ON THE POSSIBILITY OF ESTIMATING THE MEAN SIZES OF ORIENTED ICE PLATES IN A CLOUD FROM SINGLE-FREQUENCY LIDAR DATA

A.A. Popov and O.V. Shefer

Institute of Atmospheric Optics,
Siberian Branch of the Academy of Sciences of the USSR, Tomsk
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The paper analyzes the information content of the ratio of backscattering coefficients of a system of ice plates having the preferred orientation for lidar observations made along the vertical and slightly biased directions. For a system of horizontally oriented ice plates this ratio as a function of the bias angle of the sounding path with respect to the vertical has the entire descent portions with the steepness being related mainly to the mean radius of plates. If there is a flutter in orientation of the ice plates near the horizontal direction, then the backscattering coefficient ratio has, in addition to the portions of the entire descent, the horizontal portions. It is shown that the length of the horizontal portions of these curves bears the information on the flutter while the steepness of the descent portions as previously can be uniquely related to the mean radius of plates.

It has been established in the experimental study of morphology of crystal clouds which, in addition to a liquid phase, have a crystal phase that such clouds contain, to more or less extent, ice plates.[1,2] The plate crystals are well known to be oriented in space due to their aerodynamics.[3,4] We have shown in Ref. 5 that even a small number of oriented ice plates in a scattering volume provide a high–amplitude backscattered signal when a sounding direction is normal to the plate base. A lidar return which is normally reflected from a system of ice plates is called an anomalous backscattered signal.[6,7] Atmospheric crystals, which have a geometric shape different from the plates and all the more a liquid phase of clouds cannot provide backscattered signals being comparable in the order of magnitude to the anomalous ones. This fact makes it possible even in the clouds of complicated morphological composition to remotely investigate a microstructure of only that part of a crystal system, which contains the ice plates.

We have demonstrated in Ref. 5 the possibility of estimating the mean size of uniaxial orientation of plates at a single frequency. The present paper continues the research of this problem for a more complicated model of a polydisperse system of lamellar crystals. In this model the plates have no uniaxial orientation but weakly oscillate near some fixed position. The possibility of estimating the mean sizes of ice plates and their flutter using a single–frequency lidar is studied for this model.

We have shown in Ref. 8 that if we neglect reflections within the ice plates, then the backscattering coefficient β_π can be represented in the form of two factors one of which is a function of the plate size while the second factor depends on their refractive index. In particular, the formula for β_π derived on the assumption that the plates have uniaxial orientation in the atmosphere has the form

$$\beta_\pi = \beta_\pi(\beta) = A\,\frac{k^2}{\pi}\int\limits_0^\infty N(a)\left(\frac{1+\cos 2\beta}{2}\,\pi a^2 \cos b\; G(\beta)\right)^2 da, \quad (1)$$

where

$$A = A(\beta) = \frac{|R_{||}|^2 + |R_\perp|^2}{2} + \frac{I_2}{I_1}\cdot\frac{|R_{||}|^2 - |R_\perp|^2}{2}\cos 2\gamma - \frac{I_3}{I_1}\,\frac{|R_{||}|^2 - |R_\perp|^2}{2}\sin 2\gamma, \quad (2)$$

and

$$G(\beta) = \frac{2J_1(ka\sin 2\beta\cos\beta)}{ka\sin 2\beta\cos\beta}. \quad (3)$$

Here β is the bias angle of lidar axis with respect to the normal to the plate base, γ is the angle, which indicates the orientation of the vectors of the field incident on the ice plate with respect to the plane of incidence, I_1, I_2, and I_3 are the first three parameters of the Stokes vector, which characterize the polarization state of the incident radiation $R_{||}$ and $R_\perp$ are the Fresnel reflection coefficients for the plane waves with parallel and perpendicular polarizations, respectively.

Let us consider some particular cases for the coefficient A associated with different polarizations of electromagnetic fields of the incident wave. The Stokes parameters should be set as follows; $I_2 = I_3 = 0$, which corresponds to the circular polarization and $I_2/I_1 = 1$ and $I_3 = 0$ for the linear polarization. For these two particular cases from formula (2) we derive

$$A_c(\beta) = \frac{1}{2}\left(|R_{||}|^2 + |R_\perp|^2\right), \quad (4)$$

$$A_l(\beta) = |R_{||}|^2\cos^2\gamma + |R_\perp|^2\sin^2\gamma. \quad (5)$$

It should be noted that for normal incidence of the wave on the system of oriented plates the coefficients A, A_c, and A_l are identical and can be determined from the following relation:

$$A(0) = A_c(0) = A_l(0) = \left|\frac{\tilde{n}-1}{\tilde{n}+1}\right|^2, \quad (6)$$

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where $\tilde{n} = n + i\kappa$ is the complex refractive index of the plates.

If the distribution is given in a specific form, it is possible to derive an analytic formula for the coefficient of anomalous backscattering using relation (1). In Ref. 5, on the basis of the assumption that the function $N(a)$ follows γ–distribution, we have derived the following formula:

$$\beta_a(0) = \beta_\pi(0) = \left|\frac{\tilde{n}-1}{\tilde{n}+1}\right|^2 N \frac{k^2}{\pi}(\pi\,\overline{a}^{\,2})^2 \prod_{j=1}^{3}\left(1+\frac{j}{\mu+1}\right), \quad (7)$$

where $\overline{a} = a\,(1 + 1/\mu)$ is the mean radius of plates and N, μ, and a_m are the parameters of γ–distribution. As shown in Ref. 5, formula (7) adequately describes the lidar return with anomalously high amplitude which is specularly reflected from the system of oriented plates.

Let us now assume that all the ice plates weakly oscillate near some fixed position with maximum bias angle $\vartheta°$. Let us determine the backscattering coefficient for this system of plate crystals. It is denoted by $\beta_F(\beta, \vartheta)$. The goal function $\beta_F(\beta, \vartheta)$ must be expressed in terms of the above–defined backscattering coefficient $\beta_\pi(\beta)$. In addition, the following particular cases must follow from it:

$$\beta_F(\beta, 0) = \beta_\pi(\beta), \quad \beta_F(0, 0) = \beta_a\,. \quad (8)$$

For definiteness the ice plates are assumed to oscillate near the horizontal position and the angle β is counted off from the vertical. At first, let us find $\beta_F(0, \vartheta)$, i.e., the backscattering coefficient for a system of ice plates with a flutter for the vertical position of a lidar. To do this it is necessary to average the function $\beta_\pi(x)$ over a line segment $[-\vartheta, \vartheta]$. The result is

$$\beta_F(0, \vartheta) = \frac{1}{2\vartheta}\int_{-\vartheta}^{\vartheta} \beta_\pi(x)\,dx\,. \quad (9)$$

The bias angle β of the lidar axis with respect to the vertical leads to the shift of the line segment of integration toward larger numbers along the number axis at the same value. As a result, the coefficient $\beta_F(\beta, \vartheta)$ is found by averaging the function $\beta_\pi(x)$ over the line segment $[-\vartheta + \beta, \vartheta + \beta]$, i. e.,

$$\beta_F(\beta, \psi) = \frac{1}{2\vartheta}\int_{-\vartheta+\beta}^{\vartheta+\beta} \beta_\pi(x)\,dx\,. \quad (10)$$

It should be noted that one can deduce the particular cases (8) for $\beta_F(\beta, \vartheta)$ from relations (9) and (10) proceeding to the limit $\vartheta \to 0$. The 0/0 indeterminate forms, which result from passing to the limit, should be estimated according to the l'Hopital rule, and then one should use the rule of differentiation of integrals between the variable limits.

Let us form from the coefficients β_π, β_a and β_F such combinations which are independent of the refractive index. Note that only the coefficients A, A_c, and A_l depend on the refractive index. At the same time, within the small angles β the ratios of the form $A(\beta)/A(0)$, $A_c(\beta)/A_c(0)$, and $A_l(\beta)/A_l(0)$ differ weakly from unity regardless of the refractive indices of ice. The numerical calculations show that the ratio $A_c(\beta)/A_c(0)$ is most stable when β changes, i.e., the ratio of the coefficients corresponding to the circular polarization of the incident wave. In this case for the real and imaginary parts of the refractive index n and κ from the intervals [1.2; 1.5] and [0; 0.1], respectively, the relation

$$A_c(\beta)/A_c(0) = 1.000$$

is fulfilled to three significant digits beyond the decimal point with increase of the angle β from 0° to 10°. As for linear polarization, the ratio $A_l(\beta)/A_l(0)$ differs from unity by not more than 0.5% for the same values of the refractive index with increase of the angle β from 0° to 3°.

We have shown in Ref. 5 that in the case of laser sounding of a system of horizontally oriented plates the bias angle of 1° of the lidar axis with respect to the vertical results in the change of the reflected signal amplitude by several orders of magnitude. In other words, when investigating the backscattering coefficient $\beta_\pi(\beta)$, its behavior within the small angles β is of particular interest. However, for small angles β the ratio $A(\beta)/A(0)$ is independent of the refractive index, i.e., the ratio of the backscattering coefficients $\beta_\pi(\beta)/\beta_\pi(0)$ must be independent of the refractive index at the same angles. We can draw the analogous conclusion analyzing the ratio of the backscattering coefficients $\beta_F(\beta, \vartheta)/\beta_F(0, \vartheta)$ for a system of crystals with a flutter. But in this case not only one angle β but also the sum of the angles $\beta + \vartheta$ must be small. However, this condition is not stringent, since the angle of flutter of the plates, as a rule, does not exceed 1°.[3] It should be noted that the ratios $\beta_\pi(\beta)/\beta_\pi(0)$ and $\beta_F(\beta, \vartheta)/\beta_F(0, \vartheta)$ are independent of the concentration of plates in the scattering volume, because N enters as a linear factor in both the numerator and the denominator of these ratios.

In the numerical calculations we take a circular polarization of the incident radiation for definiteness. In this connection it should be noted that when replacing circular polarization by arbitrary linear polarization, the backscattering coefficients and their ratios will change by not more than 0.2% given that $\beta + \vartheta \le 2°$. As for unpolarized incident radiation, all the characteristics of backscattering studied above are formally the same as for the circular polarization.

Fig. 1 shows the ratios $\beta_\pi(\beta)/\beta_\pi(0)$ as functions of the angle β for different parameters $\overline{a}$ and μ of the system of ice plates with uniaxial orientation. Every hatched area is continuously filled with curves of $\beta_\pi(\beta)/\beta_\pi(0)$ plotted for different values of μ in the interval [1, 10]. The curve with less steepness corresponds to a greater value of μ. Actually, with increase of the parameter μ the values of radii of the plates are located close to the mean value of $\overline{a}$. Therefore, with increase of μ the number of plates with radii larger than the mean value decreases. And the plates with smaller areas provide less steepness of the characteristic $\beta_\pi(\beta)/\beta_\pi(0)$. This can easily be seen by comparing any two curves from different hatched areas. Note that within the small angles β the steepness of the characteristic $\beta_\pi(\beta)/\beta_\pi(0)$ is determined largely by the mean radii a of the plates, and to less extent by the parameters μ. This makes it possible to estimate the mean radius of the ice plates from the character of the backscattered signal variation by means of small–angle scanning of the lidar beam near the vertical even though the parameter μ of the distribution is unknown. It becomes apparent that the accuracy of determining the size $\overline{a}$ substantially increases when the parameter μ is localized in the interval narrower than [1, 10].

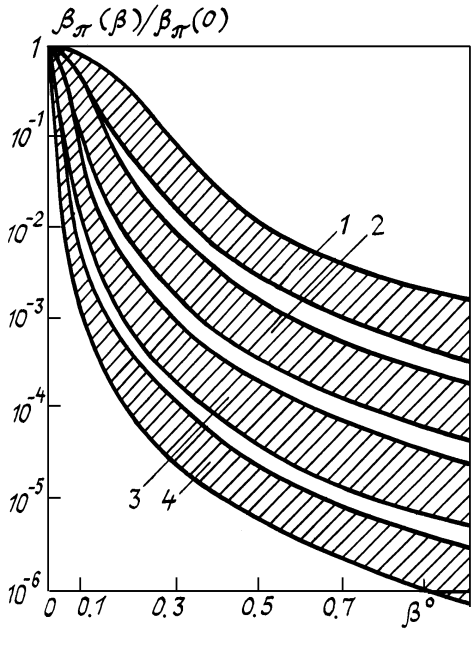

FIG. 1. Ratios of the backscattering coefficients for a system of plates with uniaxial orientation as functions of the bias angle β *of the lidar axis with respect to the vertical. The areas 1–4 correspond to the mean radii* $\overline{a}$ = 25, 50, 100, *and* 200 μm. *The upper boundary of every area corresponds to the parameter* $\mu = 10$, *the lower boundary – to* $\mu = 1$; $\lambda = 0.694$ μm.

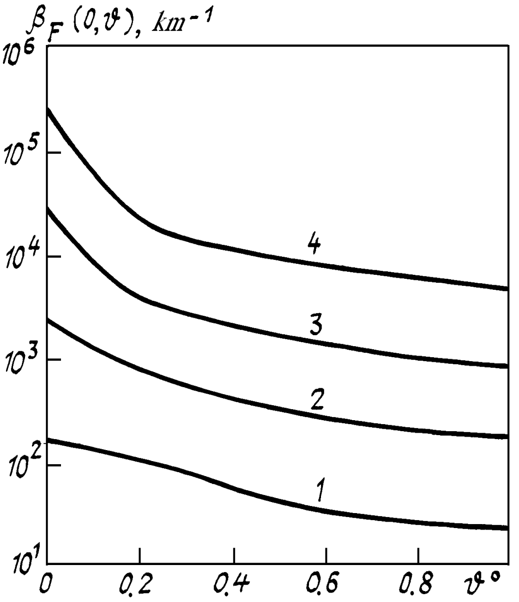

FIG. 2. Backscattering coefficients for the vertical position of the lidar axis as functions of the value of the plate flutter. 1) $N = 40\ l^{-1}$, $\overline{a} = 25$ μm; *2)* $N = 35\ l^{-1}$, $\overline{a} = 50$ μm; *3)* $N = 25\ l^{-1}$, $\overline{a} = 100$ μm; *4)* $N = 15\ l^{-1}$, $\overline{a} = 200$ μm, $\mu = 5$, $n = 1.31 + i{\cdot}10^{-3}$, *and* $\lambda_0 = 0.694$ μm.

A backscattered signal, anomalously high in amplitude, is observed in vertical sounding of a system of horizontally oriented plates. The signal anomaly is caused by uniaxial orientation of ice plates being, in fact, a semitransparent mirrors. In the presence of flutter only some of ice plates have horizontal position. On the one hand, this fact has less of a directional effect of scattering of the entire system of crystals, on the other hand, it leads to the decrease of the amplitude of the backscattered signal. This can be supported by the analysis of the plots of the backscattering coefficients $\beta_F(0, \vartheta)$ as functions of the flutter angle ϑ, shown in Fig. 2.

For the vertical position of the lidar the oscillations of the ice plates near the horizon result in decrease of the backscattering coefficients by 1–2 orders of magnitude. However, the amplitudes of the backscattered signal remain as previously anomalously high. The directional effect of scattering for a system of ice plates with flutter also remains high, but becomes less with increase of the angle y. The lobe of the scattering diagram occupies an angle 2ϑ. In addition, within this angle the scattering is practically isotropic, and passing over this limits of the angle the intensity of the scattered field starts to decrease sharply. Curves *2–6* shown in Figs. 3 and 4 are associated with this scattering mechanism. For each curve $\beta_F(\beta, \vartheta)/\beta_F(0, \vartheta)$ the length of the horizontal portion, for which scattering is isotropic, is determined roughly by the flutter angle y. In calculating with an improved accuracy, one can conclude that the flutter angle corresponds to the variation of the backscattered signal intensity by a factor of two for small bias angles of the lidar axis with respect to the vertical. However, this relationship is violated for relatively small ice plates and weak flutters (Fig. 4). It should be noted that the steepness of the descent portion of the curve $\beta_F(\beta, \vartheta)/\beta_F(0, \vartheta)$ is practically independ of the flutter value and is determined, as previously, by the mean radius of the plates.

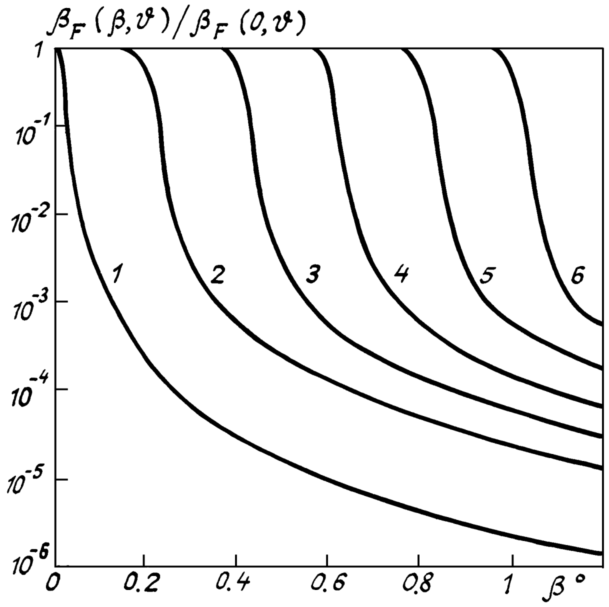

FIG. 3. Ratios of the backscattering coefficients for a system of large plates ($\overline{a}$ = 200 μm) *with flutter as functions of the bias angle* β *of the lidar axis with respect to the vertical: 1)* $\vartheta = 0°$; *2)* $\vartheta = 0.2°$; *3)* $\vartheta = 0.4°$; *4)* $\vartheta = 0.6°$; *5)* $\vartheta = 0.8°$; *6)* $\vartheta = 1°$; $\mu = 5$ and $\lambda = 0.694$ μm.

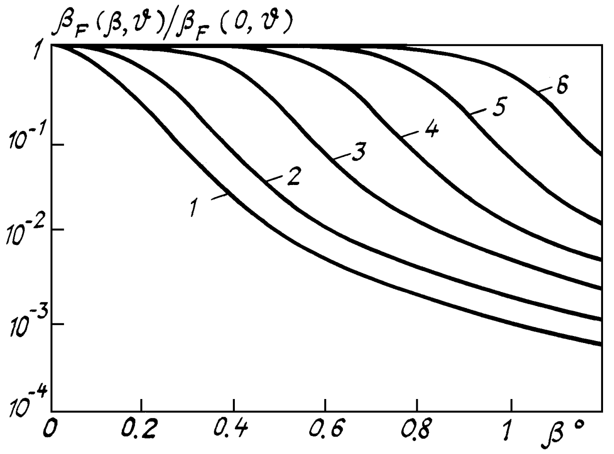

FIG. 4. Ratios of the backscattering coefficients for a system of small plates ($\overline{a}$ = 25 μm) *with flutter as functions of the bias angle β of the lidar axis with respect to the vertical: 1)* ϑ = 0°; *2)* ϑ = 0.2°; *3)* ϑ = 0.4°; *and 4)* ϑ = 0.6°; *5)* ϑ = 0.8°; *6)* ϑ = 1°; μ = 5 *and* λ = 0.694 μm.

Let us assume that the parameter μ of the distribution is well known *a priori*. Then from the data of relative measurements of the backscattering coefficient, one can estimate the flutter of the plates and their mean radius $\overline{a}$. Let the refractive index of the plates $\tilde{n}$ be known. Then from the data of absolute measurements of the backscattering coefficient $\beta_F(0, \vartheta)$ one can estimate the concentration of plate crystals N in the scattering volume.

The backscattering coefficients determined from relations (1), (7), (9), and (10) are independent of the plate thickness. This size can enter into these relations only when the reflections within the plates would be taken into account, the effect of which, as a rule, is negligible. In other words, if the backscattering coefficient depends on thickness of ice plates, this dependence is very weak. Therefore, an additional information should be used for determining the plate thickness. In this connection, it should be noted that there is an empirical relations between the thickness and the diameter of the plates.[9] Therefore, if one succeeded in retrieving the plate diameters, concrete thicknesses can be relevant for them.

In conclusion it should be pointed out that one or other concentration of the ice plates and their size as well as their position in the cloud thickness are determined by physical conditions of the atmosphere, which, at the same time, predetermine the entire structure of the cloud.

REFERENCES

1. A. Ono, J. Atm. Sci. **26**, No. 1, 138–147 (1969).
2. W.A. Cooper and G. Vali, J. Atm. Sci. **38**, No. 6, 1244–1259 (1981).
3. C.M.R. Platt, J. Appl. Meteor. **17**, 1220–1224 (1978).
4. H.R. Cho, J. Atm. Sci. **38**, No. 5, 1111–1114 (1981).
5. A.A. Popov and O.V. Shefer, Atm. Opt. **3**, No. 9, 852–856 (1990).
6. C.M.R. Platt and A.C. Dilley, J. Atm. Sci. **38**, 1069–1082 (1981).
7. C.M. R. Platt, J.C. Scoott, and A.C. Dilley, J. Atm. Sci. **44**, No. 4, 729–747 (1987).
8. A.A. Popov and O.V. Shefer, Atm. Opt. **3**, No. 1, 36–42 (1990).
9. A. Auer and D. Veal, J. Atm. Sci. **27**, No. 6, 919–926 (1970).