ON THE POTENTIAL RESOLUTION OF PASSIVE IMAGE-FORMING METHODS THOUGH TURBULENT ATMOSPHERE. I. SPECKLE-INTERFEROMETRY IN TRADITIONAL TELESCOPES

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Received October 9, 1989

A statistical model of images distorted by the atmosphere is applied together with general principles of image processing to determine the dependence of the limiting resolution of traditional telescopes with continuous apertures on the observation conditions. Quantitative estimates are obtained for typical values of system parameters. Speckle-interferometry is shown to be preferable for forming images of small-sized objects.

The turbulent atmosphere of Earth randomly perturbs light coming from targets observed through its depth and limits the actual resolution in the images thus formed to approximately one arc second, which is about one or two orders of magnitude worse than the diffraction resolution limit of modern telescopes. For several decades this actual problem of optical astronomy and location has drawn the attention of many scientific groups. As a result of their efforts a whole multitude of techniques for image forming have been suggested and just the list of reviews devoted to this question numbers more than ten titles (see Ref. 1 for example). By now not only good practical results in observing actual targets have been obtained but particular features of various groups of such techniques have been theoretically summarized. As a consequence of this the task of comparative assessment of potential capabilities and limitations has become crucial now. This and subsequent papers will treat from this point of view various groups of methods of image formation using reflected solar radiation.

The simplest approach to solving the problem of sighting a target consists in the consecutive registration and joint statistical processing of a series of short-exposure images (SEI) of a stationary target being observed. This technique was first suggested and tested by the French scientist Labeyrie.[2] He noticed that each SEI of a unresolved star is a random set of speckles, i.e., spots of light comparable in their size to the size of the Airy diffraction spot. As a result the SEI of an extended object may be regarded as a random sum of independent sub-images, corresponding to separate spots and carrying information on that object at a diffractional level of resolution in a coded form. Its processing is then reduced to the retrieval of this information and to the elimination of its random variability by averaging over a set of realizations.

In the development of the Laberyrie technique other methods were also suggested, differing in their specific processing routines, but not in their basic concept.[3–7] In astronomy these techniques are known under the general name of speckle-interferometry (SI). Their statistical analysis demonstrates[8] that processing these observation results essentially consists of measuring the correlation functions of the intensity fluctuations J with respect to the average distribution $\langle J\rangle$. Thus, the capabilities of SI are characterized by the ratio Q of the variance of the fluctuations σ_J^2 to the variance of the error of its estimate from the recorded SEI series. This error results from incomplete statistical averaging of both the intensity self-fluctuations and the recording noise. In the present study the authors will obtain and analyze expressions for the ratio Q, considering traditional telescopes with continuous apertures.

It is easy to find from the statistical model of SEI (Ref. 9) that the RMS deviation of fluctuations from their average value σ_J^2 is equal, within the accuracy of a constant of the order of one, to the variance σ_J^2. In turn, the variance depends on the squared average as

$$\sigma_J^2 = k_0\cdot k_\lambda\cdot k_T\langle J^2\rangle, \tag{1}$$

where the parameter

$$k_0 = \left[\frac{\Theta_d}{\Theta_a}\cdot\frac{\Theta_0+\Theta_a}{\Theta_0+\Theta_{ef}}\right]^2 \tag{2}$$

describes the decrease of the fluctuation contrast due to their spatial averaging in SEI, and the parameters

$$k_\lambda = \Delta\lambda_c/(\Delta\lambda + \Delta\lambda_c), \tag{3}$$

and

$$k_T = T_c/(T + T_c) \tag{4}$$

account for the effect of spatial and temporal averaging, respectively. Here Θ_0 is the angular size of the

0235-6880/90/04 356-04 $02.00

target; $\Theta_d = \lambda/D$ is the diffraction resolution of the telescope; $\Theta_a = \lambda/r_0$ is the average atmospheric resolution; Θ_{ef} is the effective resolution attained during processing of a given SEI series ($\Theta_a \geq \Theta_{ef} \geq \Theta_d$); $\Delta\lambda_c = r_0 \cdot \Theta_{ef}$ and $T_k = r_0/\sigma_v$ are the spectral (Ref. 10) and temporal[11] correlation intervals of the atmospheric distortions; λ is the average wavelength; $\Delta\lambda$ and T are the spectral and temporal intervals of SEI recording; D is the telescope aperture diameter; r_0 is the Fried parameter;[12] σ_v is the average value of fluctuations of the transport velocity for separate turbulent atmospheric layers.[11] The number of independent realizations M, used to assess the statistical characteristics of fluctuations, is determined as the product of the number of SEI It and the average number of uncorrelated sub-images It in each SEI; moreover, it may be assumed that

$$M_s = \left[\frac{\Theta_0 + \Theta_a}{\Theta_0 + \Theta_d}\right]^2. \tag{5}$$

With regard to the average SEI intensity, by determining it from the number of photons registered in single diffractional resolution element, we have for the value $\langle J \rangle$

$$\langle J \rangle = \rho_0 \cdot \Delta\lambda \cdot T \cdot \xi \cdot \eta \cdot \lambda^2 \cdot (\Theta_2/(\Theta_0 + \Theta_a))^2. \tag{6}$$

Here ξ is the transmittance of the telescope optics; η is the quantum efficiency of the recorder; and ρ_0 is the number of photons of the light emitted by the target and received in a unit solid angle, in a unit time interval, per unit aperture area, in a unit spectral bandwidth. Analyzing the laws of solar radiation reflection from cosmic objects near Earth, resolved by telescopes, one may demonstrate that ρ_0 does not depend on the size and distance from the object. A typical luminance produced by an angular area of 1(arc sec)^2 is estimated as $+3^m$ in stellar magnitudes. Note, for comparison purposes, that the experimentally measured surface luminances for Moon, Venus, and Mercury amount to 3.7^m, 2.0^m, and 3.9^m, respectively.[13]

When assessing registration noises, we will only take account of the fundamentally unremovable quantum effects, related to randomness of the number of registered photons. As a result we have for the effective noise variance σ^2_{ef}.

$$\sigma^2_{ef} = \langle J \rangle \cdot \Theta^2_{ef}/\Theta^2_d \tag{7}$$

Summarizing the foregoing comments, we find an expression for the ratio Q

$$Q = \sqrt{M} \cdot \sigma^2_J/(\sigma^2_J + \sigma^2_{ef}) = \sqrt{M_m \cdot M_s} \cdot (\gamma/(\gamma + 1)), \tag{8}$$

where the parameter $\gamma = \sigma^2_J/\sigma^2_{ef}$ characterizes the signal-to-noise ratio in the effective SEI resolution element. It follows, in particular, from this that during observations of bright targets when $\gamma \gg 1$ and $Q = \sqrt{M}$ even $M = 25$ independent realizations is sufficient for a practical reconstruction of the diffraction image.

Moreover, if one observes a bright target of small angular size with a large telescope, when $\Theta_d \leq \Theta_{ef} \ll \Theta_a$ and, hence, $M_s \gg 1$, such a reconstruction becomes possible from only one SEI.[14]

To assess the limiting capabilities of SI one needs to analyze the case $\gamma \ll 1$, since for Θ_{ef} and $\Theta_d \to 0$ we have $\gamma \to 0$ also. Then the expression (8) for the accuracy Q is transformed to

$$Q = \sqrt{M} \cdot \gamma = \sqrt{M_m} \cdot \rho_0 \cdot \lambda^2 \cdot \xi \cdot \eta \, \frac{\Delta\lambda \cdot \Delta\lambda_c}{\Delta\lambda + \Delta\lambda_c} \times$$
$$\times \frac{T \cdot T_c}{T + T_c} \cdot \left[\frac{\Theta_{ef}}{\Theta_a} \cdot \frac{\Theta_0}{\Theta_0 + \Theta_{ef}}\right]^2 \cdot \frac{\Theta_0 + \Theta_a}{\Theta_0 + \Theta_d}. \tag{9}$$

Now we analyze the obtained relationship.

1. At higher $\Delta\lambda$ and T the value of Q increases monotonically. At the same time one may assume that its maximum is essentially achieved for $T \geq 2T_c$ and $\Delta\lambda \geq 2\Delta\lambda_c$. The functions $T \cdot T_c/(T_c + T)$ and $\Delta\lambda \cdot \Delta\lambda_c/(\Delta\lambda_c + \Delta\lambda)$ may then be replaced by T_c and $\Delta\lambda_c$.

2. For $\Theta_0 \leq \Theta_a$ the accuracy Q depends on the target size Θ_0 as $\Theta^2_{ef} / \Theta_a \cdot \Theta_0$, and for objects at the resolution limit it reaches its maximum at $\Theta_0 \simeq \Theta_a$. When $\Theta_0 \gg \Theta_a$, the accuracy Q is proportional to $\Theta^2_{ef} / \Theta^2_a$ and does not depend on the target size. Therefore, SEI as an image-forming technique is preferable for small-sized targets in the visible range if $\Theta_0 \ll \Theta_a$.

3. The accuracy Q depends on the value of the Fried parameter r_0, which characterizes the spatial size of the correlation region of the atmospheric distortions. This dependence is of the order of r_0^3 for small-sized targets ($\Theta_0 < \Theta_a$), and of r_0^4 – for large-sized ($\Theta_0 > \Theta_a$).

4. The accuracy Q is directly proportional to the square root of M_m (i.e., of the number of registered SEI). This means that, provided other conditions remain identical, the following is true:

– proper selection of either the observation site or time, such that the value of r_0 is doubled, provides a possibility of reducing the required number of SEI by two orders of magnitude;

– to increase the effective resolution by a factor of two (for $\Theta_{ef} \geq 2\Theta_d$) the number of SEI must be increased by a factor of 64;

– for equal effective resolutions in the reconstructed images of large-sized ($\Theta_0 > \Theta_a$) and small-sized ($\Theta_0 \simeq \Theta_d$) targets, a factor of $(D/r_0)^2$ more SEI would be needed in the first case than in the second.

5. The limiting effective resolution Θ_{ef} is essentially independent of the value of diffraction resolution Θ_d (for $\Theta_{ef} \geq \Theta_d$). This means, in turn, that at

fixed ρ_0, r_0, η, ξ, T_c, and λ, reducing Θ_d to below a certain threshold level is meaningless.

We may estimate this limiting resolution. We obtain the following expression for Θ_{ef} from (9) for the condition that $\Theta_0 \gg \Theta_a$

$$\sqrt{M_m}\cdot\rho_0\cdot\xi\cdot\eta\cdot r_0^3\cdot T_c\cdot\Theta_{ef}^3 = Q. \quad (10)$$

For the typical values of $\lambda = 0.6$ μm, $\xi = 0.5$, $\eta = 0.2$, $T_c = 0.01$ s, $Q = 5$, $r_0 = 0.1$ m, $\rho_0 = 2\cdot 10^{26}$ m^{-3} s^{-1} scr^{-1} (i.e., the third stellar magnitude) its solution is

$$\Theta_{ef} = 3\cdot 10^{-7}/M_m^{1/6} (\text{rad}). \quad (11)$$

For $M_m = 10^3$ we obtain the value of $\Theta_{ef} = 10^{-7}$ rad from (11). This resolution, corresponding to a telescope aperture of $D = 5$ m, is apparently a practical SI limit for observations of large-sized targets. The situation is, however, quite different for small-sized targets. Thus, assuming that $\Theta_d = \Theta_{ef} = \Theta_0 \ll \Theta_a$, we obtain from (9) an equation of the form:

$$\sqrt{M_m}\cdot\rho_0\cdot\xi\cdot\eta\cdot r_0^2\cdot\lambda\cdot T_c\cdot\Theta_{ef}^2 = 8Q, \quad (12)$$

the solution of which for the same values of the parameters as above is written as

$$\Theta_{ef} = 2\cdot 10^{-7}/M_m^{1/4} (\text{rad}).$$

The stronger dependence of Θ_{ef} on the number of SEI makes it possible to achieve a resolution of $2\cdot 10^{-8}$ rad even for $M_m = 10^4$, which corresponds to a telescope diameter of $D = 25$ m. The factor of five increase in the potential resolution points once more to the preferability of employing SI to form images of small-sized targets.

In conclusion, let us point out again that the above analysis was conducted within an accuracy of factors close to unity, which also determines a measure of the analysis reliability.

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