ESTIMATION OF ERROR IN HUMIDITY OF THE TURBULENT ATMOSPHERE MEASURED USING THE RADIOACOUSTIC SOUNDING PHASE DIFFERENCE TECHNIQUE

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An equation for humidity determination by double–frequency phase method of radioacoustic sounding and numerical estimation of the effect of atmospheric turbulence on its performance are obtained. They are applicable at any ratio between relaxation and sounding frequencies. Relative deviation of humidity averaged over its turbulent fluctuations from such a value in nonturbulent atmosphere is obtained assuming weak fluctuations of speed of sound, phase differences of two frequencies and that their densities are distributed by Gaussian law. The phase difference fluctuations distribution variance is calculated for a limited Gaussian beam in a turbulent medium with Karman spectrum. For sufficiently large values of structure characteristics of temperature and wind velocity (under convective conditions at a moderate wind) the relative deviation of average humidity in this case does not exceed $2\cdot10^{-3}$% *at altitudes* 50–200 m *and the rms error, due to turbulence, in humidity measurements by the phase method does not exceed* 0.6%.

The method of double–frequency sounding determination of air humidity[1] is based on the measurement of phase difference of acoustic waves at two different frequencies, $\Delta\varphi_a$, arising due to sonic speed dispersion in humid air.[2] To measure this difference, we propose to use the integer transducer of one of the frequencies, that will allow us to find the dependence of the phase difference of acoustic waves on the difference in their speeds of propagation

$$\Delta\varphi_a = 2\pi f Z (\Delta C / C^2) . \tag{1}$$

In Eq. (1) $f = f_2$ is the frequency at which the phase difference is measured; Z is the distance from the plane of emission; $\Delta C = C_2 - C_1$ is the difference between speeds of sound; C is the average speed of sound. Equation (1) is valid if the temperature and the humidity of air change slowly with altitude, and the refractive index for the acoustic waves, n, is approximately equal to unity along the path of their propagation. In Ref. 1 the formula is presented for humidity determination from the findings on phase difference. This formula has been derived under the condition $f_r \gg f_1$ and $f_r \gg f_2$ (f_r is the relaxation frequency of humid air) and assuming that speeds of acoustic waves differ a little and Bragg condition[3] holds. The influence of turbulent fluctuations of the phase difference has been estimated here in the geometric optics approximation (GOA).

As known, the relaxation frequency of a humid air is usually estimated experimentally[2,4] or from theoretical calculations. The ANSI standard[3] is the best known among the latter. For the method under consideration, the empirical expression[4] for f_r (in hertzs) was chosen

$$f_r = 3.06 \cdot 10^4\, h^{1.3}, \tag{2}$$

where $h = (e/p)100\%$ is the molar concentration, e is the partial pressure of water vapor, and p is the atmospheric pressure. Under different meteorological conditions the value of relaxation frequency may vary from 0.22 to 200 kHz. The condition of smallness of sounding frequencies in the range 1–10 kHz in comparison with the relaxation ones holds only at air temperature $T > +15$°C, and at low relative humidity $H < 60\%$ it holds at higher temperature, i.e., within a limited range of meteorological conditions. In the present paper the formula for humidity determination and estimation of turbulence effect were obtained which are applicable at any ratio between the relaxation and sounding frequencies; the method of smooth perturbation (MSP) was used for estimating the turbulence effect.

The phenomenon of dispersion of speed of sound in air[2,3,4] causes the difference in speed of sound at different frequencies. For two frequencies, f_1 and f_2, this difference equals

$$\Delta C = \frac{C_\infty^2 - C_0^2}{2\,C}\left[\frac{f_2^2}{f_r^2 + f_2^2} - \frac{f_1^2}{f_r^2 + f_1^2}\right] + \Delta W, \tag{3}$$

where C_∞ and C_0 are the speeds of sound at a frequency $f \gg f_r$ and at $f \ll f_r$, respectively, ΔW is the difference between the wind velocity projections on the direction of acoustic waves propagation. If the acoustic waves of different frequencies are emitted simultaneously in a given direction, ΔW can be considered to be equal to zero along the whole path of their propagation. Having solved Eq. (1) with respect to h taking into account Eqs. (2) and (3) and removing the restrictions $f_r \gg f_1$ and $f_r \gg f_2$, we obtain the formula for the air humidity determination

$$h = \exp\left\{0.385 \ln\left[\frac{\pi f z (C_\infty^2 - C_0^2)}{2\,C^3\,\Delta\varphi_a}(f_2^2 - f_1^2) - \frac{f_1^2 + f_2^2}{2} \pm \right.\right.$$
$$\left.\left. \pm\left\{\left[\frac{f_1^2+f_2^2}{2} - \frac{\pi f z (C_\infty^2 - C_0^2)}{2C^3\Delta\varphi_a}(f_2^2 - f_1^2)\right]^2 - f_1^2 f_2^2\right\}^{1/2}\right] - 7.947\right\}. \tag{4}$$

applicable under any meteorological conditions. In Eq. (4) the plus sign determines the higher value of humidity at one and the same phase shift.

When measuring the humidity by vertical radioacoustic sounding from data on the phase difference $\Delta\varphi_e = (w_{D_2} - w_{D_1})(Z/C)$ between Doppler shifts of radiofrequencies w_{D_1} and w_{D_2} ($w_{D_i} = (4\pi f_e/C_e)C_i$; $i = 1, 2$; C_e is the speed of radiowave), formula (4) is applicable under the Bragg condition $2f_e/C_e = f/C$ (Ref. 3), when $\Delta\varphi_e = \Delta\varphi_a$. Similar formula is applicable when the Bragg condition does not hold. If $f_r \gg f_1$ and $f_r \gg f_2$, Eq. (4) takes a simpler form, derived in Ref. 1,

$$h = \exp\left\{0.385 \ln\left[\frac{\pi f z (C_\infty^2 - C_0^2)}{C^3 \Delta\varphi_a}(f_2^2 - f_1^2)\right] - 7.947\right\}. \quad (5)$$

When sounding the turbulent atmosphere, the turbulent fluctuations of sonic speed, C_1 and C_2, occur,[5] that brings about the turbulent fluctuations of the average sonic speed C and phase differences $\Delta\varphi_a$ and $\Delta\varphi_e$.

To estimate the effect of atmospheric turbulence on the accuracy of air humidity determination from phase difference in double–frequence radioacoustic sounding, equations (4) and (5) should be statistically averaged. Assuming that turbulent fluctuations C', $\Delta\varphi'_a$ (or $\Delta\varphi'_e$) are small and distributed according to Gaussian law, we apply the linearization of logarithms in Eqs. (4) and (5) with respect to fluctuations C', $\Delta\varphi'_a$ (or $\Delta\varphi'_e$) and the theorem about the mean value of exponential function of normally distributed fluctuating variable with zero mean. We take into account the correlation terms using linear approximation

$$\left\langle \exp 1.155 \frac{\Delta\varphi' C'}{\Delta\varphi_0 C}\right\rangle \simeq 1 + 1.155 \frac{\langle\Delta\varphi' C'\rangle}{\Delta\varphi_0 C}. \quad (6)$$

In this case the relative deviation of humidity $\langle h\rangle$ averaged over turbulent fluctuations from that in nonturbulent atmosphere, h_0, equals

$$\frac{\langle h\rangle - h_0}{h_0} \simeq \left(1 + 1.155 \frac{\Delta\varphi' C'}{\Delta\varphi_0 C}\right) \times$$

$$\times \exp\left\{0.074\left[\frac{\langle\Delta\varphi'^2\rangle}{\Delta\varphi_0^2} + \beta^2 \frac{\langle C'^2\rangle}{C^2}\right]\gamma^2(h_0)\right\} - 1 \simeq$$

$$\simeq 0.074\left[\frac{\langle\Delta\varphi'^2\rangle}{\Delta\varphi_0^2} + \beta^2 \frac{\langle C'^2\rangle}{C^2}\right]\gamma^2(h_0) + 1.155 \frac{\Delta\varphi' C'}{\Delta\varphi_0 C} \quad (7)$$

at small turbulent fluctuations C' and $\Delta\varphi'$. Here C and $\Delta\varphi_0$ are the unperturbed values of sonic speed and phase difference ($\Delta\varphi_0 = \Delta\varphi_a$ or $\Delta\varphi_0 = \Delta\varphi_e$),

$$\gamma(h_0) = (f_r^2 + f_2^2)(f_r^2 + f_1^2) / (f_r^4 - f_1^2 f_2^2),$$

and $\beta = 3$, if Bragg condition holds, otherwise $\beta = 2$. For $f_r \gg f_1$ and $f_r \gg f_2$, when formula (5) is valid, $\gamma(h_0)$ is about unity. When the value of relaxation frequency is close to the values of sounding frequencies (at air temperature below 15°C or low relative humidity), the coefficient $\gamma(h_0)$ essentially differs from unity. Figure 1 shows the coefficient γ vs. relative humidity H.

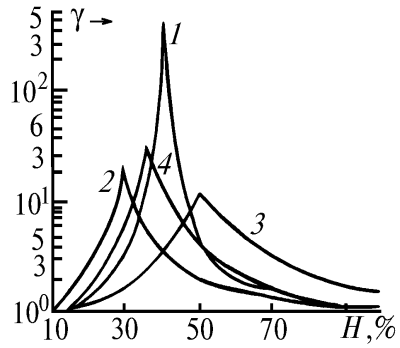

FIG. 1. Coefficient γ vs. relative humidity H at air temperature 0°C: *frequencies are* 3.4 *and* 6.8 kHz *(1),* 2.5 *and* 5 kHz *(2),* 5 *and* 10 kHz *(3), and* 2.5 *and* 10 kHz *(4).*

Ignoring the turbulent fluctuations of atmospheric pressure, the relative deviation of average absolute, e, and relative, H, humidity can be determined by the equality (7) too, because

$$\frac{\langle e\rangle - e_0}{e_0} = \frac{\langle H\rangle - H_0}{H_0} = \frac{\langle h\rangle - h_0}{h_0},$$

where $H = (p/e_s)$, in %, e_s is the water vapor saturation pressure.

To find the variance of distribution of sonic speed fluctuations $\langle C'^2\rangle$, let us use the relation[5]

$$\langle C'^2\rangle = 2 C^2 C_n^2 L_0^{2/3}, \quad (8)$$

where C_n^2 is the structure characteristic of the acoustic refractive index, L_0 is the outer scale of turbulence, in the ground layer it equals $0.4 Z$ (Ref. 5).

The variance of distribution of weak turbulent fluctuations of difference of acoustic phase at two frequencies, $\langle\Delta\varphi'^2\rangle$, can be calculated on the basis of relationship for time autocorrelation function of phase for a limited Gaussian beam[6] in a turbulent medium with Karman spectrum. The dependence of the spectrum and the structure constant of the acoustic refractive index C_n^2 on the parameters of moving atmosphere has been refined in Ref. 7. One may start from the assumption that acoustic antenna produces an acoustic wave with the plane phase front in the aperture plane.[4] Under the condition $\Delta k_a \gg k_a$, where $\Delta k_a = 2\pi f \Delta C/C^2$ is the difference of wave numbers of acoustic waves of f_1 and f_2 frequencies, calculated after frequency transformation, the authors of this paper have derived the relation for the variance of distribution of fluctuation of phase difference of wave beams

$$\langle\Delta\varphi'^2\rangle = 2\pi^2 (\Delta k_a)^2 \int_0^Z d\xi \int_0^\infty d\kappa\, \kappa\, \Phi_n(\kappa) \times$$

$$\times \exp\left\{-\alpha_2 \frac{Z-\xi}{k_a}\kappa^2\right\}\left\{1 + \cos\left[\alpha_1 \frac{Z-\xi}{k_a}\kappa^2\right]\right\}, \quad (9)$$

where $\alpha_1 = \dfrac{W^2 + \xi/Z}{1+\Omega^2}$, $\alpha_2 = \dfrac{W(1-\xi/Z)}{1+\Omega^2}$, $\Omega = 2\pi a^2/Z\lambda$ is the Fresnel number of the transmitting acoustic antenna with the aperture radius a for the sounding wavelength $\lambda = \lambda_2$, $\Phi_n(k)$ is the Karman spectrum[7,8] of the acoustic refractive index n:

$\Phi_n(\kappa) = 0.033\, C_n^2\, (\kappa^2 + k_0^2)^{-11/6}$, $\kappa_0 = 2\pi / L_0$.

For $\Omega \gg 1$ (plane wave) $\alpha_1 = 1$ and $\alpha_2 = 0$, and for $\Omega \ll 1$ (spherical wave) $\alpha_1 = \xi/Z$ and $\alpha_2 = 0$. In this limiting cases, formula (9) determines the variance of distribution of fluctuations of phase difference between plane and spherical waves[8]. For numerical estimation of $\langle\Delta\varphi'^2\rangle/\Delta\varphi_0^2$ we transform Eq. (9) to the form

$$\langle\Delta\varphi'^2\rangle / \Delta\varphi_0^2 = 0.326\, [\mathcal{L}(Z, \lambda, a) / Z^2], \tag{10}$$

where

$$\mathcal{L}(Z, \lambda, a) = \int_0^Z d\xi C_n^2(\xi) \int_0^\infty dy (y + y_0)^{-11/6}\, e^{-Ay}(1 + \cos B y),$$

in m^2; $y_0 = k_0^2$; $A = \dfrac{\Omega\,(Z-\xi)^2}{k_a\,(1+\Omega^2)\,Z}$; $B = \dfrac{(Z-\xi)\,(Z\,\Omega^2 + \xi)}{k_a\,(1+\Omega^2)\,Z}$; C_n^2 is the structure characteristic of the acoustic refractive index,[7] in m$^{-2/3}$, Z is the sounding altitude, in m. Note that at any positive y

$$\left|e^{-Ay}\right| < 1, \quad \left|1 + \cos B\, y\right| < 2.$$

Thus, the estimation of $\langle\Delta\varphi'^2\rangle/\Delta\varphi_0^2$ derived for wave beams at weak phase fluctuations using smooth perturbation method is always smaller than that derived in approximation of geometric optics approach applicable under the condition $(\kappa^2(Z-\xi)/k_a \ll 1$, when $e^{-Ay} \simeq 1$ and $(1 + \cos By) \simeq 2$. The upper limit of the variance of phase difference fluctuations determined by the geometric–optics method

$$\langle\Delta\varphi'^2\rangle / \Delta\varphi_0^2 < 3.64 \cdot 10^{-2}\, C_n^2\, L_0^{5/3} / Z \tag{11}$$

is independent of sounding frequency, the antenna size, and wind velocity.[1]

To estimate the correlation function of the phase difference fluctuations and fluctuation of sonic speed, the effect of inhomogeneity of the acoustic refractive index along the path of propagation should be taken into account. In geometric optics approximation the fluctuations of phase difference $\Delta\varphi'$ are linearly related to fluctuations of the acoustic refractive index n' (Ref. 5):

$$\Delta\varphi' = \Delta k_0 \int_0^Z n'(\kappa)\, d\kappa, \tag{12}$$

where $\Delta k_0 = 2\pi f \Delta C/C^2$, $n' = -C'/C$. Since the variance of phase difference fluctuations is determined by the integral

$$\langle\Delta\varphi'^2\rangle = (\Delta k_0)^2 \int_0^Z \int_0^Z \langle n'(\kappa)\, n'(\kappa')\rangle\, d\kappa\, d\kappa', \tag{13}$$

the correlation function can be related to the derivative of it

$$\frac{\Delta\varphi'\, C'}{\Delta\varphi_0\, C} = -\frac{1}{Z}\int_0^Z \langle n'(\kappa) n'(Z)\rangle d\kappa = -\frac{1}{2\Delta\varphi_0 \Delta k_0}\frac{\partial}{\partial Z}\langle\Delta\varphi'^2\rangle =$$

$$= -\frac{\langle\Delta\varphi'^2\rangle}{2\,\Delta\varphi_0^2}\,\frac{\dfrac{\partial}{\partial Z}(C_n^2\, Z^{8/3})}{C_n^2\, Z^{5/3}}, \tag{14}$$

having used Eq. (11).

Over dry underlying surface the structure constant of the humidity fluctuations and correlation of temperature and humidity is several orders of magnitude smaller than those of fluctuations of temperature and wind velocity. In this case for C_n^2 calculation in Eqs. (8), (11), and (14) the following formula[7]

$$C_n^2 = 1/4(C_T^2 / T^2 + 7.33\, C_V^2 / C^2) \tag{15}$$

should be used.

Let us consider now the case of sufficiently large values of structure characteristics determined by empirical equations[3] (under convective conditions at a moderate wind):

$$C_T^2 = C_{T_0}\,(Z / Z_0)^{-4/3};$$
$$C_V^2 = C_{V_0}\,(0.03 + 0.97\,(Z / Z_0)^{-2/3}), \tag{16}$$

where $C_{T_0} = 62$ deg^2 m$^{-2/3}$, $C_{V_0} = 1.54$ m$^{4/3}$s^{-2}, $Z_0 = 10$ m.

Let us use the relation (15) and altitude dependences (16) to estimate the correlation function (14)

$$\langle\Delta\varphi'\, C'\rangle / \Delta\varphi_0\, C \simeq -1.2\,\langle\Delta\varphi'^2\rangle / \Delta\varphi_0^2, \tag{17}$$

and the variance of fluctuations of phase shift and sonic speed

$$\langle\Delta\varphi'^2\rangle / \Delta\varphi_0^2 \lesssim (1.1 \div 2.1) \cdot 10^{-7};$$
$$\langle C'^2\rangle / C^2 \simeq (1.5 \div 2.9) \cdot 10^{-5} \tag{18}$$

at an altitude 50–200 m at air temperature from +20 to 40°C and relative humidity $H > 30\%$. As a result, the turbulent shift of the average humidity h according to Eq. (7) and numerical estimates (17) and (18) is mainly determined by the effect of sonic speed fluctuations and equals to

$$|(\langle h\rangle - h_0) / h_0| \lesssim (1.0 \div 1.9) \cdot 10^{-3}\%.$$

In conclusion let us calculate the rms deviation $\sqrt{\langle(h-h_0)^2\rangle}/h_0$ under the same meteorological conditions in a turbulent atmosphere under the condition $f_r \gg f_1$ and $f_r \gg f_2$

$$\frac{\sqrt{\langle(h-h_0)^2\rangle}}{h_0} = \frac{1}{h_0}\left[\left(\frac{\partial h}{\partial\Delta\varphi}\right)^2\langle\Delta\varphi'^2\rangle + 2\,\frac{\partial h}{\partial\Delta\varphi}\frac{\partial h}{\partial C}\,\langle\Delta\varphi' C'\rangle + \right.$$

$$\left. + \left(\frac{\partial h}{\partial C}\right)^2\langle C'^2\rangle\right]^{1/2} = 0.385\sqrt{\frac{\langle\Delta\varphi'^2\rangle}{\Delta\varphi_0^2} + 3\,\frac{\langle\Delta\varphi'\, C'\rangle}{\Delta\varphi_0\, C} + 9\,\frac{\langle C'^2\rangle}{C^2}},$$

where $\dfrac{\partial h}{\partial C}$ and $\dfrac{\partial h}{\partial\Delta\varphi}$ are the derivatives of the humidity h. At an altitude 50–200 m and air temperature from 20 to 40°C we obtain from numerical estimates (17) and (18)

$$\sqrt{\frac{\langle(h-h_0)^2\rangle}{h_0}} \lesssim (0.45 \div 0.62)\ \%.$$

If there are only temperature inhomogeneities in the atmosphere then the absolute value of variance of fluctuations of the phase shift $<\Delta\varphi'>$ increases with increasing altitude more slowly than the square phase shift $\Delta\varphi_0^2$, because at an altitude below 1 km the structure function C_n^2 decreases rapidly with altitude. Under this conditions the rms error and turbulent shift of the average humidity decrease proportionally to $Z^{-1/3}$ and $Z^{-2/3}$, respectively. However, such conditions are very rare. Under the effect of wind velocity inhomogeneities the rms error and turbulent shift of humidity increase proportionally to $\sqrt{d + bZ^{2/3}}$ and $d + bZ^{2/3}$, where d and b are the functions of air temperature. Thus, for the developed turbulence (under convective conditions at moderate wind) the relative deviation of the average humidity and rms error of humidity measurements by phase difference due to turbulence in double–frequency radioacoustic sounding are essentially smaller than those in the amplitude radioacoustic sounding.[9]

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