CORRELATION FUNCTION OF THE LONG–WAVE RADIATION INTENSITY UNDER THE CONDITIONS OF BROKEN CLOUDS: SOLUTION TECHNIQUE

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A closed system of equations for the correlation function of tong-wave radiation is derived and a method for solving it based on the use of a broken clouds model constructed using Poisson point fluxes is proposed.

A cloud field, considered as an ensemble of clouds which scatter and emit thermal radiation, is a stochastic formation. A radiation field transformed by such a cloud field also becomes random, thus making it necessary to use statistical methods to study the interaction of cloud fields and radiation fields. Many natural processes, such as heating of the atmosphere and the underlying surface of the Earth, the melting of snow, and so on, are dependent not only on the average amount of incident radiation energy but also depend on its spatial and temporal variability. Therefore, the solution of many practical problems requires comprehensive information on the statistical properties of radiation field or, at least, on their mean value, variance, and correlation function.

It is known that calculations of long-wave radiation transfer in the atmosphere must take into account the absorption of radiation by atmospheric gases. Usually, atmospheric transmission functions are used for an approximate account of this absorption. In our further discussion we shall deal only with the interaction of radiation and the cloud material and therefore the methods developed here for calculating the statistical characteristics of the radiance field are only applicable to studies within the atmospheric transparency windows.

The calculations of the mean intensity of thermal radiation were made using formulas derived in Ref. 1. It was assumed there that the scattering of long-wave radiation by clouds is negligible. The applicability limits of this approach were studied in Ref. 2, where a statistical modeling algoritm was constructed for the purpose of estimating the mean intensity of the thermal radiation and the role of multiple scattering in the formation of the long-wave radiance field is suggested.

SOLUTION TECHNIQUE

Let us assume that except for the cloud field the atmosphere is horizontally homogeneous and is characterized by the temperature $T(z)$. It is also assumed that the atmosphere is in a state of thermodynamic equilibrium. The underlying surface is assumed to be a blackbody with temperature $T_s = T(0)$. The optical model of the broken clouds is defined in the layer Λ ($h \le z \le H$) in the form of random scalar fields of the extinction coefficient $\sigma\kappa(\mathbf{r})$, the photon survival probability $\lambda\kappa(\mathbf{r})$, and the scattering phase function $g(\boldsymbol{\omega}, \boldsymbol{\omega}')\kappa(\mathbf{r})$, where $\boldsymbol{\omega} = (a, b, c)$ is the unit direction vector of propagation, $\kappa(\mathbf{r})$ is the indicator function of the random set $G \in \Lambda$ in which the cloud material is present. The mathematical model of the field $\kappa(\mathbf{r})$ is constructed using Poisson point fluxes.[3] In this model the value of $\langle\kappa(\mathbf{r})\rangle = p$ is the absolute and $V(\mathbf{r}_1, \mathbf{r}_2) = (1 - p)\exp(-A(\boldsymbol{\omega})|\mathbf{r}_1 - \mathbf{r}_2|) + p$ the conditional probability of cloud occurrence. Here $A(\boldsymbol{\omega}) = A(|a| + |b|)$, where $A = (1.65(N - 0.5)^2 + 1.04)/D$, $N = p$ is the cloud intensity factor, and D is the characteristic horizontal size of a clouds.

For an ordered sequence of points $\{\mathbf{r}_i\}$, whose coordinates form monotonic sequences, the n–dimensional probability of cloud occurrence can be factored and the following equation[3] for decoupling of correlations is valid

$$\langle\kappa(\mathbf{r}_1)\,\kappa(\mathbf{r}_2)\,R[\kappa]\rangle = V(\mathbf{r}_1\mathbf{r}_2)\,\langle\kappa(\mathbf{r}_2)R[\kappa]\rangle, \qquad (1)$$

where $R[\kappa]$ is a functional depending on the values of $\kappa(\mathbf{r})$ along the segmented line passing through the points $\{\mathbf{r}_i\}$, $i = 2, ..., n$, and the brackets denote averaging over the ensemble. Neglecting the absorption of radiation by aerosol and atmospheric gases, within the limits of the layer Λ the random intensity $I(\mathbf{r}, \boldsymbol{\omega})$ satisfies the stochastic radiative transfer equation for the intensity of unscattered radiation $\varphi(\mathbf{r}, \boldsymbol{\omega})$ and diffuse radiation $i(\mathbf{r}, \boldsymbol{\omega})$ can be written in the form

$$\varphi(\mathbf{r}, \omega) + \frac{\sigma}{|c|}\int_{E_z} k(\mathbf{r}')\,\varphi(\mathbf{r}', \boldsymbol{\omega})\,d\xi$$

$$= \frac{(1 - \lambda)\sigma}{|c|}\int_{E_z} k(\mathbf{r}')B(\xi)d\xi + I_z(\boldsymbol{\omega}), \qquad (2)$$

$$i(\mathbf{r}, \omega) + \frac{\sigma}{|c|}\int_{E_z} k(\mathbf{r}')i(\mathbf{r}', \boldsymbol{\omega})d\xi = \frac{\sigma}{|c|}\int_{E_z} k(\mathbf{r}')\Phi_i(\mathbf{r}', \boldsymbol{\omega})d\xi, \qquad (3)$$

$$\Phi_i(\mathbf{r}', \boldsymbol{\omega}) = \lambda\int_{4\pi} g(\boldsymbol{\omega}, \boldsymbol{\omega}')(i(\mathbf{r}', \boldsymbol{\omega}) + \varphi(\mathbf{r}', \boldsymbol{\omega}))d\boldsymbol{\omega}', \qquad (4)$$

where

$$E_z = \begin{cases} (h, z), \ c > 0, \\ (z, H), \ c < 0, \end{cases} \qquad I_z(\boldsymbol{\omega}) = \begin{cases} I_h^{\uparrow}(\boldsymbol{\omega}), \ c > 0, \\ I_H^{\downarrow}(\boldsymbol{\omega}), \ c < 0, \end{cases}$$

$\mathbf{r}' = \mathbf{r} + \dfrac{\xi - z}{c}\,\boldsymbol{\omega}$, $I_z(\boldsymbol{\omega})$ is the intensity of radiation from external sources at the boundaries of the cloud layer, and $B(z) = B(T(z))$ is the Planck function.

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Note that if light scattering in the layers above and below the cloud layer is not taken into account, the influence of the aerosol and gaseous components of the atmosphere can be easily taken into account by means of the boundary conditions.

The solution of Eq. (2) has the form

$$\varphi(\mathbf{r},\,\omega) = \frac{(1-\lambda)\sigma}{|c|}\int_{E_z} B(\xi)\mathrm{k}(\mathbf{r}')\,j(\mathbf{r}')\,d\xi + I_z(\omega)\,j(\mathbf{r}), \qquad (5)$$

where the function $j(\mathbf{r}) = \exp\left(-\frac{\sigma}{|c|}\int_{E_z}\mathrm{k}(\mathbf{r}')d\xi\right)$ can be interpreted as the random intensity of unscattered radiation at the point $\mathbf{r}$ assuming that a monodirectional source of radiation of unit power emitted along the direction ω is located at the point $\mathbf{r}^{(0)} = (x^{(0)},\, y^{(0)},\xi) = \mathbf{r}\,\frac{z-\xi}{c}\,\omega$, where $\xi = h$ if $c > 0$ and $\xi = H$ if $c < 0$.

Let the correlation function of the long-wave radiation intensity be represented as $\langle I(\mathbf{x}_1)I(\mathbf{x}_2)\rangle = \langle\varphi(\mathbf{x}_1)\varphi(\mathbf{x}_2)\rangle + \langle\varphi(\mathbf{x}_1)i(\mathbf{x}_2)\rangle + \langle i(\mathbf{x}_1)\varphi(\mathbf{x}_2)\rangle + \langle i(\mathbf{x}_1)i(\mathbf{x}_2)\rangle$. Here $\mathbf{x}_i = (\mathbf{r}_i,\,\omega)$, $i = 1, 2$ is a point from the phase space X of coordinates and directions. Let us now derive equations for each of these components (their physical meanings are obvious) and either solve them or construct algorithms of their solution by the Monte Carlo method.

Let the points $\mathbf{r}_1$ and $\mathbf{r}_2$ and directions ω_1 and ω_2 be chosen so that the conditions

$$x_2^{(0)} \le x_2 \le x_1^{(0)} \le x_1 \text{ and } y_2^{(0)} \le y_2 \le y_1^{(0)} \le y_1 \qquad (6)$$

are satisfied. The inequality signs in one or both of these relations can be replaced by the opposite ones.

1. THE FUNCTIONS $\langle\varphi(\mathbf{x}_1)\varphi(\mathbf{x}_2)\rangle$ AND $\langle\varphi(\mathbf{x}_1)i(\mathbf{x}_2)\rangle$

Let us first write down Eq. (5) at the points $\mathbf{x}_1$ and $\mathbf{x}_2$, then multiply them and average over the ensemble of $\kappa(\mathbf{r})$

$$\langle\varphi(\mathbf{x}_1)\varphi(\mathbf{x}_2)\rangle = \frac{\sigma^2(1-\lambda)^2}{|c_1||c_2|}\int_{E_{z_1}} d\xi_1 \times$$

$$\times \int_{E_{z_2}} d\xi_2 B(\xi_1)B(\xi_2)\langle\kappa(\mathbf{r}_1')\kappa(\mathbf{r}_2')j(\mathbf{r}_1')j(\mathbf{r}_2')\rangle +$$

$$+ I_z(\omega_1)\frac{\sigma(1-\lambda)}{|c_2|}\int_{E_{z_2}} B(\xi)\,\langle\mathrm{k}(\mathbf{r}_2')\,j(\mathbf{r}_2')\,j(\mathbf{r}_1)\rangle\,d\xi +$$

$$+ I_z(\omega_2)\frac{s(1-\lambda)}{|c_1|}\int_{E_{z_1}} B(\xi)\,\langle\mathrm{k}(\mathbf{r}_1')\,j(\mathbf{r}_1')\,j(\mathbf{r}_2)\rangle\,d\xi +$$

$$+ I_z(\omega_1)I_z(\omega_2)\,\langle j(\mathbf{r}_1)\,j(\mathbf{r}_2)\rangle\,, \qquad (7)$$

As will be shown below the construction of the Monte Carlo algorithms for calculating the functions $\langle i(\mathbf{x}_1)\varphi(\mathbf{x}_2)\rangle$ and $\langle i(\mathbf{x}_1)i(\mathbf{x}_2)\rangle$, requires a knowledge of the correlations $\langle\kappa(\mathbf{r}_1)\varphi(\mathbf{x}_1)\varphi(\mathbf{x}_2)\rangle$ and $\langle\kappa(\mathbf{r}_1)\varphi(\mathbf{x}_1)i(\mathbf{x}_2)\rangle$, which can be found using Eq. (5). We now multiply the expression for $\varphi(\mathbf{x}_1)\varphi(\mathbf{x}_2)$ by $\kappa(\mathbf{r}_1)$ and average it taking into account formula (1)

$$\langle\kappa(\mathbf{r}_1)\varphi(\mathbf{x}_1)\varphi(\mathbf{x}_2)\rangle = \frac{\sigma^2(1-\lambda)^2}{|c_1||c_2|}\int_{E_{z_1}} d\xi_1 \times$$

$$\times \int_{E_{z_2}} d\xi_2 B(\xi_1)B(\xi_2)V(\mathbf{r}_1,\,\mathbf{r}_1')\,\langle\kappa(\mathbf{r}_1')\,\kappa(\mathbf{r}_2')\,j(\mathbf{r}_1')\,j(\mathbf{r}_2')\rangle\ +$$

$$+ I_z(\omega_1)\frac{\sigma(1-\lambda)}{|c_2|}\int_{E_{z_2}} B(\xi)\;V(\mathbf{r}_1,\,\mathbf{r}_1')\,\langle\kappa(\mathbf{r}_2')\,j(\mathbf{r}_2')\,j(\mathbf{r}_1)\rangle\,d\xi\ +$$

$$+ I_z(\omega_2)\frac{\sigma(1-\lambda)}{|c_2|}\int_{E_{z_1}} B(\xi)\;V(\mathbf{r}_1,\,\mathbf{r}_1')\,\langle\kappa(\mathbf{r}_1')\,j(\mathbf{r}_1')\,j(\mathbf{r}_2)\rangle\,d\xi +$$

$$+ I_z(\omega_1)\;I_z(\omega_2)\,\langle\mathrm{k}(\mathbf{r}_1)\,j(\mathbf{r}_1)\,j(\mathbf{r}_2)\rangle\,. \qquad (8)$$

From Eqs. (7) and (8) it follows that the correlations $\langle\varphi(\mathbf{x}_1)\,\varphi(\mathbf{x}_2)\rangle$ and $\langle\kappa(\mathbf{r}_1)\,\varphi(\mathbf{x}_1)\,\varphi(\mathbf{x}_2)\rangle$ can be determined provided that the statistical characteristics of function $j(\mathbf{r})$ are well known. These characteristics have been obtained in Ref. 4 for arbitrary distribution of points $\mathbf{x}_i$ lying in one and the same plane.

By multiplying expression (5) by $i(\mathbf{x}_2)$ and $\kappa(\mathbf{r}_1)i(\mathbf{x}_2)$ and averaging, takingEq. (1) into account, we have

$$\langle\varphi(\mathbf{x}_1)i(\mathbf{x}_2)\rangle = \frac{\sigma(1-\lambda)}{|c_2|}\int_{E_{z_1}} B(\xi)\,\langle\mathrm{k}(\mathbf{r}_1')\,j(\mathbf{r}_1')\,i(\mathbf{x}_2)\rangle\,d\xi +$$

$$+ I_z(\mathbf{w}_1)6j(\mathbf{r}_1)i(\mathbf{x}_2)c\,, \qquad (9)$$

$$\langle\kappa(\mathbf{r}_1)\varphi(\mathbf{x}_1)i(\mathbf{x}_2)\rangle =$$

$$= \frac{\sigma(1-\lambda)}{|c_1|}\int_{E_{z_1}} B(\xi)\;V(\mathbf{r}_1,\,\mathbf{r}_1')\,\langle\mathrm{k}(\mathbf{r}_1')\,j(\mathbf{r}_1')\;i(\mathbf{x}_2)\rangle\,d\xi +$$

$$+ I_z(\omega_1)\,\langle\mathrm{k}(\mathbf{r}_1)\;j(\mathbf{r}_1)\;i(\mathbf{x}_2)\rangle\ . \qquad (10)$$

According to equations (9) and (10), the sought-after correlations are expressed in terms of the functions $\langle j(\mathbf{r}_1)\;i(\mathbf{x}_2)\rangle$ and $\langle\mathrm{k}(\mathbf{r}_1)\;j(\mathbf{r}_1)\;i(\mathbf{x}_2)\rangle$, for which simple analytical formulas have been obtained[5] wich are valid only at $\omega_1 = \omega_\perp = (0.0 \pm 1)$. In the general case of an arbitrary direction ω_1 we have

$$S(\mathbf{r}_1, \mathbf{r}_2) = \langle k(\mathbf{r}_1)\, j(\mathbf{r}_1)\, i(\mathbf{x}_2)\rangle = pv(z_1)\langle i(z_2, \omega_2)\rangle +$$
$$+ pv_A(z_1)(u(z_2, \omega_2) - \langle i(z_2, \omega_2)\rangle)\exp(-Ax\Delta x - Ay\Delta y), \quad (11)$$

$$\langle j(\mathbf{r}_1) i(\mathbf{x}_2)\rangle = \langle j(z_1)\rangle\langle i(z_2, \omega_2)\rangle +$$
$$+\frac{p}{1-p}(v(z_1) - \langle j(z_1)\rangle)(u(z_2, \omega_2) - 6i(z_2, \omega_2)\rangle)\times$$
$$\times \exp(-Ax\Delta x^{(0)} - Ay\Delta y^{(0)}), \quad (12)$$

$$\Delta x = |x_1 - x_2|,\ \Delta y = |y_1 - y_2|,$$
$$\Delta x^{(0)} = |x_1^{(0)} - x_2|,\ \Delta y^{(0)} = |y_1^{(0)} - y_2|, \quad (13)$$

$$pv(\tilde{z}) = \langle k(\mathbf{r})\, j(\mathbf{r})\rangle = \sum_{i=1}^{2} D_i e^{-\lambda_i \tilde{z}},$$
$$\tilde{z} = \begin{cases}(z-h)/c, & c > 0,\\ (z-H)/c, & c < 0,\end{cases} \quad (14)$$

$$v_A(\tilde{z}) = \sum_{i=1}^{2}\frac{D_i\lambda_i}{\lambda_i - A(\omega)}\exp(-(\lambda_i - A(\omega))\tilde{z}),$$
$$\lambda_{1,2} = \frac{\sigma + A(\omega)}{2} \mp \frac{\sqrt{(\sigma + A(\omega))^2 - 4A(\omega)\sigma p}}{2},$$
$$D_1 = \frac{\lambda_2 - \sigma}{\lambda_2 - \lambda_1},\quad D_2 = 1 - D_1,$$

and for the functions $\langle i(z, \omega)\rangle$ and $pu(\mathbf{r}, \omega) = \langle\kappa(\mathbf{r}) i(\mathbf{r}, \omega)\rangle$ we have now constructed algorithms[5] of statistical modeling. Correlation functions of the intensity and the fluxes of diffuse solar radiation are calculated[5,8] using the Monte Carlo method. In these algorithms the random weight is determined by the function $S(\mathbf{r}_1, \mathbf{r}_2)$ and therefore in calculating the random weight it is necessary to take into account the dependence of $S(\mathbf{r}_1, \mathbf{r}_2)$ on ω_1.

2. THE CORRELATIONS $\langle i(\mathbf{x}_1)\varphi(\mathbf{x}_2)\rangle$ AND $\langle i(\mathbf{x}_1) i(\mathbf{x}_2)\rangle$

Let us denote the random intensities $i(\mathbf{x}_2)$ and $\varphi(\mathbf{x}_2)$ as $f(\mathbf{x}_2)$. From expressions (3), (4), and (6) it follows that $i(\mathbf{r}', \omega_1) f(\mathbf{x}_2) = R[\kappa(\mathbf{r})]$ is a functional which depends explicitly on $\kappa(\mathbf{r})$ everywhere up to the point $\mathbf{r}'$ and implicitly on $\kappa(\mathbf{r})$ via the functional Φ_i everywhere in the layer Λ. Therefore formula (1) can be considered only as an approximation.

We now write Eq. (3) at the point $\mathbf{x}_1$, multiply it by $f(\mathbf{x}_2)$ and $\kappa(\mathbf{r}) f(\mathbf{x}_2)$ and average it, taking Eq. (1) into account:

$$\langle i(\mathbf{x}_1) f(\mathbf{x}_2)\rangle + \frac{\sigma p}{|c_1|}\int_{E_{z_1}} Y(\mathbf{r}_1', \omega_1, \mathbf{x}_2)d\xi =$$
$$= \frac{\sigma p}{|c_1|}\int_{E_{z_1}} \Phi_y(\mathbf{r}_1', \omega_1, \xi_2)d\xi; \quad (15)$$

$$Y(\mathbf{x}_1, \mathbf{x}_2) + \frac{\sigma}{|c_1|}\int_{E_{z_1}} V(\mathbf{r}_1, \mathbf{r}_1')Y(\mathbf{r}_1', \omega_1, \mathbf{x}_2)d\xi =$$
$$= \frac{\sigma}{|c_1|}\int_{E_{z_1}} V(\mathbf{r}_1, \mathbf{r}_1')\Phi_y(\mathbf{r}_1', \omega_1, \mathbf{x}_2)d\xi; \quad (16)$$

$$pY(\mathbf{x}_1, \mathbf{x}_2) = \langle\kappa(\mathbf{r})\, i(\mathbf{x}_1)\, f(\mathbf{x}_2)\rangle;$$
$$py(\mathbf{x}_1, \mathbf{x}_2) = \langle\kappa(\mathbf{r}_1)\,\varphi(\mathbf{x}_1)\, f(\mathbf{x}_2)\rangle;$$

$$\Phi_y(\mathbf{x}_1, \mathbf{x}_2) = \lambda\int_{4\pi} g(\omega, \omega')Y(\mathbf{r}_1, \omega', \mathbf{x}_2) + y(\mathbf{r}_1, \omega', \mathbf{x}_2)d\omega'. \quad (17)$$

Formally, Eq. (16) does not differ from the equation for the function $u(\mathbf{x}) = \langle\kappa(\mathbf{r})I(\mathbf{x})\rangle/p$ (see Ref. 2) with the exception of the additional variable $\mathbf{x}_2$ in Eq. (12), which can be considered as a parameter. This circumstance allows as to avoid cumbersome calculations and write down the following integral equation for the function

$W(\mathbf{x}_1, \mathbf{x}_2) = Y(\mathbf{x}_1, \mathbf{x}_2) + y(\mathbf{x}_1, \mathbf{x}_2)$:

$$W(\mathbf{x}_1, \mathbf{x}_2) = \frac{\lambda}{|c_1|}\int_{E_z} d\xi\int_{4\pi}\sum_{i=1}^{2} D_i\lambda_i\exp\left(-\lambda_i\frac{|z_1 - \xi|}{|c_1|}\right)\times$$
$$\times g(\omega, \omega')W(\xi, \omega', \mathbf{x}_2)d\omega' + y(\mathbf{x}_1, \mathbf{x}_2). \quad (18)$$

By substituting Eq. (18) into Eq. (16) and making the transformations which are described in defail in Ref. 2, it is possible to write the function $\langle i(\mathbf{x}_1) f(\mathbf{x}_2)\rangle$ in the following form:

$$\langle i(\mathbf{x}_1) f(\mathbf{x}_2)\rangle = \frac{\lambda\sigma p}{|c_1|}\int_{E_z} d\xi\int_{4\pi}\sum_{i=1}^{2} D_i\exp\left(-\lambda_i\frac{|z_1 - \xi|}{|c_1|}\right)\times$$
$$\times g(\omega, \omega')W(\xi, \omega', \mathbf{x}_2)d\omega'. \quad (19)$$

Now let us consider the algorithm for estimating the linear functional $\langle i(z_*, \omega_*) f(\mathbf{x}_2)\rangle$ using the Monte Carlo method. Since the radiation detector is localized where as the radiation source is distributed over the phase space of coordinates and directions, we shall use the method of conjugate trajectories.[2]

We write the adjoint transfer equation

$$\omega\nabla I^*(\mathbf{r}, -\omega) + \sigma\kappa(\mathbf{r})I(\mathbf{r}, -\omega) =$$
$$= \lambda\sigma\int_{4\pi} g(-\omega, -\omega')\,\kappa(\mathbf{r})\, I^*(\mathbf{r}, -\omega')\, d\omega + p(\mathbf{r}, -\omega), \quad (20)$$

with boundary conditions

$$I^*(\mathbf{r}, -\omega)\big|_{\substack{z=h\\ c>0}} = I^*(\mathbf{r}, -\omega)\big|_{\substack{z=H\\ c<0}} = 0, \quad (21)$$

and source density

$$\mathrm{p}(\mathbf{r}, -\boldsymbol{\omega}) = \delta(z - z_*)\, \delta(\boldsymbol{\omega} + \boldsymbol{\omega}_*). \tag{22}$$

The integral equation for the function

$U^*(z, -\boldsymbol{\omega}) = \langle \kappa(\mathbf{r}) I^*(\mathbf{r}, -\boldsymbol{\omega})\rangle / p$

has the form[7]

$$U^*(\mathbf{x}^*) = \int_x k(\mathbf{x}', \mathbf{x}^*) U^*(\mathbf{x}')\mathrm{d}\mathbf{x}' + v(\tilde{z}_*)\mathrm{d}(\boldsymbol{\omega} + \boldsymbol{\omega}_*), \tag{23}$$

where

$$k(\mathbf{x}', \mathbf{x}^*) = \frac{\lambda g(-\boldsymbol{\omega}, -\boldsymbol{\omega}')\sum_{i=1}^{2} D_i \lambda_i \exp(-\lambda_i |\mathbf{r} - \mathbf{r}'|)}{|\mathbf{r} - \mathbf{r}'|^2} \times$$

$$\times\left(\delta \frac{\mathbf{r} - \mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|} + \boldsymbol{\omega} \right),$$

and

$$\mathbf{x}^* = (\mathbf{r}, -\boldsymbol{\omega}). \tag{24}$$

Using the optical reciprocity theorem[6] one can show that

$\langle i(z_*, \boldsymbol{\omega}_*) f(\mathbf{x}_2) \rangle =$

$$= \frac{\lambda\sigma p}{|c_*|} \int_{E_{z_*}} \mathrm{d}\xi \int_{4\pi} \mathrm{d}\boldsymbol{\omega}' W(\mathbf{x}_1', \mathbf{x}_2) \int_{4\pi} \mathrm{g}(\boldsymbol{\omega}', \boldsymbol{\omega}'') v(\xi)\mathrm{d}(\boldsymbol{\omega}'' - \boldsymbol{\omega}_*)\mathrm{d}\boldsymbol{\omega}'' =$$

$$= \frac{\lambda\sigma p}{|c_*|} \int_{E_{z_*}} \mathrm{d}\xi \int_{4\pi} \mathrm{d}\boldsymbol{\omega}' U^*(\xi, -\boldsymbol{\omega}') \int_{4\pi} \mathrm{g}(-\boldsymbol{\omega}', -\boldsymbol{\omega}'') y(\xi, -\boldsymbol{\omega}'', \mathbf{x}_2)\mathrm{d}\boldsymbol{\omega}''. \tag{25}$$

The possibility of using the Monte Carlo method to estimate the functional $\langle i(z_*, \boldsymbol{\omega}_*) f(\mathbf{x}_2)\rangle$ is ensured by the convergence in the space L_1 of the Neumann series of Eq. (23).

Let us define the Markov chain $\{\mathbf{x}_i\}$ with initial density $\pi(\mathbf{x}_0) = \sum_{i=1}^{2} D_i \lambda_i \exp(-\lambda_i \tilde{z}_0)$ and transition density $k^*(\mathbf{x}', \mathbf{x}^*)/\lambda$. Then

$$\langle i(z_*, \mathrm{w}_*) f(\mathbf{x}_2) \rangle = M \sum_{n=0}^{N_1} Q_n y(\mathbf{r}_n, -\boldsymbol{\omega}_{n+1}, \mathbf{x}_2), \tag{26}$$

where M is the symbol of mathematical expectation over the ensemble of realizations, N_1 is the random number of the last state, and the random weight

$$Q_0 = \frac{\sigma p v(\tilde{z}_0, \boldsymbol{\omega})\delta(\boldsymbol{\omega} + \boldsymbol{\omega}_*)}{\pi(\tilde{z}_0, \boldsymbol{\omega})}, \quad Q_n = \lambda Q_{n-1},$$

where $\mathbf{r}_0 = (x_0, y_0, z_0) = \mathbf{r}_* + \dfrac{z_0 - z_*}{c}\, \boldsymbol{\omega}$ is the point where the first collision taken place.

In order to calculate the functional $\langle i(z_*, \boldsymbol{\omega}_*) f(\mathbf{x}_2)\rangle$ it is necessary to model the trajectories starting from the point $\mathbf{r}_* = (0, 0, z_*)$ with the initial direction $\boldsymbol{\omega}_*$ and then to calculate the value of $y(\mathbf{r}_n, \boldsymbol{\omega}_{n+1}, \mathbf{x}_2)$ at every collision point. Note that the function $y(\mathbf{x}_1, \mathbf{x}_2)$ is determined by expression (17), and therefore if $f(\mathbf{x}_2)\ \varphi(\mathbf{x}_2)$, $py(\mathbf{x}_1, \mathbf{x}_2) = \langle \kappa(\mathbf{r}_1)\varphi(\mathbf{x}_1)\varphi(\mathbf{x}_2)\rangle$ whereas at $f(\mathbf{x}_2) = i(\mathbf{x}_2)$ $py(\mathbf{x}_1, \mathbf{x}_2) = \langle \kappa(\mathbf{r}_1)\varphi(\mathbf{x}_1)\ i(\mathbf{x}_2)\rangle$. The correlation $y(\mathbf{x}_1, \mathbf{x}_2)$ can be obtained from Eqs. (8) and (10).

Solution of the problem of thermal radiation transfer through nonisothermal clouds requires a knowledge of the temperature profile within the cloud. Because of the complicated shape of $B(T(z))$ it is necesary to use numerical integration in the solution of Eqs. (7)–(10). On the other hand, if one assumes that the clouds are isothermal, then Eqs. (7)–(10) can be considerably simplified and for the sought-after correlations we obtain

$$\langle \varphi(\mathbf{x}_1)\, \varphi(\mathbf{x}_2)\rangle = \langle \varphi(\mathrm{z}_1, \boldsymbol{\omega}_1)\rangle \langle \varphi(\mathrm{z}_2, \boldsymbol{\omega}_2)\rangle +$$
$$+ (I_z(\boldsymbol{\omega}_1) - (1 - \lambda)B_c)\, (I_z(\boldsymbol{\omega}_2) - (1 - \lambda)B_c) \times$$
$$\times (\langle j(\mathbf{r}_1)\, j(\mathbf{r}_2) \rangle - \langle j(z_1) \rangle \langle j(z_2) \rangle); \tag{27}$$

$$\langle \varphi(\mathbf{x}_1) i(\mathbf{x}_2) \rangle = \langle \varphi(\mathrm{z}_1, \boldsymbol{\omega}_1) \rangle \langle i(\mathrm{z}_2, \boldsymbol{\omega}_2) \rangle +$$
$$+ (I_z(\boldsymbol{\omega}_1) - (1 - \lambda)B_c)\, (j(\mathbf{r}_1) i(\mathbf{x}_2) - \langle j(z_1)\rangle \times \langle i(\boldsymbol{\omega}_2) \rangle), \tag{28}$$

$$\langle \mathrm{k}(\mathbf{r}_1)\, \varphi(\mathbf{x}_1)\, \varphi(\mathbf{x}_2) \rangle = p\Psi(\mathrm{z}_1, \boldsymbol{\omega}_1) \langle \varphi(\mathrm{z}_2, \boldsymbol{\omega}_2) \rangle +$$
$$+ (\Psi(\mathrm{z}_2, \boldsymbol{\omega}_2) - \langle \varphi(\mathrm{z}_2, \boldsymbol{\omega}_2) \rangle)\, p\Psi_A(\mathrm{z}_1, \boldsymbol{\omega}_1) \times$$
$$\times \exp(-Ax\, \Delta x - Ay\, \Delta y), \tag{29}$$

$$\langle \mathrm{k}(\mathbf{r}_1)\, \varphi(\mathbf{x}_1)\, i(\mathbf{x}_2) \rangle = p\Psi(\mathrm{z}_1, \boldsymbol{\omega}_1) \langle j(\mathrm{z}_2, \boldsymbol{\omega}_2) \rangle +$$
$$+ (u(\mathrm{z}_2, \boldsymbol{\omega}_2) - \langle i(\mathrm{z}_2, \boldsymbol{\omega}_2) \rangle)\, p\Psi_A(\mathrm{z}_1, \boldsymbol{\omega}_1) \times$$
$$\times \exp(-Ax\, \Delta x - Ay\, \Delta y), \tag{30}$$

$$\Psi(\mathrm{z}, \boldsymbol{\omega}) = \langle \kappa(\mathbf{r})\varphi(\mathbf{x})\rangle / p =$$
$$= (1 - \lambda)B_c + v(\tilde{z})(I_z(\boldsymbol{\omega}_2) - (1 - \lambda)B_c), \tag{31}$$

$$\Psi_A(\mathrm{z}, \boldsymbol{\omega}) = (1 - \lambda)B_c + v_A(\tilde{z})\, (I_z(\boldsymbol{\omega}_2) - (1 - \lambda)B_c), \tag{32}$$

$B_c = B(T_c)$, T_c is the of clouds-temperature. Note that at $A(\boldsymbol{\omega}) = 0$, $\Psi_A(z, \boldsymbol{\omega}) = \Psi(z, \boldsymbol{\omega})$.

Substituting Eqs. (29) and (30) in Eq. (26) we obtain

$$\langle j(\mathbf{x}_1)\, \varphi(\mathbf{x}_2) \rangle = \langle i(z_1, \boldsymbol{\omega}_1)\rangle \langle \varphi(z_2, \boldsymbol{\omega}_2)\rangle + (\Psi(z_2, \boldsymbol{\omega}_2) -$$
$$- \langle \varphi(z_2, \boldsymbol{\omega}_2)\rangle) M \sum_{n=0}^{N_1} Q_n \Psi_A(z_n, -\boldsymbol{\omega}_{n+1}) \exp(-Ax\Delta x_0 - Ay\Delta y_0), \tag{33}$$

$$\langle i(\mathbf{x}_1) i(\mathbf{x}_2)\rangle = \langle i(z_1, \boldsymbol{\omega}_1)\rangle \langle i(z_2, \boldsymbol{\omega}_2)\rangle + (u(z_2, \boldsymbol{\omega}_2) -$$
$$- \langle i(z_2, \boldsymbol{\omega}_2)\rangle)\, M \sum_{n=0}^{N_1} Q_n \Psi_A(z_n, -\boldsymbol{\omega}_{n+1}) \exp(-Ax\Delta x_0 - Ay\Delta y_0), \tag{34}$$

$\Delta x_0 = |x_0 - x_2|$, $\Delta y_0 = |y_0 - y_2|$.

Thus, using the stochastic equation of radiative transfer we have obtained the equations for the correlation function of the long–wave radiation intensity and have developed methods and algorithms for their solution.

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