EFFECT OF TURBID MEDIUM SCATTERING PHASE FUNCTION ANISOTROPY: ON LASER BEAM PARAMETERS

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The angular and spatial parameters of a laser beam are estimated analytically from the transfer equation in the small-angle approximation. It is shown that the angular anisotropy of scattered radiation can be accounted for simply by calculating the rms error in an elementary act of scattering.

Laser radiation propagating in turbid media typically displays significant dependence upon the optical properties of such media.

We estimate the energy and spatial-angular parameters of the laser beam propagating through a turbid medium by using for this purpose the small-angle approximation of the scalar version of the radiation transfer equation. Here the isotropic medium containing large scattering particles is assumed to be described by three optical parameters: its extinction factor ε, its scattering coefficient σ, and its scattering phase function $x(\gamma)$.

The small-angle approximation solution of the transfer equation has the form[1,2,3]

$$F(\bar{\xi}, L, \bar{\eta}) = F_0(\bar{\xi}, \bar{\eta} + \bar{\xi}L) \exp\left[-\int_0^L \left\{ \varepsilon(z) - \frac{\tilde{\sigma}(z) F_{ph}(z, |\bar{\eta} + \bar{\xi}(L - z)|)}{2} \right\} dz \right], \qquad (1)$$

where $F_0(\bar{\xi}, \bar{\eta} + \bar{\xi}L)$ is the Fourier transform of the radiation source luminance; $F_{ph}(z, |\bar{\eta} + \bar{\xi}(L - z)|)$ is the Fourier transform of the scattering phase function of the medium; z is the longitudinal coordinate axis, coinciding with the geometrical center of the beam; $\bar{\xi}, \bar{\eta}$ are the spatial and angular frequencies, respectively, of the Fourier transform of the radiation intensity in the plane $z = L$.

In the most typical practical situation, when the optical parameters of the medium are randomly varying along the beam path, the radiation flux transferred through a flat round area, symmetrical with respect to the beam axis, can be presented as

$$P = \int_{S_{rec}} \int \int_{-\infty}^{\infty} F(\bar{\xi}, z, 0) \exp(-i\bar{\xi}\bar{r})\, d^2r \cdot d^2\xi, \qquad (2)$$

where S_{rec} is the area of the receiving surface.

Exact integration of the expression (2) is, generally speaking, impossible. Practical results are obtained by approximate calculations. Note that, in contrast to free space, the spatial diffraction of radiation in the axial zone of a narrow light beam is clearly observed in a stratified turbid medium. We may consider the beam effective cross section as the parameter characterizing such diffraction. For the given case the effective cross section is most naturally defined as the ratio of the energy flux density in the beam to that at its axis. Then, by definition, we have[3]

$$S_{ef} = \frac{4\pi^2 F_0(0, L, 0)}{\int\int_{-\infty}^{\infty} F(\bar{\xi}, L, 0)\, d^2\xi}, \qquad (3)$$

where S_{ef} is the effective cross-section of the beam in the $z = L$ plane.

On the other hand.

$$F_{ef} = \pi R^2_{ef} \qquad (4)$$

where R_{ef} is the effective beam radius.

Substituting (1) into (3) for a source with a Gaussian luminance distribution in the $z = 0$ plane and sin exponential scattering phase function[2], we have

$$x(\gamma) = \frac{2}{\mu^2(z)} \exp[-\gamma/\mu(z)] \qquad (5)$$

which yields

$$S_{ef} = \frac{2\pi \cdot \exp\left[\int_0^L \{\varepsilon(z)\}\, dz \right]}{\int_0^{\infty} \xi \exp\left[-\xi^2 \frac{r_p^2}{4} - \int_0^L \left\{ \varepsilon(z) - \frac{\tilde{\sigma}(z)}{[\{\mu(z)\xi(L-z)\}^2 + 1]^{3/2}} \right\} dz \right] d\xi}, \qquad (6)$$

where $\tilde{\sigma}(z)$ is the effective scattering coefficient $\tilde{\sigma} = \sigma(z)(1 - x_0)$, x_0 is the isotropic backscattering part of the scattering phase function.

Integrating the denominator of (6) by parts and replacing variables as in Ref. 2, we obtain

$$R^2_{ef} = r_p^2 \frac{Q + 1}{Q + \exp[-B(Q + 1)]}. \qquad (7)$$

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where $r_p^2 = r_{in}^2 + \alpha_{in}^2 L^2$;

$$Q = \frac{r_p^2}{\int^L \tilde{\sigma}(z) \langle \gamma^2(z) \rangle (L-z)^2 dz} ; \quad B = \int \tilde{\sigma}(z) dz ;$$

and r_{in} is the initial beam radius; α_{in} is the initial angular divergence of the beam, $\langle \gamma^2(z) \rangle$ is the mean square of the angle of ray deviation in each elementary act of scattering for the phase function of the form (5) $\langle \gamma^2(z) \rangle = 6\mu^2(z)$.

As can be seen, in a turbid medium the effective beam radius is given by the product of the beam size in free space and a certain function characterizing the radiation diffraction. This result enables one to simplify greatly the procedure of assessing beam parameters for the actual conditions.

By analogy with the effective cross section, one can consider the effective angular divergence of the beam. We define it as the ratio of the effective radius to the propagation distance. We have then

$$\alpha_{ef}^2 = \alpha_{in}^2 \frac{Q + 1}{Q + \exp[-B(q+1)]} , \tag{8}$$

where α_{ef} is the effective angular divergence of the beam in the medium.

From the above expressions we can obtain for the recorded beam power

$$P = \pi P_0 \left(1 - \exp\left[-\frac{r_0^2}{R_{ef}^2} \right] \right) \times$$
$$\times \exp\left[-\int_0^L \{ \varepsilon(z) - \tilde{\sigma}(z) \} dz \right] , \tag{9}$$

where r_0 is the radius of the receiving surface which intercepts the incident flux.

The case when the beam is symmetrical with respect to the receiving surface is exceptionally idealized. As a rule, the beam axis does not coincide with the receiver geometrical center. To estimate in a convenient manner the effect of the beam spatial displacement on the signal level one can introduce a function of the form

$$f(\varphi) = \exp\left[-\frac{(x-\alpha)^2 + (y-\beta)^2}{r_0^2} \right] , \tag{10}$$

satisfying the normalization condition

$$\int \int_{-\infty}^{\infty} f(\varphi)\, dx\, dy = \pi r_0^2 .$$

Here α, β are the parameters characterizing the beam displacement $\varphi = \sqrt{\alpha^2 + \beta^2}$.

This function opens up the possibility: for taking account of the effect of the beam displacement on the signal level, and for extending the integration to an infinite plane, thus simplifying the calculations considerably. As a result we have

$$P = \pi P_0 \frac{r_0^2}{(R_{ef}^2 + r_0^2)} \exp\left[\frac{\varphi^2}{(R_{ef}^2 + r_0^2)} \right] \times$$
$$\times \exp\left[-\int_0^L \{ \varepsilon(z) - \tilde{\sigma}(z) \} dz \right] , \tag{11}$$

Comparing the obtained relationships, we see that (11) is not equal to (9) for $\varphi = 0$; this is the consequence of the simplifications we assumed while introducing the function (10). However, in practice, for $r_0/R_{ef} \ll 1$, the above discrepancy is negligibly small.

The analysis of the above results demonstrates that to estimate the power of the optical signal transmitted through a turbid medium, one has only to take account of the combined effect of spatial diffraction and extinction of radiation, both due to multiples scattering in the direction of beam propagation. These conclusions find their support in comparisons of theoretical vs. experimental data.[2,3]

in the medium

The processes considered in this paper relate to stationary radiation sources. Moreover, if the source intensity varies noticeably at a temporal scale considerably longer than the average lifetime for photons in the medium, the light field there can be considered quasistationary,[4] so that there remains no need to account for temporal distortions of the pulse shape produced by scattering of the photon pathways in the medium.[5]

These results can find application to calculating laser beam characteristics in the real atmosphere, and also to assessing the required field of view for receiving optical systems.

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