ON THE ENERGY PARAMETERS OF ECHO-SIGNALS IN SLANT SENSING OF A FOAM-COVERED SEA SURFACE THROUGH THE ATMOSPHERE

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The power of the echo-signal in pulsed sensing of the sea surface partially covered with foam through the atmosphere is studied. Analytical expressions are derived for the average received power and for the echo-pulse delay and width in slant sensing of the sea surface through both the optically transparent and dense aerosol atmospheres. It is shown that the presence of foam on the sea surface significantly distorts the shape of the received echo-signal.

The energy parameters of the echo–signal in nadir sensing of the sea surface partially covered with foam were considered in Ref. 1. Below we study the echo–signal power in slant sensing of the sea surface partially covered with foam.

Let the radiation wavelength be small compared with the characteristic heights of the irregularities and the curvature radii of the sea surface. Analogous to Ref. 1, we may write the expression for the power received by a lidar

$$P(t) = (1 - S_f)P_0(t) + S_f P_f(t), \tag{1}$$

where S_f is the relative fraction of the sea surface covered with foam and whitecaps; $P(t)$, $P_0(t)$, and $P_f(t)$, are, respectively, the average radiation power recorded when sensing the sea surface partially covered with foam, foam–free sea surface, and sea surface continuously covered with foam.

Analogous to Ref. 1, for the model of the foam–free sea surface we take the model of a randomly uneven locally specular surface, and for the model of the sea surface continuously covered with foam we consider two models: the model of a randomly uneven, locally Lambertian surface and the model of a flat Lambertian surface.

The value of $P_f(t)$ for the model of a flat Lambertian surface is well known. Let us find the average radiation powers received by a lidar in slant sensing through the atmosphere for the models of a randomly uneven surface with locally Lambertian and locally specular scattering phase functions of elementary sections, assuming that the distribution of elementary section slopes is identical to that of the sea wave slopes. We also assume that the source and receiver are separated.

Analogous to Ref. 1, we may write an integral expression for the echo–signal power in slant sensing of the randomly uneven surface S (neglecting shading of some elements of the surface by other elements, assuming the probability density functions for heights and slopes of the sea surface to be Gaussian, and considering for simplicity that the optical axes of the source and the receiver lie in the same XOZ plane):

$$P_{0,f}(t) \simeq \int\limits_{-\infty}^{\infty} d\zeta\, W(\zeta) \int\limits_{S_0} E_s(R'_\zeta)\, E_r(R''_\zeta)\, K_{0,f}\left(\overline{\gamma_x^2}, \overline{\gamma_y^2}, R_x, R_y\right) \times$$

$$\times f\left(t - \frac{L_s + L_f}{c} - \frac{R_x(\cos\psi + \cos\chi)}{c} + \frac{\zeta(R)(\sin\psi + \sin\chi)}{c} - \frac{R_x^2 + R_y^2}{2c}\left(\frac{1}{L_s} + \frac{1}{L_f}\right)\right) dR. \tag{2}$$

Here, for a locally Lambertian surface we have

$$K_f\left(\overline{\gamma_x^2}, \overline{\gamma_y^2}, R_x, R_y\right) = K_f\left(\overline{\gamma_x^2}, \overline{\gamma_y^2}\right) =$$

$$= \frac{A}{\pi} \frac{a \exp\left(\frac{1}{2a}\right)}{4\left(\overline{\gamma_x^2}, \overline{\gamma_y^2}\right)^{1/2}} \sum_{k=0}^{\infty} \frac{a^{-k}}{k!} \left(\frac{\beta}{2}\right)^{2k} \times$$

$$\times \left\{\cos\psi\, \cos\chi\, a^{1/4} \frac{\Gamma(2k + 2)}{\Gamma(k + 1)} W_{-k-3/4,k+3/4}\left(\frac{1}{a}\right) - \right.$$

$$- \cos\psi\, \cos\chi\, a^{-1/4} \frac{\Gamma(2k + 3)}{\Gamma(k + 2)} \left(\frac{\beta}{2}\right) W_{-k-5/4,k+5/4}\left(\frac{1}{a}\right) +$$

$$\left. + 2\sin\psi\, \sin\chi\, a^{-1/4} \frac{\Gamma(2k + 1)}{\Gamma(k + 1)} W_{-k-1/4,k+1/4}\left(\frac{1}{a}\right)\right\};$$

$$a = 4\left(\frac{1}{\overline{\gamma_x^2}} + \frac{1}{\overline{\gamma_y^2}}\right)^{-1}; \quad \beta = \frac{\Delta a}{2}; \quad \Delta = \frac{1}{2\overline{\gamma_x^2}} - \frac{1}{2\overline{\gamma_y^2}};$$

and for a locally specular surface we have

$$K_0\left(\overline{\gamma_x^2}, \overline{\gamma_y^2}, R_x, R_y\right) = \exp\left\{ - \frac{1}{2\overline{\gamma_x^2}}\left[\frac{q_x}{q_z} + \frac{R_x}{q_z} \times \right.\right.$$

$$\left.\left. \times \left(\frac{\sin^2\psi}{L_s} + \frac{\sin^2\chi}{L_f}\right)\right]^2 - \frac{1}{2\overline{\gamma_y^2}}\left[\frac{R_y}{q_z}\left(\frac{1}{L_s} + \frac{1}{L_r}\right)\right]^2 \right\} \times$$

$$\times K_0\left(\overline{\gamma_x^2}, \overline{\gamma_y^2}\right);$$

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$$K_0\left(\overline{\gamma_x^2}, \overline{\gamma_y^2}\right) = \frac{q^4}{q_z^4}\frac{V^2}{8\pi\left(\overline{\gamma_x^2}, \overline{\gamma_y^2}\right)^{1/2}};$$

$q_x = (\cos\psi + \cos\chi)$; $q_z = -(\sin\psi + \sin\chi)$; $q^2 = q_x^2 + q_z^2$;

$R'_\zeta = (R_x\sin\psi + \zeta(R)\cos\psi, R_y)$; $R''_z = (R_x\sin\chi + \zeta(R)\cos\chi, R_y)$, $R = \{R_x, R_x\}$ is the radial distance in the plane S_0 (the projection of the surface S on the plane $z = 0$); $\gamma^2_{x,y}$ is the variance of the slopes of the sea surface; $W_{n,m}(x)$ is the Whittaker function; $\Gamma(k)$ is the gamma function; $f(t)$ describes the shape of the sensing pulse; $\zeta(R)$ is the height of the randomly uneven surface S at the point R; $W(\zeta)$ is the probability density function of the heights of the randomly uneven surface S; $E_s(R)$ and $E_r(R)$ are the illumination (for the case of continuous exposure) by the real and fictitious sources (with the parameters of the receiver) in the atmosphere in the planes perpendicular to the source and receiver optical axes, respectively;[2] L_s and L_r are the distances from the source and the receiver to the center of the observation sector lying on the surface S_0; φ and χ are the illumination and the observation angles, respectively, which are counted off from the OX axis in the plane $z = 0$; A is the albedo of an elementary section of the surface; V^2 is the Fresnel coefficient of a flat sea surface. We assume in the derivation of Eq. (2) that the refractive index of water is constant over the illuminated section of the sea surface while φ differs weakly on χ (Ref. 3), so that $V^2(r) \approx V^2 = \left(\frac{n-1}{n+1}\right)^2$, where n is the refractive index of water.

The formula for $K_f\left(\overline{\gamma_x^2}, \overline{\gamma_y^2}\right)$ has been derived in the approximation $\beta \ll 1$, which holds well for a wide range of conditions of the wind–driven sea waves.

Using the expressions for $E_s(R)$ and $E_r(R)$ (see Ref. 2) for the narrow illuminating beam, we derive the following analytical expressions for an average (over an ensemble of surfaces) received echo–signal power in sounding of the randomly uneven surface through the aerosol atmosphere:

$$P_{0,f}(t) \simeq \frac{a_s a_r}{L_s^2 L_r^2}(C_s + C_r)^{-1/2}\, 2\sqrt{\pi}\, \nu^{-1/2} \times$$
$$\times\ \overline{\omega}^{-1/2} \exp\left\{-(t')^2\alpha\right\} K_{0,f}\left(\overline{\gamma_x^2}, \overline{\gamma_y^2}\right) F_{0,f}\,. \qquad (3)$$

In the derivation of Eq. (3) we assume that the illuminated spot size and the receiver observation sector on the surface are much larger that σ, neglecting the last term in the expression for $f(t)$ and assuming the Gaussian shape of the sensing pulse of the form

$$f(t) = \frac{2}{\sqrt{\pi}}\exp\left\{-\frac{4t^2}{\tau_s^2}\right\}$$

Here the subscript 0 refers to the locally specular surface and the subscript f – to locally Lambertian surface,

$$F_f = 1;\quad F_0 = \exp\left\{-\frac{q_x^2}{q_z^2}\frac{1}{\overline{\gamma_x^2}} - t'\beta + C\right\};$$

$$\alpha = \frac{4}{\tau_s^2}\left\{1 - \frac{4q_x^2}{\nu\tau_s^2c^2} - \frac{4}{\tau_s^2c^2}\frac{1}{\omega\nu^2}(q_x\kappa - q_z\nu)^2\right\};$$

$$\beta = \frac{8\hat{q}}{\tau_s^2c\nu}\left[q_x + \frac{\kappa}{\omega\nu}(q_x\kappa - q_z\nu)^2\right];$$

$$\hat{q} = \frac{q_x}{q_z^2\, 2\,\overline{\gamma_x^2}}\left(\frac{\sin^2\psi}{L_s} + \frac{\sin^2\chi}{L_r}\right);$$

$$C = \hat{q}\left(\frac{\kappa^2\hat{q}}{\nu^2\omega} + \frac{\sin^2\psi}{L_s} + \frac{\sin^2\chi}{L_r}\right);$$

$$\kappa = C_s\sin\psi\cos\psi + C_r\sin\chi\cos\chi + \frac{4q_xq_z}{\tau_s^2c^2};$$

$$\nu = \frac{1}{2\,\overline{\gamma_x^2}\,q_z^2}\left(\frac{\sin^2\psi}{L_s} + \frac{\sin^2\chi}{L_r}\right)^2 + C_s\sin^2\psi + C_r\sin^2\chi + \frac{4q_x^2}{\tau_s^2c^2};$$

$$w = \frac{1}{2\sigma^2} + C_s\cos^2\psi + C_r\cos^2\chi + \frac{4q_z^2}{\tau_s^2c^2} - \frac{\kappa^2}{\nu};$$

$$\overline{\omega} = 2\sigma^2\omega;\quad t' = t - \frac{L_s + L_r}{c};$$

σ^2 is the variance of the heights of the sea surface.

The values of a_s, a_f, C_s, and C_r were given in Ref. 1 for the optically transparent and dense aerosol atmospheres.

As σ^2 and $\overline{\gamma_{xy}^2} \to 0$, formula (3) for $P_f(t)$ transforms into the formula for a plane Lambertian surface while the formula for $P_0(t)$ transforms into the formula for a flat specular surface.

We will now evaluate the effect of foam on the echo–signal shape. Using Eqs. (1) and (3), we derive the following formula for an average echo–signal power in slant sensing of the foam–covered sea surface:

$$P(t) \simeq b_1\, b_2\, G[z(t)], \qquad (4)$$

where

$$G(z) = \exp\{-z^2 - zd\} + \frac{b_3}{b_2}\exp\{-z^2R\}; \qquad (5)$$

$$z = t'\sqrt{\alpha};\quad d = \frac{\beta}{\sqrt{\alpha}};$$

$$b_1 = \frac{a_s a_r}{L_s^2 L_r^2}(C_s + C_r)^{-1/2}\frac{2}{\sqrt{\pi}}\nu^{-1/2}\,\overline{\omega}^{-1/2};$$

$$b_2 = (1 - S_f)\frac{q^4}{q_z^4}\frac{V^2}{8\left(\overline{\gamma_x^2}\ \overline{\gamma_y^2}\right)^{1/2}}\exp\left\{-\frac{q_x^2}{q_z^2}\,2\,\overline{\gamma_x^2}\right\}.$$

For the model of foam in the form of a randomly uneven locally Lambertian surface we have

$$b_3 = S_f \pi K_f \left(\overline{\gamma_x^2}, \overline{\gamma_y^2} \right); \quad R = 1.$$

For the model of foam in the form of a flat Lambertian surface we have

$$b_3 = S_f A \sin\psi \sin\chi \ \overline{\omega}^{1/2}; \quad R = \frac{\alpha_0}{\alpha}; \quad \alpha_0 = \alpha(s = 0).$$

Figure 1 shows the calculational results of the shape of the echo–signal received from the sea surface for various driving wind velocities U. The calculations were performed using Eq. (5) for the models of foam in the form of a randomly uneven locally Lambertian surface (solid lines) and a flat Lambertian surface (dashed lines) with the following values of the parameters: $\varphi = \chi = 60°$, $L_s = L_r = 10$ km, $\alpha_r = 2.9\cdot10^{-2}$, $\tau_s = 10^{-10}$ s, $\alpha_s = 10^{-3}$, $\mu = 0$ (the parameter μ characterizes the atmospheric optical density[1,2]), and $U = 14$ (curve *1*) and 18 m/s (curve *2*).

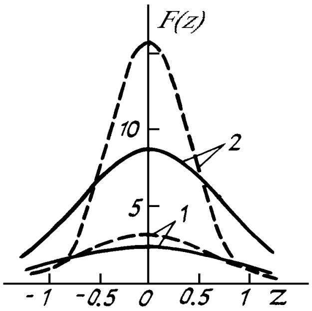

FIG. 1. Shape of echo–pulse received from the sea surface

Here and below the values $\overline{\gamma_{xy}^2}$ were calculated from the Cox and Munk formulas, and the following formulas were used for S_f and σ (see Refs. 5 and 6): $S_r = 0.09U^3 - 0.3296U^2 + 4.54U - 21.33$; $\sigma = 0.016U^2$, where U is the driving wind velocity in m/s.

It can be seen from the figure that both the presence of foam and the foam model which is employed in the calculations affect the amplitude and the shape of the echo–signal in slant sensing with a narrow laser beam. The effect of the model of foam is strongly manifested only for narrow laser beams when the illuminated spot on the sensed surface becomes comparable with the heights of the irregularities.

Formula (5) describes the echo–signal shape in slant sensing of the foam–covered sea surface.

We will now estimate the delay T and the width t of the echo–pulse. We define T and τ (analogous to Ref. 6) as follows:

$$T = \frac{\int_{-\infty}^{\infty} dt\, tP(t)}{\int_{-\infty}^{\infty} dt\, P(t)}; \quad \tau^2 = \frac{\int_{-\infty}^{\infty} dt\, (t - T)^2 P(t)}{\int_{-\infty}^{\infty} dt\, P(t)}. \qquad (6)$$

After some quite cumbersome calculations from Eq. (2) (for the case of monostatic sensing, i.e., $L_s = L_r = L$, $\varphi = \chi$) we arrive at the expressions

$$T = T_0 K_0 + T_f K_f; \qquad (7)$$

$$\tau^2 = \tau_0^2 K_0 + \tau_f^2 K_f, \qquad (8)$$

where

$$T_0 = \frac{2L}{c} + \frac{1}{2cL}\left[\frac{1}{a_1} + \frac{1}{a_3 + a_4 a_2^{-1}}\right] - \frac{2L}{c}\,\mathrm{cotan}^2\psi\, \frac{(2\sigma^2)^{-1} + C_s + C_f}{a_2 + L^2 2\overline{\gamma_x^2}\,(C_s + C_f)(2\sigma^2)^{-1}} + \frac{\cos^2\psi}{L^2\left(2\overline{\gamma_x^2}\right)^2 cL(a_3 + a_4 a_2^{-1})^2};$$

$$\tau_0^2 = \frac{\tau_s^2}{8} + \frac{4s^2}{c^2 z \sin^2\psi} + \frac{2\,\mathrm{cotan}^2\psi}{c^2(C_s + C_f)z} + \frac{4\sigma^2\sin^2\psi}{c^2L^2 2\overline{\gamma_x^2}\,(C_s + C_f)z} + \frac{1}{2c^2L^2}\left(\frac{1}{a_1^2} + \frac{1}{(a_3 + C_s + C_f)^2\sin^4\psi}\right) - \frac{4\cos^2\psi}{\sin^4\psi\, c^2L^2\left(2\overline{\gamma_x^2}\right)(a_3 + C_s + C_f)^2} \times \left(1 - \frac{1}{2\sin^2\psi\left(1 + 2L^2\overline{\gamma_x^2}\,(C_s + C_f)\right)}\right);$$

$$z = 1 + \left[1 + L^2 2\overline{\gamma_x^2}\,(C_s + C_f)\right]^{-1};$$

$$a_1 = C_s + C_f + \left(\frac{2}{q_z L}\right)^2 \frac{1}{2\overline{\gamma_x^2}}; \quad a_2 = (2\sigma^2)^{-1} + (C_s + C_f)\cos^2\psi;$$

$$a_3 = \left(\frac{2\sin^2\psi}{q_z L}\right)^2 \frac{1}{2\overline{\gamma_x^2}}; \quad a_4 = (C_s + C_f)\frac{\sin^2\psi}{2\sigma^2};$$

$$a_0 = \exp\left\{\frac{q_x^2\sin^4\psi}{q_z^4 L^2(\overline{\gamma_x^2})^2 p}\right\}; \quad K_0 = a_0(a_0 + a_f)^{-1};$$

$$K_f = a_f(a_0 + a_f)^{-1}; \ p = a_3 + p_0;$$

$$p_0 = \sin^2\psi(C_s + C_f) - \sin^2\psi\cos^2\psi\,(C_s + C_f)^2 a_2^{-1}.$$

For the model of foam in the form of a randomly uneven locally Lambertian surface we have:

$$a_f = \frac{b_3}{b_2}\left(\frac{a_1}{C_s + C_f}\right)^{1/2}\left(\frac{p}{p_0}\right)^{1/2};$$

$$T_f = \frac{2L}{c} + \frac{1}{2cL(C_s + C_f)}\left[1 + \frac{1 + 2\sigma^2(C_s + C_f)\cos^2\psi}{\sin^2\psi}\right];$$

$$\tau_f^2 = \frac{\tau_s^2}{8} + \frac{4\sigma^2}{c^2\sin^2\psi} + \frac{2\cot^2\psi}{c^2(C_s + C_f)z} + \frac{1 + \sin^{-4}\psi}{2c^2L^2(C_s + C_f)^2};$$

For the model of foam in the form of a flat Lambertian surface we have:

$$a_f = \frac{S_f A \sin\psi \, (a_1 P)^{1/2}}{b_2(C_s + C_f)} ; \quad T_f = \frac{2L}{c} + \frac{1 + \sin^{-2}\psi}{2cL\,(C_s + C_f)} ;$$

$$\tau_f^2 = \frac{\tau_s^2}{8} + \frac{2\cot^2\psi}{c^2(C_s + C_f)} + \frac{1 + \sin^{-4}\psi}{2c^2L^2(C_s + C_f)^2} ;$$

where (T_0, τ_0) and (T_f, τ_f) are the delay and width of the echo–signal reflected from the foam–free sea surface and the sea surface continuously covered with foam, respectively.

Figures 2 and 3 show the calculated widths of the echo–pulse reflected from the sea surface for the various driving wind velocities U. The calculations were carried out based on Eq. (8) for the models of foam in the form of a randomly uneven locally Lambertian surface (solid lines) and in the form of a flat Lambertian surface (dashed lines) with the following values of the parameters: L = 10 km, $a_f = 2.9\cdot10^{-2}$, $\tau_s = 10^{-10}$ s, $a_s = 10^{-3}$, $\varphi = 60^\circ$ (curves *1*) and 80° (curves *2*), $\mu = 0$ (Fig. 2) and $3\cdot10^{-3}$ (Fig. 3).

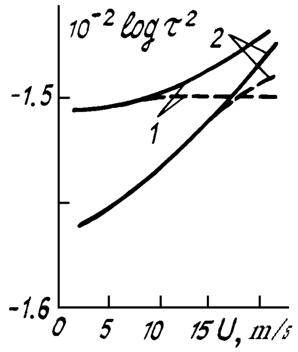

FIG. 2. Durattion of echo–pulse received from the sea surfacet trough the clear atmosphere.

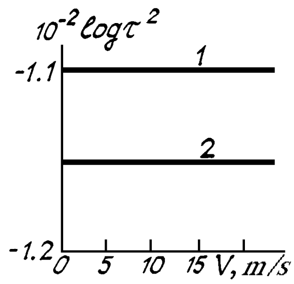

FIG. 3 Durattion of echo–pulse received from the sea surface through the dense aerosol atmosphere.

It can be seen from Fig. 2 that the width of the echo–pulse depends strongly on both the driving wind velocity and the model of foam in sensing through the optically transparent aerosol atmosphere. However, the latter dependence is manifested only for the high driving wind velocity. The interesting phenomenon is observed in slant sensing for the model of foam in the form of a flat Lambertian surface. In this case the echo–signal width at $\varphi = 60^\circ$ and for high driving wind velocities is less than that at $\varphi = 80^\circ$.

This is explained by the fact that the main contribution to the echo–signal (at $\varphi = 80^\circ$) comes from the component which is caused by the signal reflection from the foam–free sea surface. Atmospheric turbidity (see Fig. 3) results in the sharp increase of the value of τ and reduction in the effect of foam on the echo–signal. The echo–signal width in this case depends weakly on the model of foam (solid and dashed lines are superimposed in Fig. 3).

The results presented here complete a number of our works pertaining to the study of the signal powers recorded by lidars in sensing the foam–covered sea surface.[1,7] Generalizing the results of studies performed we can draw the following conclusions:

1. The presence of foam on the sea surface has an appreciable effect on the amplitude of the echo–signal for all the operating conditions of sounding systems (pulsed and continuous sensing, monostatic and bistatic sensing, and slant and nadir sensing).

2. The effect of the model of foam on the echo–signal is manifested, as a rule, only for high driving wind velocity and sufficiently narrow laser beams (i.e., when the illuminated spot on the surface being sensed becomes comparable with the heights of the irregularities).

3. The effect of the atmosphere on the echo–signal depends in a complicated manner on the operating condition of the sounding system and the model of foam. Atmospheric turbidity results in a sharp reduction in the effect of foam on the echo–signal shape in pulsed sensing.

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