessity appears of developing the algorithms for calculating the derivatives $\partial K/\partial r$, which strongly oscillate about zero level. Calculation of the integrals of a function with such properties is a non-trivial problem and requires application of special quadrature formulas. In the simplest case, one can perform discretization of the problem using the procedure considered below.

Let us define a uniform grid for setting $S(r)$ at a given number of nodes n with the step $\Delta = R/n$, on which we shall approximate the distribution $S(r)$ by piecewise continuous function (spline) following the formula

$$S(r) = \sum_{j=1}^{n} S_j N_j(r), \qquad (12)$$

where $S_j = S(r_j)$ $(S(0) = 0)$. The basis functions $N_j(r)$ have the form

$$N_j(r) = N_0[(r - r_j)/\Delta], \; j = 1, 2, \ldots, n, \qquad (13)$$

where

$$N_0(r) = \begin{cases} 1 - |r|, & r \le 1, \\ 0, & r > 1. \end{cases}$$

At such an approximation the set of functions $S(r)$ joins the set of the vectors S with the non-decreasing components

$$0 < S_1 \le S_2 \le \ldots \le S_n \le C, \qquad (14)$$

where C is the upper boundary of the total cross section of particles. Substituting Eq. (12) into Eq. (6) and making some other transformations we obtain the finite difference analog of Eq. (6):

$$\sum_{j=1}^{n} Q_j(\lambda) S_j = \varepsilon(\lambda), \qquad (15)$$

where

$$Q_j(\lambda) = \bar{K}_{j-1}(\lambda) - \bar{K}_j(\lambda), \quad j = 1, \ldots, n; \qquad (16)$$

$$\bar{K}_j(\lambda) = \frac{1}{r_{j+1} - r_j} \int_{r_j}^{r_{j+1}} K(\lambda, r)\mathrm{d}r, \quad j = 0, 1 \ldots, n-1; \qquad (17)$$

$$\bar{K}_n(\lambda) = 0.$$

The functions $\bar{K}_j(\lambda)$ $(j \ne n)$ represent the averaged values of the kernel $K(\lambda, r)$ on the intervals $[r_j, r_{j+1}]$. It is seen from Eqs. (15)–(17) that in the finite difference procedure considered there is no need in calculating the integrals containing the derivatives $\partial K/\partial r$.

1.4. Estimation of the upper bound of the integral distribution

In practical realization of the considered method it is necessary to have data on the total geometric cross section of particles determined by the $S(R)$ value or, at least, on any of its estimate from above. This would be an easy task if one knows the admissible range of variations of the mean extinction efficiency factor $\bar{K}(\lambda) = \varepsilon(\lambda)/S(R)$. The following value can be chosen as such an estimate in Eq. (14):

$$C = \gamma_s S(R) = \varepsilon(\lambda)/\bar{K}_{\min}(\lambda), \qquad (18)$$

where $\gamma_s = \bar{K}/\bar{K}_{\min}$ characterizes the measure of closeness between C and $S(R)$, and $\bar{K}_{\min}(\lambda)$ is the minimum value of the mean efficiency factor on the set of admissible models of microstructure. Besides, one should select the minimum value with respect to λ from the set of C values given by Eq. (18).

The set of spectral dependences of the mean efficiency factor $\bar{K}(\lambda)$ calculated using the "haze H"[5] model of microstructure at different values of the refractive index is shown in Fig. 1 as an example. It is seen from Fig. 1 that the efficiency factor $\bar{K}(\lambda)$ at all wavelengths takes the minimum values at the minimum value of the real part of the refractive index. The ratio $\gamma_s = \bar{K}/\bar{K}_{\min}$ is most close to unity in the left-hand side of the spectral interval. For example, if the true value of the complex refractive index is equal to $(1.5 - i\cdot 0)$, then the estimate made without the account of the absorption at $\lambda = 0.31$ μm is $\gamma_s(0.31) = 1.05$. If admitting that the imaginary part of the refractive index can be within the limits $0 \le \kappa \le 0.05$, then the estimate $\gamma_s(0.31)$ increases up to 1.14.

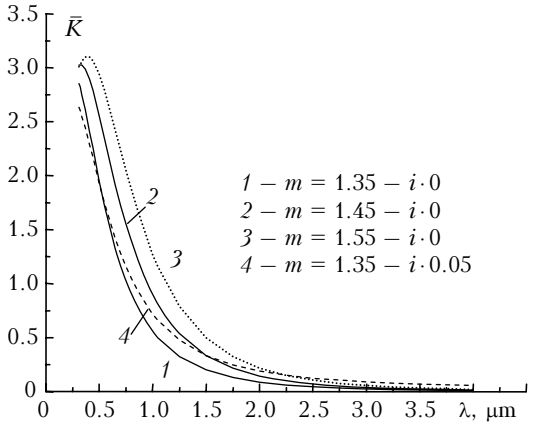

Fig. 1. The effect of the refractive index on the spectral dependences of the mean extinction efficiency factor $\bar{K}(\lambda)$ for a model polydisperse aerosol of the type of haze H.

The tendencies in the behavior of γ_s revealed remain the same for more complicated models of the microstructure, when particles of coarse aerosol fraction are present in the medium together with the particles of submicron size. Figure 2 shows the dependences of the mean efficiency factor $\bar{K}(\lambda)$ for the model, in which the particles of coarse fraction are represented by a wide lognormal distribution with the mean radius $\bar{r}_s = 1.23$ μm at different value p of its relative contribution to the total extinction coefficient $\varepsilon(\lambda)$ at the wavelength $\lambda = 0.55$ μm. In this case the complex refractive index was taken to be the same for both fractions and equal to $1.35 - i\cdot 0$.

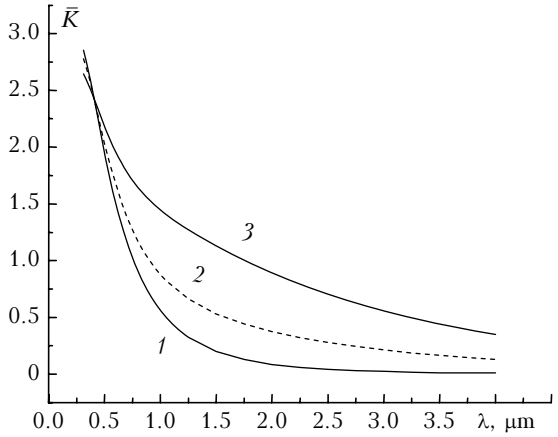

Fig. 2. Variability of the spectral dependence of the mean extinction efficiency factor $\bar{K}(\lambda)$ at variations of the relative contribution p of the coarse fraction to the total extinction coefficient $\varepsilon(0.55)$: $p = 0$ (*1*); 0.2 (*2*); 0.5 (*3*).

It is seen from Fig. 2 that in the vicinity of the wavelength $\lambda = 0.41$ μm all curves intersect, i.e., the choice of the model of microstructure does not affect the value of the efficiency factor $\bar{K}(\lambda)$ at this wavelength. The estimate γ_s for $\lambda = 0.41$ μm changes within the limits 1.25–1.35 for the range of admissible values of the imaginary part of the refractive index $0 \le \kappa \le 0.05$.

Thus, it follows from the presented estimates that at variations of the microphysical parameters of the aerosol model in quite a wide range, the value of the proportionality coefficient in the ratio $C = \gamma_s S(R)$ between the total geometric cross section of aerosol particles $S(R)$ and the estimate of its upper bound C obtained from Eq. (18) does not exceed the value 1.35, at least.

2. Simulation of the inverse problem for the extinction coefficient of submicron aerosol

Closed numerical experiment has been carried out in order to estimate the effectiveness of the proposed technique in solving the inverse problem. The aerosol distribution of the haze H type in the submicron size range is considered in this paper as

the model. The results obtained by numerical simulation using a more complicated model comprising two aerosol fractions, submicron (s) and coarse (c), are considered in the second part of this paper.[6]

The accuracy of retrieval of the integral size distribution function $S_{\downarrow}(r)$ is studied in the numerical experiment, as well as the respective parameters of the microstructure: the mean radius of particles $\bar{r}_s$ [Eq. (11)] and the volume packing factor V [Eq. (10)]. Solution to the inverse problem was found by minimizing the functional of discrepancy (9), under the restrictions (14), using the method of conditional gradient.[3]

Special attention was paid to the study of the effect the amount of initial data and the errors in them, as well as the errors in *a priori* setting of the refractive index produce on the inversion results. The problems of algorithmic realization of the minimization procedure were considered in detail. To peak generally, the boundaries of the size of scattering particles are among the unknown parameters, which are set *a priori*. Earlier, in Section 1.4 recommendations have been given on the *a priori* setting of the upper boundary of the total geometric cross section of particles. The effect of this boundary on the solution quality is considered below.

In the numerical simulations the values of the extinction coefficient $\varepsilon_{0i} = \varepsilon_0(\lambda_i)$ were calculated using a preset model of the aerosol microstructure. Then the random error was introduced according to the following rule:

$$\varepsilon_i = \varepsilon_{0i} + \varepsilon_{0,1}\delta(2\eta_i - 1)\,. \qquad (19)$$

The specific values of the wavelengths from the spectral range [0.31; 4.0] μm were selected according to the capabilities of the instrumentation.[1] The value δ in Eq. (19) characterizes the level of the relative error, and η_i is the realization of the random value uniformly distributed over the range [0, 1).

The examples of retrieval of the integral size distribution function $S_{\downarrow\delta}(r)$ for the model of the medium formed by only submicron particles with the complex refractive index $m = 1.5 - i\cdot 0$ are shown in Fig. 3 for $\delta = 0.05$ and 0.1.

Let us note some peculiarities of the results obtained by inversion. First, it is related to the upper boundary R of the particle size in solving the inverse problem. It is known that the efficiency of retrieving the differential aerosol size distributions essentially depends on the correct selection of the range of definition of the solution sought. In this example, it was not assumed *a priori* that particles belong to the submicron range, so the value $R = 4.15$ μm was selected with a certain reserve. Nevertheless, the retrieved functions $S_{\downarrow\delta}(r)$ were set on the range $r < 0.75$–0.8 μm. The contribution of particles of greater size to the total cross section is less than 1%.

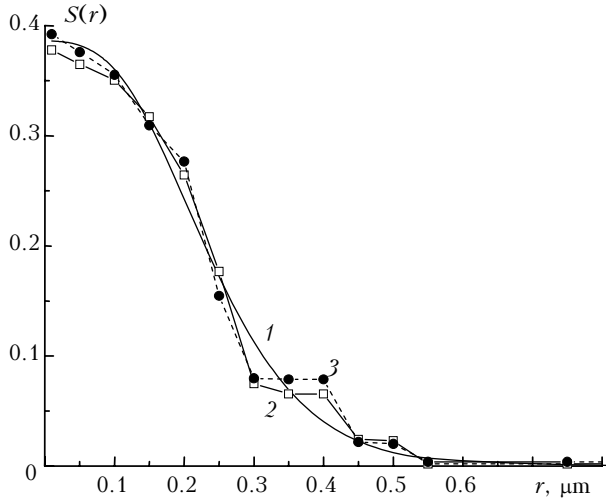

Fig. 3. Results of retrieval of the function $\varepsilon(\lambda)$ at the known value of the refractive index and different error in measurement: (*1*) model distribution; (*2, 3*) distributions $S_{\downarrow\delta}(r)$, retrieved at $\delta = 0.05$ and 0.1, respectively.

Now let us consider how the *a priori* set upper boundary C of the total cross section of particles affects the retrieved function $S_{\downarrow\delta}(r)$. The model calculations on retrieval of the integral size distribution function $S_{\downarrow\delta}(r)$ at different *a priori* setting the upper boundary C in the inequalities (14) were carried out using the estimates of γ_s obtained in Section 1.4. It was revealed from the analysis of the results obtained that the choice of the value C mainly affects the retrieval of the function $S_{\downarrow\delta}(r)$ at small r values, i.e., in the range where the particles are optically less active. The dependences $S_{\downarrow\delta}(r)$ in the range $r < 0.12$ μm retrieved for γ_s from the range 1.1–1.4 are shown in Fig. 4 as an example.

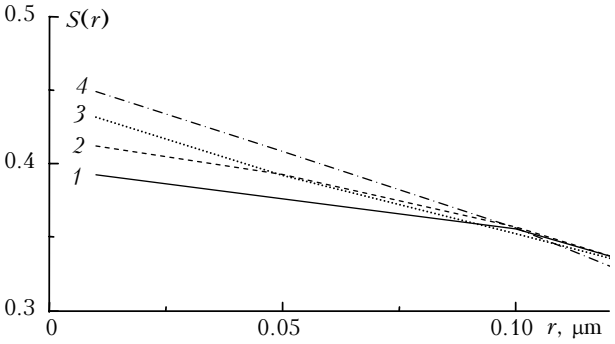

Fig. 4. The shape of the distribution $S_{\downarrow\delta}(r)$ at small r, retrieved in the numerical experiment as a function of the parameter γ_s: γ_s = 1.1 (*1*); 1.2 (*2*); 1.3 (*3*); 1.4 (*4*).

At $r \to 0$ the divergence of the curves presented in Fig. 4 monotonically increases. This leads to the increase of the error in retrieval of the total cross section of particles $S_\delta(R) = S_{\downarrow\delta}(0)$ due to

overestimation of the contribution coming from the smallest particles.

The dependence of $S_\delta(R)$ on γ_s shown in Fig. 5 is practically linear.

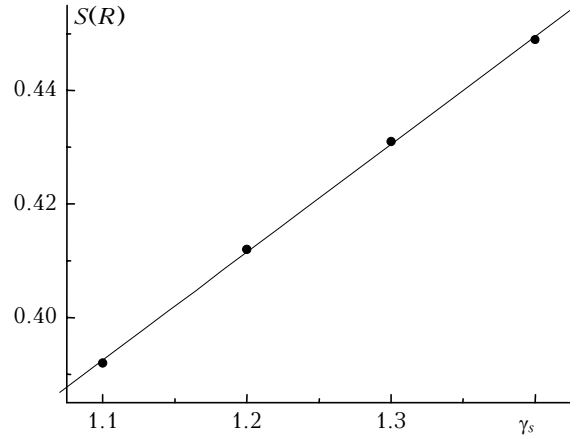

Fig. 5. The total cross section $S(R)$ as a function of the parameter γ_s.

It follows from Fig. 5 that the error in retrieval of the total cross section $S_\delta(R)$ at the maximum value $\gamma_s = 1.4$ increases by 15%. One should note that the choice of γ_s and the respective changes of the retrieved values $S_\delta(R)$ practically do not affect the level of the discrepancy obtained, which does not exceed 5% in the range $0.31 \le \lambda \le 0.55$ μm. So one can conclude that the variations in the behavior of the retrieved dependences $S_{\downarrow\delta}(r)$ shown in Fig. 5 are caused, first of all, by the *a priori* data on the upper boundary C introduced. The effect of the value γ_s, and, at the same time, the upper boundary C on the accuracy of retrieval of the parameters of microstructure V and $\bar{r}_s$ is shown in the Table below.

Table. The parameters of haze microstructure retrieved by numerically inverting the spectral dependences ε(λ)

Parameter	Precise data	δ=0.05, γ_s=1.1	δ=0.10, γ_s =1.1	δ=0.10, γ_s=1.4	δ=0.10, n=1.45	δ=0.10, n=1.55	δ=0.10, κ=0.05
				R = 4.15 μm			
V	0.124	0.127	0.132	0.135	0.144	0.124	0.121
$\bar{r}_s$, μm	0.241	0.251	0.252	0.225	0.275	0.233	0.234
				R = 0.55 μm			
V	0.119	0.121	0.122	0.125	0.130	0.114	0.121
$\bar{r}_s$, μm	0.234	0.242	0.236	0.211	0.255	0.216	0.234

The effect of the errors in the refractive index. In the above section, when retrieving the aerosol microstructure from optical measurements, the refractive index of particles was assumed known exactly, and was equal to $1.5 - i \cdot 0$. However, in practice, as a rule, the value of the refractive index is known with some error, it is necessary to estimate the effect of the errors in *a priori* setting the refractive index on the accuracy of retrieval of the aerosol microstructure. The results of retrieving the integral distribution $S_{\downarrow\delta}(r)$ at the error in setting the real part of the refractive index $\Delta n = \pm 0.05$ and the random measurement error of $\delta = 0.1$ are shown in Fig. 6. The following general conclusion can be drawn from comparison of the dependences (Fig. 6) with the results obtained by solving the inverse problem with the precise refractive index (see Fig. 3). The effect of the errors in the real part of the refractive index is qualitatively revealed as the shift of the curves $S_{\downarrow\delta}(r)$ with respect to abscissa and ordinate axes. The errors in setting the imaginary part of the refractive index analogously affect the retrieved distribution $S_{\downarrow\delta}(r)$.

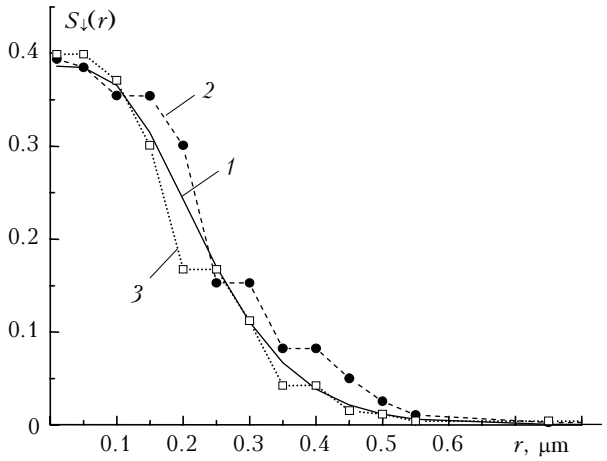

Fig. 6. The effect of *a priori* choice of the refractive index on the results obtained by inverting the function ε(λ): model distribution with the refractive index $n = 1.5$ (*1*); distributions $S_{\downarrow\delta}(r)$, retrieved at the error in measurement $\delta = 0.1$ (*2, 3*) [$n = 1.45$ (*2*); 1.55 (*3*)].

Retrieval of the parameters of microstructure. The final purpose of the study is retrieval of the parameters of microstructure, i.e., the mean radius $\bar{r}_s$ and the volume packing factor V. The model calculations of these parameters by formulas (10) and (11) are presented in the Table for $R = 4.15$ μm and 0.55 μm. The first value refers to the case when the *a priori* information on the correspondence of the particle ensemble to the submicron range is absent. The second one defined the upper boundary R of optically active particles, for which the contribution to extinction of coarse particles was within the measurement error.

The data given in the Table for two R values are close to each other, but there are some differences. Both the packing factor and the mean radius of particles decreased, though insignificantly, because of ignoring particles with the size greater than 0.55 μm. Columns 3–5 contain the retrievals of the parameters of microstructure using exact refractive index, and columns 6–8 show the results at additional error in setting real and imaginary parts of

the refractive index. Column 5 characterizes the maximum effect of *a priori* uncertainty in setting the total geometric cross section $S(R)$ represented by the parameter γ_s (see Eq. (18)).

Let us note the main characteristics of the accuracy of retrieval of the parameters of microstructure for $R = 4.15$ μm. The error in calculating the volume packing factor V does not exceed 6% (at the exact value of the refractive index) and 16% at $|\Delta n| = 0.05$ for measurements $\varepsilon(\lambda)$ with the error $\delta = 0.1$. The absolute error in retrieving the mean radius of particles under the same conditions is about 0.01 and 0.03 μm, respectively. The uncertainty in the choice of the real part of the refractive index stronger affects the retrieval of both parameters than its imaginary part.

As to the data at $R = 0.55$ μm, one can note, in general, that narrowing the range of definition of the retrieved function to the boundaries out of which real distribution is equal to zero, improves the accuracy of estimation of both the volume packing factor and the mean radius of particles.

Conclusion

It is proposed to use the integral aerosol size distributions for retrieval of the parameters of aerosol microstructure from optical measurements with multi-wavelength sun photometers. It is shown that within such an approach an approximate stable solution of the inverse problem can be found by minimizing the functional of discrepancy on the set of monotonic bounded functions without any additional restriction of the sought solution and attraction of special regularization procedures. This enables one to easily automate the process of processing large amount of measurement data in the regime of continuous observations.

Refusing from retrieval of a detailed information on aerosol microstructure in the form of differential particle size distribution function does not hamper obtaining data on the moments of different orders, including the mean particle radius and the volume packing factor, from the inverted data. In many practical applications, these data are sufficient for the control of the state of atmospheric aerosol.

The efficiency of the proposed technique in solving the inverse problem for measurements of the spectral dependences of the light extinction by submicron aerosol is shown by the methods of numerical simulation. It is shown that the volume packing factor of submicron aerosol is determined with the error less than 6% and the accuracy of retrieval of the mean radius of particles reaches 0.01 μm.

Acknowledgments

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