FACTORS ENGENDERING THE EDGE WAVE AFFECTED BY ABSORPTIVITY, THICKNESS, AND SHAPE OF A DIFFRACTION SCREEN

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It has been experimentally established that above a body surface a zone is formed in which light rays deviate in two different directions on both sides of the initial propagation direction. The efficiency of light diffraction has been found to decrease with distance from a screen and its edges and fractures. A portion of the light energy of the edge wave component propagating from the screen has been found to convert into the energy of the edge wave component propagating in the geometric shadow with the screen covered in soot. It has been demonstrated that the edge light is produced by rays deviated in the diffraction zone and by rays reflected from the screen edge, with the latter being conceptually a Sommerfeld edge wave component. It has been found that the phase shift between the fundamental and Sommerfeld components of edge light results in their amplification on illuminated side and attenuation in the shadow to the extent that total values of flux of the edge rays propagating on both sides of a shadow boundary of a thin weakly absorbing screen turn out to be approximately equal. The effects of thickness, shape, and absorptivity of the screen on the edge light have been estimated. The hypothesis that the wave amplitude diffuses through the light wave front has been demonstrated to be false.

As is well known, the problem of the edge wave existence has attracted attention since the very beginning of investigations into diffraction of light. In spite of the fact that the edge light was discovered already by Newton, available information about the edge wave gave no clear–cut idea of the point of wave origin and factors engendering this wave, as well as of the effects of absorptivity, shape, and thickness of diffraction screens. Thus Sommerfeld,[1] Maey,[2] and Rabinovich[3] considered the screen edge to be a source of the edge wave, whereas Newton[4] and Young and Malyuzhents (see Ref. 5) believed that the edge wave is formed in a zone adjacent to a screen edge.

To elucidate the conditions of formation of edge light propagating from a thin screen (a blade), let us perform experiments using a scheme shown in Fig. 1, where S is the slit 30 μm wide, S' is its image matched with the slit s_b formed by blades, s_0 is the slit bounding an incident light beam, curves *1* and *2* characterize the distribution of the light intensity J over the width of the slit S' and over the input plane of a photomultiplier, O is the window (5. 5 mm wide) placed at the input of the photomultiplier, Ob is the objective with a focal length of 50 mm. The width t of the slit s_b is 98 μm, and the width of S' is 23 μm, i.e, a few times less than t given that the light flux coming from the screen is equal to 0.93 of the incident light flux. The slit s_0 (3.1–3.7 mm wide) transmitted only the rays forming the central maximum from S in the focal plane of the objective. The slit S is illuminated by a parallel beam of green light with λ = 0.53 μm selected from radiation of a filament lamp or emitted by a He–Ne laser. The incident beam width at the photomultiplier input varies from 6 to 6.5 mm as a function of λ at the distance L = 118 mm from s_b. Angular beam half–width γ is 1.4–1.57°.

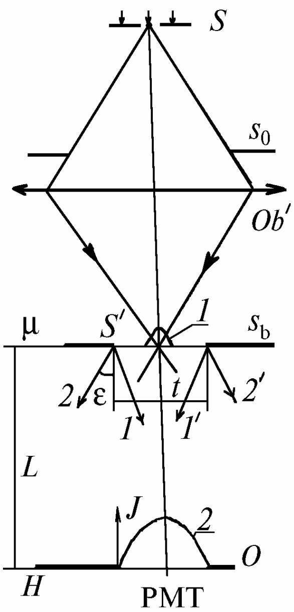

FIG. 1. Scheme of investigation of a light propagation pattern near the screen edge.

As can be seen from curve *2* in Fig. 2, describing the distribution of the light intensity $J = \Delta\Phi_{inc}/\Delta\mu$ over the width of S', the maximum intensity in the incident light beam have the rays passing through the center of S' (here $\Delta\Phi_{inc}$ is the change in the radiation flux incident on s_b when it is covered by one of the screens forming this slit for the distance Δμ).

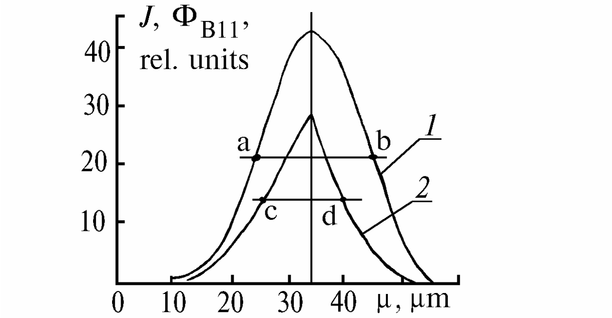

FIG. 2. Plots of the light intensity distribution over the width of the image S' of a light source and variation of the flux of the edge rays as functions of S' position relative to the screen edge.

In the initial position, S' is at the center of s_b. When s_b moves along the μ axis in such a way that S' approaches the left edge of the slit, after a while horizontal bands (B_1 and B_2) can be seen in the field of vision on both sides of the incident beam projection. These bands are formed by edge rays *1* and *2* propagating in the direction of incident light and in the shadow from the left screen of s_b. As is seen from the data in Table I and curve *1* in Fig. 2, the brightness of bands gradually decreases on the periphery and its behavior correlates with that of the light intensity distribution over the width of S' as it moves.

TABLE I.

H, mm	Φ_{B1i}, rel. units	Φ_{B2i}, rel. units
5.65	118	105
11.15	21	20
16.65	10	9
22.15	6	6.5
27.65	4	4.2
33.15	3.4	–
38.65	3	–

Table I gives the values of the light flux Φ_B of the edge rays recorded in the input plane of the photomultiplier in the intervals (5.5 mm wide) arranged consecutively along the band starting from the projection of unperturbed incident beam, and H is the distance from the beam axis to the center of the corresponding interval related to the diffraction angle of the edge rays by the dependence $H = L \tan\varepsilon$. As is seen, a major portion of flux of edge rays *1* and *2* is concentrated in the first interval.

Curve *1* characterizes the variation of flux $\Phi_{B11(21)}$ of edge rays *1* and *2* in the first interval as S' approaches the left edge of the slit and goes beyond the screen forming this edge.

Figure 2 demonstrates that the maxima of J and Φ_{B11} occur on the common vertical axis, in what follows that Φ_{B11} reaches its maximum when the incident light intensity is maximum at the point of origin of the edge rays, that is, when the center of S' is at this point. When s_b moves to the left, mirror symmetrical bands appear formed by rays *1'* and *2'*.

Let us move the photomultiplier to the right along the H axis to receive Φ_{B11}, then move s_b to the right until Φ_{B11} maximizes. If the edge rays had their origin at the slit edge, Φ_{B11} and $\Phi_{B1'1}$ would maximize with the center of S' and these edges coincident. However, the displacement $\Delta\mu_1$ of s_b from the position with maximum $\Phi_{B1'1}$ to the position with maximum Φ_{B11} turned out to be less than t by 9.3 μm. Hence edge light comes from the points removed at the distance $h = (t - \Delta\mu_1)/2 = 4.7$ μm from the slit edges rather than from the edges.

This fact testifies that a zone exists in optically less dense medium (in air) above the surfaces of screens forming the slit, in which the incident light rays deviate from the initial propagation direction and so form the edge rays. Furthermore, light entering into the diffraction zone deviates in the directions on both sides of the initial propagation direction, because rays *2* and *2'* are formed together with rays *1* and *1'*. Since slit edges are not ideally smooth and sharp, a certain portion of incident light will be scattered by irregularities and curvatures of these edges in the process of reflection. Edge light produced in this manner spreads in various directions in contrast to the edge light engendered by diffraction of the incident rays in the diffraction zone which propagates along the horizontal axis. Light scattered by the slit edges is seen predominately at large angles, when B_1 and B_2 exhibit low brightness but nevertheless, they are distinctly pronounced.

As S' approaches the slit edges, the flux of edge rays *1* and *1'* at the input of the photomultiplier maximizes in the band interval 3.5 mm wide located immediately behind the incident beam projection when the distance between the center of S' and the slit edges is 1.8 μm larger than that with maximum flux observed in the interval of the same width but displaced at the distance ΔH = 16. 5 mm from the first interval in the direction toward the end of B_1. That is, the efficiency of ray diffraction within the zone is no longer constant but increases from its boundary to the slit edge. For this reason the edge light intensity decreases rapidly as the angle of deflection from the original propagation direction increases.

Rays passing along the zone boundary do not deviate and hence form the boundary rays of edge light produced by the rays of a parallel incident beam. A diverging light beam coming from S' is formed by superposition of parallel beams of equal widths S' propagating at different angles relative to each other within angular divergence. As a consequence, the flux of the edge rays under examined conditions is formed by fluxes engendered by diffraction of rays of parallel beams within the zone and shifted relative to each other. Since the intensity of edge rays increases as diffraction angles decrease, and maximum intensity in the incident beam have axial and nearly–axial parallel beams, a major portion of the edge flux turns out to be concentrated within the projection of incident light and hence cannot be measured.

An analysis of the available data on the distribution of the light intensity over the incident beam width in the input plane of the photomultiplier and on the behavior of the edge light amplitude as a function of the diffraction angles[6,7] shows that maximum contribution to the edge flux on the periphery of the incident beam projection comes from the rays propagated initially at an angle of 45′ to the beam axis. Their diffraction angle within the zone is $(\gamma_{inc} - 45') = 49'$. In the light of the foregoing, the parameter h is not the entire width of the diffraction zone but characterizes the thickness of its layer adjacent to the screen edge within which incident rays deviate at the angles larger than 49′. In connection with decreasing efficiency of diffraction toward the zone boundary, its entire width should be much greater than h.

As S' approaches the slit edge, Φ_{B21} maximizes when the distance from the center of S' to the screen edge is 2.5 μm smaller than at the instant with maximum Φ_{B11}. This circumstance may be explained by less effective diffraction of rays in the direction toward the screen as compared with the efficiency of their diffraction in the direction from the screen on the same zone levels.

The ratio of the maximum value of Φ_{B11} to that of Φ_{B21} in different experiments varied within the limits 1–1.1 for natural light.

The values of the edge ray flux propagating in the shadow and in the opposite direction are equal to 14.8 and 16.6 rel. units, correspondingly, for the incident flux $\Phi_{inc} = 414$ rel. units. Hence the flux of incident rays engendering B_1 and B_2 should be $(\sqrt{14.8} + \sqrt{16.6})^2 = 63$ rel. units. The incident flux decreases by $\Delta\Phi_{inc} = 119.6$ rel. units when the central part of S' is covered by the screen for the distance $\Delta\mu = 5$ μm. On this basis we conclude that the edge rays propagating beyond the incident beam projection come from the diffraction zone layer with the approximate thickness $\Delta\mu(63/119.6) = 2.7$ μm.

In Ref. 6, based on the experimental investigations into the edge light intensity distribution over the plane S' obtained with the screen Sc located up against the axis of an incident beam (see Fig. 3), it was concluded that the edge light amplitude was inversely proportional to the distance from the observation point to the incident beam axis or inversely proportional to the angle of deviation of the edge rays from the initial propagation direction. In these experiments the incident beam width in the screen plane was much larger than S'. This casts doubt on the nature of investigated light: was it edge light or was it engendered by an open side of the wave front in line with the Huygens–Fresnel principle?

Let the slit s_1 be placed at the distance μ_1 from the beam axis, and the slit s_2 be matched with the center of the maximum max_1 recorded from s_1 against the criterion of maximization of the flux transmitted through it. In so doing, $H_1/\mu_1 = (l + L)/l$. In this case light incident on s_1 beyond S' comes actually from the edge of the screen Sc.

Since the maximum intensity of the edge rays cannot exceed the incident light intensity, formula (1) derived in Ref. 6 becomes invalid for small values of μ and ε. In the above–mentioned experiments this formula was correct for $\mu \geq 55$ μm ($l = 21.9$ mm) and $\varepsilon \geq 8.6'$. It cannot be verified for smaller values of μ due to the presence of directly transmitted light.

The distances between the points a, b and c, d taken at half maxima of the curves Φ_{B11} and J in Fig. 2 differ approximately by 7 μm. It is not difficult to understand that they must be equal at very small depth of the zone. Hence the difference between the distances is likely to be the thickness of the zone layer from which the incident rays fall within the interval $B_{11(21)}$ after diffraction.

The flux of the edge rays propagating in the direction from the screen decreases, and simultaneously the edge flux in the shadow (Table II) increases with a thin screen (blade) covered with a layer of soot 15 μm thick. In this case the total flux $(\Phi_{B11} + \Phi_{B21})$ remains unchanged, that is, soot causes conversion of a portion of the edge flux energy on the illuminated side into the flux of the edge rays in the shadow. We can easily understand this if we bear in mind that a part of rays of incident light is reflected from the screen, thereby engendering rays *1'* and *2'* (see Fig. 4). To propagate in the shadow, rays *2'* must be reflected under the edge pole. Because directly transmitted light cannot penetrate this region, it becomes apparent that these rays first deviate toward the screen in the diffraction zone.

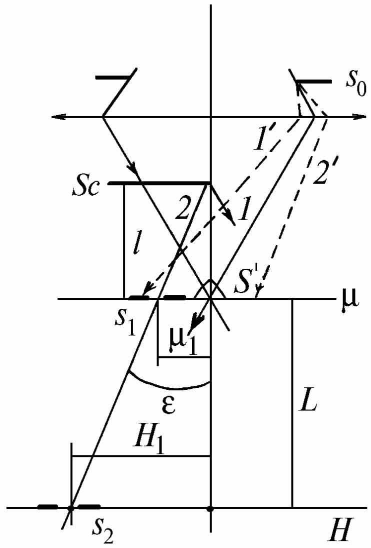

FIG. 3. Scheme of investigation of the edge wave pattern at small diffraction angles.

TABLE II.

Φ_{B11}, rel. units	Φ_{B21}, rel. units	Comment
15	15	Blade
9.5	20	Blade covered in soot

After ray *2'* has lost half wave by reflection, the rays *2* and *2'* become opposite in phase and hence the edge flux in the shadow decreases.

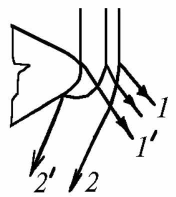

FIG. 4. Scheme of the edge light formation with a thin screen.

The rays *1* and *1'*, first opposite in phase,[6] match in phase after ray *1'* has lost half wave by reflection, and produce increased illumination to the right of the incident beam projection in comparison with the illumination produced by rays *1*. The rays *1'* and *2'* are absorbed by the screen covered in soot. As a result, only rays *2* propagate in the shadow, whereas rays *1* propagate on the other side. Because of this the illumination in the shadow increases, whereas on the other side, on the contrast, it decreases. To increase Φ_{B11} from 9.5 to 15 rel. units, the flux of rays *1'* must be equal approximately to $(\sqrt{15} - \sqrt{9.5})^2 = 0.626$ rel. units. To decrease Φ_{B21} from 20 to 15 rel. units, the flux of rays *2'* must be equal to $(\sqrt{20} - \sqrt{15})^2 = 0.36$ rel. units.

The foregoing shows that 1) edge light coming from the screen is produced by the incident rays deviated in the diffraction zone and reflected from the screen edge, with the latter being principally Sommerfeld's component of the edge light, 2) edge ray flux coming from the diffraction zone is the major portion of the total edge flux.

As the soot layer thickness decreases, the effect of the energy conversion between the edge light components becomes progressively less pronounced. Thus Φ_{B11} = 13 rel. units and Φ_{B21} = 14.8 rel. units when the layer thickness is of the order of 2–3 μm.

Let a thick screen with a flat face from the side of transmitted light, for example, a glass rectangular prism with lengths of the cathetus faces being equal to 10.6 μm, be placed instead of the slit s_b (see Fig. 1). The prism is positioned so that its front face lies in the plane S', whereas the face AB (see Fig. 5) is parallel to the incident beam axis. As a result of this replacement, the edge light propagating from the screen amplifies several times, as can be seen from Table III, where $\Phi_{B1.b}$ and $\Phi_{B1.p}$ are the values of the edge ray flux recorded in the intervals (5.5 mm wide) arranged consecutively along the band B_1 and coming from blade and prism, respectively. The effect of amplification of the edge flux testifies the decrease of the ray diffraction efficiency in the diffraction zone not only toward its boundary, but also along the face AB. Hence ray diffraction is most efficient in the zone near the prism edges.

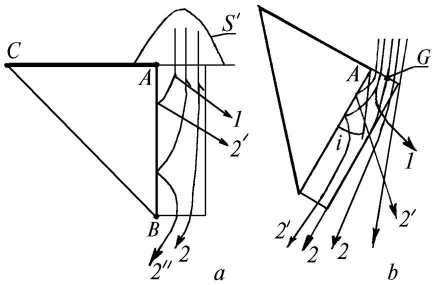

FIG. 5. Scheme of edge ray formation with a thick screen with flat faces.

TABLE III.

H, mm	$\Phi_{B1.b}$, rel. units	$\Phi_{B1.p}$, rel. units
5.5	12.2	36.4
11	2.3	5.5
16.5	1.1	2.3
22	0.7	0.9

The edge rays *2*′, after diffraction in the region of the prism edge A in the prism shadow, are incident on the face AB and reflect from it. After their reflection, the rays propagate in less effective regions of zone incapable of restoring their original direction. Therefore, they leave this zone propagating at various angles to the direction of propagation of edge rays *1* formed in the region of the edge A and propagating to the right of the shadow boundary (SB).

It is evident that the angles are the larger, the closer the rays *2*′ approach the edge A before the reflection.

The rays *1* and *2*′, being opposite in phase at the initial moment, matched approximately in phase owing to a loss of half wave by reflection and hence reinforce each other.

When the prism is rotated about the edge A at an angle i clockwise (see Fig. 5*b*), only the rays *2*′ entering into the zone within the region AG reach the face and reflect from it. The ray *2* after deviation at the point G grazes. The other rays *2*, being external to it, deviate at smaller angles and propagate to the left from the incident beam propagation direction at the angles smaller than i. As i increases, the point G moves toward more efficient region of the diffraction zone. As a consequence, the flux of the reflected rays will decrease, whereas the flux of the rays *2* will increase. At the same time, the region of overlap of beams of rays *2*′ and *1* decreases. The decrease of the number of reflected rays and of the region of their overlap with rays *1* causes the decrease of the total flux of rays propagating to the right from the incident beam.

Behavior of the values of flux Φ_{B11} and Φ_{B21} of the edge rays as a function of i is shown in Fig. 6*a*, where curve *1* is for Φ_{B11} and curve *2* – for Φ_{B21}. It is seen from the figure that at $i \geq 9°$ the edge flux Φ_{B11} becomes practically constant and, as the experiments have shown, equal to the flux of the edge rays propagating from a thin screen. That is, at $i \geq 9°$ the edge flux to the right of the incident beam is formed only by rays *1*, and in this respect thick screen is equivalent to a thin screen.

For edge rays *2* the difference between thick and thin screens is still pronounced at $i \geq 9°$, especially at large diffraction angles. It is evident from lesser values of the flux Φ_{B2i} recorded in the intervals arranged consecutively along B_2 in comparison with the analogous values of the flux Φ_{B1i} of the edge rays *1* (see Table IV) recorded at i = 13.9°. The essence of this difference is that the rays *2* after refraction in the effective region of the zone still reach the face AB and reflect from it even at such values of i (Fig. 5*b*). However, in this case reflected rays *2*′ are superimposed on the rays *2* refracted in less effective region of the zone rather than on the rays *1* owing to large values of i. Being in phase at the initial moment, after a loss of half wave by reflection they turn out to be opposite in phase and attenuate each other.

The effect of amplification of the edge ray flux propagating from the prism due to superposition of the edge rays *1* and the reflected rays *2*′ is confirmed by results of experiments with the face AB covered in soot. In this case the effect of absorption of the rays *2* incident on the face results in attenuation of the examined edge light. This is supported by Fig. 6*b*, in which the curves characterize the intensity J_{e1} of the edge light propagating from the prism at different distances from the incident beam axis with pure face AB (curve *1*) and face covered in soot (curve *2*).

TABLE IV.

Filament lamp light at λ = 0.53 μm				Laser radiation at λ = 0.6328 μm, with the electric vector lying in the plane of incidence			
ε°	H, mm	Φ_{B1i}, rel. units	Φ_{B2i}, rel. units	ε°	H, mm	Φ_{B1i}, rel. units	Φ_{B2i}, rel. units
2.7	5.4	20	14.9	2.9	5.8	25.1	19.5
5.5	11	4.6	1.6	5.7	11.4	5	2.6
8.3	16.6	2.1	0.47	8.5	17	2.3	0.6
11.1	22.2	1.35	0.05	11.3	22.6	1.1	0.05

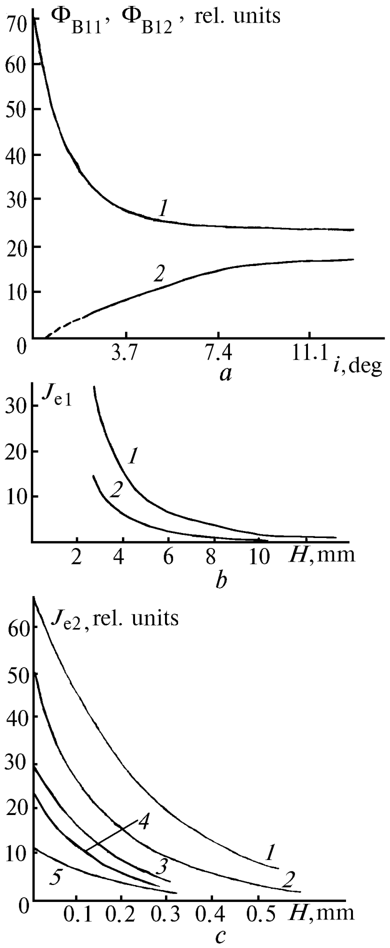

FIG. 6. Behavior of the edge light for a thick screen with flat faces as a function of i and state of the absorbing surface.

In contrast to attenuation of the edge light propagating to the right of the prism, the face covered in soot results in considerable amplification of the edge flux in the shadow, as is seen from Fig. 6*c*, in which curve *5* shows the intensity J_{e2} of the edge rays in the shadow of the prism at the distance L = 99.5 mm from the edge A (as a function of the distance to the shadow boundary) without soot, curve *2* – the intensity with prism face covered in soot, and curve *1* – the intensity with thin screen. The essence of this phenomenon is as follows. The edge rays *2* (Fig. 5*a*) after diffraction in less effective region of the zone meet with the rays *2''* diffracted in somewhat more effective region of this zone and for this reason capable of reaching the prism face and reflecting from it near the edge B. After reflection they propagate within the zone getting more effective toward the edge B, in contrast to the rays *2*, cannot overcome further diffraction toward the face, and owing to this enter the shadow. Being in phase at the initial moment, these rays after a loss of the half wave by reflection of one of them turn out to be opposite in phase. As a result, light in the shadow of the prism without soot has maximum intensity. With the face covered in soot the flux of rays *2* reaches the shadow without attenuation due to absorption of rays *2''*. Since the rays *2* and *2''* deviate within low effective region of the zone, the edge light in the prism shadow is seen predominantly at small angles with respect to the shadow boundary. Ratio of the light intensity at the shadow boundary to the intensity of light incident on this boundary increases from 0.048 to 0.29 without soot on the prism face.

When the width of the wave front near the edge A is much greater than that of S', prism covered in soot also causes increase in the illumination produced in the shadow.[8] This fact together with the explanation of experimental behavior of the light intensity additionally confirms that the open side of the wave front does not produce illumination.

For laser radiation and prism with pure face $\Phi_{B11p}(i{=}0)/\Phi_{B11p}(i{=}13.9°) = 2.6$, $\Phi_{B11s}(i{=}0)/\Phi_{B11s}(i{=}13.9°) = 3.16$.

At i = 13.9°, Φ_{B11p}/Φ_{B11s} = 0.984 and Φ_{B21p}/Φ_{B21s} = 1.2, that is, approximately equal to the corresponding ratios for a thin screen.

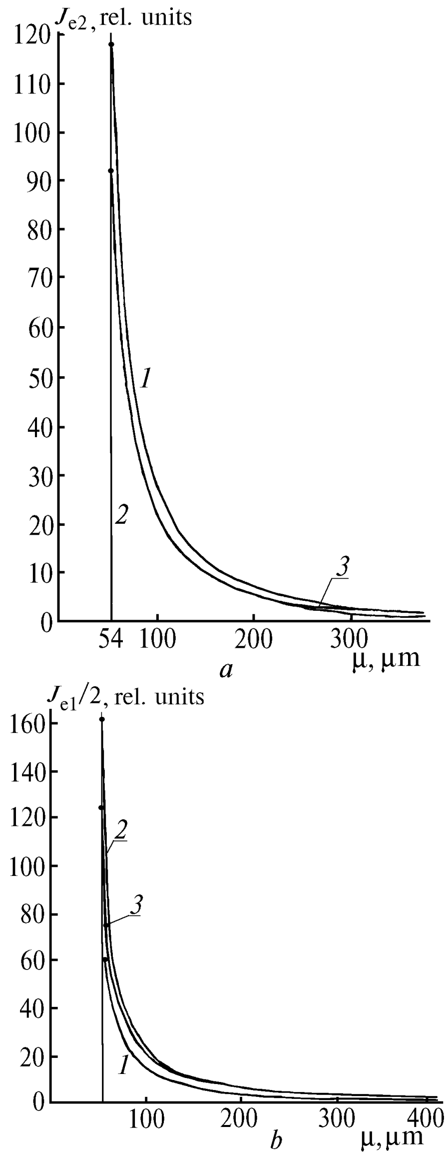

FIG. 7. Distribution of the light intensity of the edge flux coming from a cylindrical screen.

Figure 7*a* shows the behavior of the edge rays J_{e2} in the shadow of cylindrical screens obtained in the experiments based on the scheme shown in Fig. 3 (l = 35.5 mm). Curves *1*, *2*, and *3* characterize J_{e2} as a function of the distance to the S' axis in the shadow of a blade, polished aluminium rod 5.8 mm in diameter, and cylinder made of stainless steel 30 mm in diameter.

As is seen, with rod and cylinder used as a screen, the edge light intensities obey the same law up to $\mu = 0.25$ mm ($\varepsilon = 0.4°$) analogous to that of J_{e2} in the blade shadow.

Hence the conclusion by Fok[9] that the light intensity in the shadow obeys the exponential law is contrary to fact.

The edge light intensity to the right of S' (J_{e1}) is shown in Fig. 7*b*, where curves *1*, *2*, and *3* characterize J_{e1} coming from a blade, an aluminium rod, and a cylinder. In accordance with this curves, J_{e1} coming from the rod and the cylinder (in contrast to the edge light intensities in the shadow) is much higher than J_{e1} from the blade, and the law of the intensity variation is analogous to that of J_{e1} coming from the blade.

The reasons for the increase in the intensity of edge light coming from cylindrical screens to the right of S' and for its decrease in the shadow can be understood from Fig. 8. The rays *2* and *2″* propagating in the shadow after their refraction in the zone toward the screen turn out to be opposite in phase, due to a loss of the half wave by the rays *2″* due to reflection. Therefore, illumination in the shadow decreases.

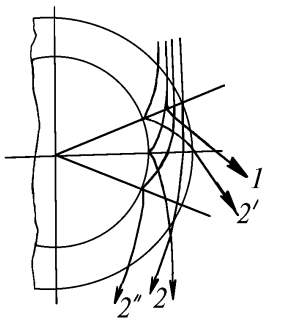

FIG. 8. Scheme of the edge light formation with cylindrical screens.

The rays *1* and *2′*, being opposite in phase at the initial moment, deviate in two different directions in the zone, turn out to be in phase due to a loss of the half wave by ray *2′* after reflection, and reinforce each other.

Due to the increase in the intensity of rays *2′* engendered by the increasing area of a reflecting surface, the light intensity to the right of S' turns out to be considerably higher than the intensity of corresponding rays coming from blade.

With screens covered in soot illumination in the shadow is produced only by rays *2*, whereas on the other side – by rays *1*. That is why illumination increases in the shadow and decreases to the right of S'.

There are two ideas that seems to be suitable for explanation of the diffraction zone existence above a body surface. They are the Newton hypothesis that light might interact with screen and the Young––Malyuzhents assumption that the amplitude diffuses into the wave front.

In case of diffusion of the amplitude, the following phenomena would be observed:

1. In the process of propagation of the incident wave along absorbing surface, wave amplitude in the region adjacent to the surface would be strongly attenuated due to absorption of light,[5] and diffusion of the amplitude in the shadow after the wave front passage of the distance of absorption would be less pronounced as compared with the case of light propagation above weakly absorbing surface. Actually the light intensity in the shadow of a glass prism covered in soot is higher than that without soot (Fig. 6*c*) as well as than that in the shadow of a prism with deposited aluminum film (curve *3*) and of the Johanson gauge block (steel) 9 mm thick (curve *4*).

2. In the case of propagation of the incident wave front along a flat weakly absorbing surface of a thick screen (a prism), amplitude distribution would preserve its form and hence the light intensity in the shadow of a prism would be approximately equal to that in the shadow of a thin screen. As is seen from curves *1* and *5* of Fig. 6*c*, it is several times less.

These facts show the hypothesis that the amplitude diffuses through the wave front is not verified by experiment.

As a consequence, only the Newton hypothesis remains from the available hypotheses that explain the factors engendering light diffraction in the diffraction zone.

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