Free energy balance at the upper boundary of the atmosphere

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Photon statistics in the processes of entropy and the free energy transfer by electromagnetic field in the atmosphere are discussed in this paper. It is shown that multi-equilibrium (generalized Plank's statistics) photon number distribution for monochromatic radiation is due to minimum free energy of the field. A method for simulating free energy fluxes in the atmosphere and free energy balance at its upper boundary is developed, using outgoing thermal radiation and reflected solar radiation. The presence of an extremum in the free energy import to the planet is shown within the model of gray atmosphere under the condition of the energy balance at its upper boundary. The extremum is caused by the optical depth τ of the planet atmosphere and is realizable at τ value, close to a current value for the Earth atmosphere. The assessed annual mean flux of the free energy imported by the Earth is about 59.13 W/m^2 or $3 \cdot 10^{16}$ W/m^2 over the globe. This magnitude determines an integral power of the heat engine of our planet, while its ratio to the power of incoming solar radiation can be considered as the efficiency of this heat engine. Simultaneous satellite monitoring of radiation balance and free energy balance at the upper boundary of the atmosphere could be used to monitor possible variations of integral power of the Earth heat engine and global warming process.

Introduction

The problem of estimating the entropy production by the Earth climate system is under active discussion in the present-day literature.[1–11] One of the key lines in such investigations is an assessment of the magnitude of the entropy production on the Earth from the balance of radiation entropy at the upper boundary of the atmosphere under the assumption of the annually-average stationarity of the climate system.[3–6,8–10] However, for deeper insight into the processes proceeding in the climate system, investigation of free energy balance in the atmosphere is of interest along with the study of radiation and entropy balances.

It is well-known that a thermodynamic variable, characterizing the ability of any physical system to do mechanical work, is its free energy,[12–14] incoming to the Earth with solar radiation. The energy is consumed for maintaince of processes of general atmospheric and ocean circulation and biomass growing; it partially is stored in biota, chemical links, in photosynthesis. Similar to the radiation balance, the free energy is defined by the difference between energy fluxes of incoming solar radiation and outgoing heat radiation; its balance can be determined as the difference between free energy of incoming and outgoing radiations.

The free energy balance of the planet is evidently determined by characteristics of its albedo for solar radiation and atmospheric transmission for outgoing thermal radiation. The import of free energy to the Earth allows the sustention of planetary circulation, as well as photosynthesis processes and the planetary life. This means that the annually-average Earth free energy budget should be much greater than zero, and its magnitude should characterize the flux of the imported free energy consumed for work, i.e., the power and the efficiency of the Earth heat engine.

In this paper, a method for determining free energy fluxes of atmospheric radiation is suggested. The annually-average free energy budget at the upper boundary of the atmosphere is assessed and the integrated power and the efficiency of the Earth heat engine are calculated.

1. Photon statistics. The entropy and radiation free energy

Free energy of an equilibrium system at the temperature T is defined as the difference between total system energy and its part, consumed for recording information (entropy) in the system.[15] Thus, before determining the radiation free energy, its entropy should be revealed.

Based on the general entropy definition,[12,13] the entropy S_ν for monochromatic electromagnetic radiation can be written as

$$S_\nu = -k\sum_{n=0}^{\infty} p_\nu(n)\ln p_\nu(n), \qquad (1)$$

where $p_\nu(n)$ is the distribution function of the photon number $n = 0, 1, 2, \ldots$ in the frequency field mode ν; k is the Boltzmann constant.

It follows from Eq. (1) that the entropy of electromagnetic field is totally determined by the

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photon statistics of the considered radiation. Distribution functions of the photon numbers are certainly known only for radiation of two ideal physical systems: Poisson statistics of ideal laser radiation

$$p_\nu^{\text{Poisson}}(n) = e^{-<n_\nu>}\frac{<n_\nu>^n}{n!} \text{ [Refs. 16 and 17]}$$

and Planck (equilibrium thermal) statistics of blackbody radiation with the temperature T:

$$p_\nu^{\text{thermal}}(n) = \frac{<n_\nu(T)>^n}{(<n_\nu(T)>+1)^{n+1}} \text{ [Ref. 18]},$$

where $<n_\nu>$ and $<n_\nu(T)>$ are the mean photon number in the mode of laser and thermal radiations, respectively.

In the processes of resonance absorption and radiation, photon statistics, initially Planck or Poisson, can essentially change.[19,20] Therefore, real photon statistics of solar radiation and especially of the Earth radiation heat flux should be taken into account in calculations of both entropy balance and radiation free energy at the upper atmospheric boundary. In general case, the latter can essentially differ from the equilibrium Planck photon statistics of the blackbody radiation.

In this work, a more common distribution of the photon number in the field mode is considered. It may be called multi-equilibrium photon statistics in the case when the photon number distribution function $p_\nu(n)$ takes the form of generalized heat distribution function (or generalized Planck distribution).[21,22] In this case, in contrast to the blackbody with the common equilibrium temperature T for all modes, each mode of the field ν is characterized by its own equilibrium temperature T_ν [Ref. 22]:

$$p_\nu(n) = \frac{<n_\nu>^n}{(<n_\nu>+1)^{n+1}}, \quad <n_\nu>=\frac{1}{\exp(h\nu/kT_\nu)-1}, \qquad (2)$$

where $<n_\nu> = \sum_{n=0}^{\infty} np_\nu(n)$ is the average photon number in the mode. For the filed, described by the multi-equilibrium photon number distribution function, the monochromatic field free energy F_ν can evidently be defined similarly to the case of blackbody equilibrium radiation[12,13]:

$$F_\nu = E_\nu - T_\nu S_\nu . \qquad (3)$$

The only difference is that the common equilibrium temperature T for all field modes is replaced by the proper equilibrium temperature T_ν of each mode ν. Here $E_\nu = h\nu<n_\nu>$ is the total energy of the field mode; $T_\nu = \frac{h\nu}{k\ln(1+1/<n_\nu>)}$ is the mode brightness temperature (coinciding with the equilibrium one). The monochromatic radiation entropy S_ν in this case is defined, according to Eqs. (1) and (2), by the well-known classical equation[21,22]

$$S_\nu = k\{(<n_\nu>+1)\ln(<n_\nu>+1) - <n_\nu>\ln<n_\nu>\}. \qquad (4)$$

As a result, the following short equation for monochromatic radiation free energy F_ν (3) with the photon statistics $p_\nu(n)$, described by Eq. (2), can be obtained as a function of average photon number in a mode:

$$F_\nu = -h\nu\frac{\ln(<n_\nu>+1)}{\ln(1+1/<n_\nu>)}. \qquad (5)$$

The integral flux of free energy of the field F, propagating in a chosen direction, is calculated by multiplying Eq. (5) by the number of spectral modes $\frac{2\nu^2}{c^2}d\nu$ for nonpolarized radiation $\nu, \nu+d\nu$ [Ref. 18] with following integration over all frequencies and the spatial angle:

$$F = \int_\Omega d\Omega \int_0^\infty d\nu \frac{2\nu^2}{c^2}F_\nu. \qquad (6)$$

Using Eq. (5), the following equation for integral flux of free energy F can be obtained:

$$F = -\int_0^{2\pi} d\varphi \int_0^{\pi/2} \cos\theta\sin\theta d\theta \int_0^\infty d\nu \frac{2h\nu^3}{c^2}\frac{\ln(<n_\nu>+1)}{\ln(1+1/<n_\nu>)}. \qquad (7)$$

When the dependence of $<n_\nu>$ on the angles θ and φ is negligible in Eq. (7), obtain

$$F = -\pi\int_0^\infty d\nu \frac{2h\nu^3}{c^2}\frac{\ln(<n_\nu>+1)}{\ln(1+1/<n_\nu>)}. \qquad (8)$$

In classical (Planck) case of isotropic blackbody radiation, when all modes have an equal temperature T_ν, i.e., $T_\nu = \text{const} = T$, a well-known equation for free energy fluxes of blackbody radiation $F = -(1/3)\sigma T^4$ [Refs. 12 and 13] follows from Eq. (8); here $\sigma = 5.67\cdot10^{-8}$ W/(m$^2\cdot$K^4) is the Stefan–Boltzmann constant.

Summarizing the above described, it is necessary to point out that the formal use of the generalized Planck distribution (2) allows one to introduce the so-called brightness temperature for any radiation[12,22]:

$$T_\nu = \frac{h\nu}{k\ln(1+1/<n_\nu>)}; \quad <n>_\nu = c^2R_\nu/(2h\nu^3)$$

(R_ν is the spectral brightness of monochromatic radiation), with any photon statistics. However, thus defined variable T_ν has a sense of the equilibrium temperature of the field mode only in case when photon statistics of the given radiation is actually described by the multi-equilibrium distribution function (2).

2. Principle of minimum of radiation free energy

It can be shown that such form (2) of the photon number distribution function follows from the principle of minimum of free energy flux F (6). To do this, consider the problem of definition of the photon number distribution function $p_\nu(n)$, minimizing the flux functional of field free energy F. Equating the functional variation δF to zero

$$\delta F = \delta \int_\nu d\Omega \int_0^\infty d\nu \frac{2\nu^2}{c^2} F_\nu = 0, \quad (9)$$

obtain the condition $\delta F_\nu = \sum_{n=0}^{\infty} \frac{\partial F_\nu}{\partial p_\nu(n)} \delta p_\nu(n) = 0$ at any $\delta p_\nu(n)$, or $\frac{\partial F_\nu}{\partial p_\nu(n)} = h\nu n + kT_\nu(1 + \ln p_\nu(n)) = 0$ for any $n \neq 0$.

The photon number distribution function $p_\nu(n) = C\exp\left[-\frac{h\nu}{kT_\nu}n\right]$ follows from the above, where C is the normalization factor, defined by the condition of normalization to the distribution function $\sum_{n=0}^{\infty} p_\nu(n) = 1$. With accounting for the normalization, the photon number distribution function, at which F from Eq. (6) takes its extremum value, has the form

$$p_\nu(n) = \left(1 - \exp\left[-\frac{h\nu}{kT_\nu}\right]\right)\exp\left[-\frac{h\nu}{kT_\nu}n\right]. \quad (10)$$

Positive sign of the second derivatives of functional (6) with respect to the photon number distribution function $\frac{\partial^2 F_\nu}{\partial p_\nu(n)^2} = \frac{1}{p_\nu(n)} > 0$ indicates that the found extremum is really the minimum.

Since the average photon number for the monochromatic radiation, described by distribution function (2), $<n_\nu> = \sum_{n=0}^{\infty} n p_\nu(n) = \frac{1}{\exp(h\nu/kT_\nu) - 1}$ then hence, distribution (10), minimizing the radiation free energy flux F (6), can be rewritten as

$$p_\nu(n) = \frac{<n_\nu>^n}{(<n_\nu> + 1)^{n+1}}. \quad (11)$$

Equation (11) is the same as Eq. (2) for the multi-equilibrium distribution function of the photon number.

As it is known from the literature, Eqs. (2) and (11) accurately describe the photon statistics of grey body[5,8] and are applicable to the reflected sunlight[6] in case when the photon number distribution function of the initial solar radiation is approximated by the Planck photon number distribution function of the blackbody radiation.[6,23]

For a self-organizing system, like our planet, the photon statistics of the outgoing radiation can be of great importance, since different photon statistics allow different free energy import and export at the same radiation (energy) balance. A constant free energy flux comes to the Earth with sunlight; a part of the flux returns into the outer space when reflecting from clouds and the earth's surface. In addition, free energy is exported with earth's radiation heat flux. The magnitude of these free energy fluxes (at zero radiation balance) are controlled by the photon statistics of outgoing thermal radiation. The free energy flux from Sun passing through the atmosphere can be considered as constant (at constant earth's albedo); therefore, it is natural to assume that the free energy import by the Earth is maximal when the free energy sink from the planet back to the outer space is minimal. In this case, the photon number distribution function for outgoing thermal radiation should be close to the distribution function, defined by Eqs. (2) and (11).

3. The balance of radiation free energy in the atmosphere

To evaluate the balance of the radiation free energy at the upper boundary of the atmosphere depending on the atmosphere optical depth and planet albedo, consider the following model.

1. Solar radiation is the blackbody radiation with the temperature T = 5785 K [Refs. 5 and 23].

2. The planet albedo A is frequency-independent.

3. The outgoing heat flux of planet radiation is described by the model of grey atmosphere with the optical depth τ [Refs. 4 and 24].

4. Photon statistics of the planet radiation heat flux is described by distribution (11).

Equation (7) is valid for free energy fluxes of any electromagnetic field, described by photon statistics (2) and (11) in cases of the grey body radiation passing through the atmosphere[4,7] and the reflected sunlight.[5] Thus, this model allows the use of multi-equilibrium function of photon number distribution (11) for describing the free energy of atmosphere-passing and reflected solar radiations, as well as heat flux radiated by a planet with grey atmosphere.

Within the above model of a planet with black surface and grey atmosphere, consider the free energy balance $\downarrow\Delta F$ at the upper boundary of the atmosphere at zero radiation (energy) balance. Similar to the radiation balance, define the balance of the radiation free energy at the upper boundary of the atmosphere (free energy import by the planet) as the difference between the free energy of passing solar radiation (with accounting for the reflection due to the planet albedo) and those of outgoing thermal radiation:

$$\downarrow\Delta F = (1/4)\downarrow F_s^{in} - \uparrow F_{thrmal}^{out}. \quad (12)$$

Factor 1/4 takes into account the distribution of free energy flux, coming from the Sun, all over the spherical surface of the planet.

To calculate the free energy flux of outgoing thermal radiation, use Eq. (7), preliminary determining the magnitude of $<n_\nu>$. The equation

$$<n_\nu> = (1-u)<n_\nu>_s + u<n_\nu>_a \quad (13)$$

follows from the radiation transfer equation in the model of grey atmosphere for the average number of photons in the mode of outgoing thermal radiation of the planet.[5] Here $<n_\nu>_s$ is the average photon number in the mode of the planet surface radiation filed; $<n_\nu>_a$ is the average photon number in the mode of atmosphere radiation field; $u = 1 - e^{-\tau}$, τ is the optical depth of the grey atmosphere. Multiplying Eq. (13) by $\frac{2h\nu^3}{c^2}d\nu$ and integrating over the frequency and spatial angle, obtain[5]:

$$\sigma T_e^4 = (1-u)\sigma T_s^4 + u\sigma T_a^4. \quad (14)$$

Here T_e, T_s, and T_a are the equivalent planet radiation temperature, surface temperature, and effective atmosphere temperature, respectively. The surface temperature T_s in the model of grey atmosphere is connected with the equivalent planet radiation temperature T_e by the equation[24]

$$T_s = T_e(1+\tau)^{1/4} = T_e\{1-\ln(1-u)\}^{1/4}. \quad (15)$$

As a result, obtain from Eqs. (14) and (15)

$$T_a = T_e u^{-1/4}\{1+(u-1)(1-\ln(1-u))\}^{1/4}. \quad (16)$$

Equations (15) and (16) allow $<n_\nu>_s$ and $<n_\nu>_a$ in Eq. (13) to be expressed as functions of T_s and T_a, respectively:

$$<n_\nu>_s = 1/\{\exp[h\nu/kT_s]-1\}$$

and

$$<n_\nu>_a = 1/\{\exp[h\nu/kT_a]-1\}. \quad (17)$$

The condition of zero radiation balance at the upper boundary of the atmosphere assumes an equality of the fluxes of incoming solar radiation and outgoing thermal one[23,24]:

$$(I_s/4)(1-A) = \sigma T_e^4,$$

where $I_s = (r_{sun}^2/L_{es}^2)I_0$ is the solar constant on the Earth orbit (r_{sun} is the Sun radius; L_{es} is the distance from the Earth to the Sun); I_0 is the solar constant near the Sun. Using the condition along with Eqs. (7), (13), and (17), as well as neglecting the dependence on θ and φ, express the free energy flux of outgoing thermal radiation $\uparrow F_{thermal}^{out}$ as a function of the planet atmosphere optical depth τ and the parameter $\alpha = \frac{r_{sun}^2}{L_{es}^2}(1-A)$:

$$\uparrow F_{thermal}^{out} = -\frac{\pi k^4 I_0 \alpha}{2c^2h^3\sigma}\int_0^\infty dx\, x^3 \frac{\ln\left\{\left(\frac{1-u}{e^{ax}-1}+\frac{u}{e^{bx}-1}\right)+1\right\}}{\ln\left\{1+1/\left(\frac{1-u}{e^{ax}-1}+\frac{u}{e^{bx}-1}\right)\right\}}, \quad (18)$$

where

$$x = \frac{h\nu}{kT_{solar}};\quad a = \{\alpha/4[1-\ln(1-u)]\}^{-1/4};$$

$$b = u^{1/4}\{(\alpha/4)[1+(u-1)(1-\ln(1-u))]\}^{-1/4};$$

T_{solar} is the temperature of the equilibrium solar radiation.

In case of calculating the incoming free energy flux of solar radiation $\downarrow F_s^{in}$, it is necessary to set $<n_\nu> = \alpha<n_\nu>^{solar}$ ($<n_\nu>^{solar} = 1/\{\exp[h\nu/kT_{solar}]-1\}$) in Eq. (7). Since we neglect the dependence on θ and φ, the free energy flux of solar radiation, distributed all over the spherical surface of the planet, is expressed as

$$\frac{1}{4}\downarrow F_s^{in} = -\frac{\pi k^4 T_{solar}^4}{2c^2h^3}\int_0^\infty dx\, x^3 \frac{\ln\left\{\frac{\alpha}{e^x-1}+1\right\}}{\ln\left\{1+1/\left(\frac{\alpha}{e^x-1}\right)\right\}}. \quad (19)$$

Here, as in Eq. (18), $x = h\nu/(kT_{solar})$; $T_{solar} = 5785$ K.

As is seen from Eqs. (18) and (19), the planet-imported free energy flux $\downarrow\Delta F$ (12) is a function of α and u, which, in their turn, depend on the magnitude of solar constant on the given orbit I_s (for the Earth $I_s \approx 1370$ W/m^2), defined by the squared ratio of the Sun radius to the distance from the Sun to the orbit r_{sun}^2/L_{es}^2, the planet albedo A (for the Earth $r_{sun}^2/L_{es}^2 = 0.0000215$, $A \approx 0.3$), and on the grey atmosphere optical depth τ (for the Earth with grey atmosphere[24] $T_e = 255.2$ K, $T_s = 288.2$ K, $T_a = 278.7$ K, and $\tau \approx 0.63$), respectively. Note, that such defined flux of imported free energy $\downarrow\Delta F = \downarrow\Delta F(\alpha,\tau)$ has an extremum with respect to τ, as well as with respect to α under the condition $r_{sun}^2/L_{es}^2 \sim 1$.

The dependence of the free energy flux $\downarrow\Delta F$ (12), imported by the Earth, on the atmosphere optical depth τ is shown in Fig. 1.

The dash line shows the maximally possible flux of free energy, imported by the planet, $\downarrow\Delta F_{max} \approx 63.75$ W/m^2, corresponding to two asymptotic cases: a planet with black surface free of atmosphere ($\tau = 0$) and with black atmosphere ($\tau \to \infty$). Note, that the imported free energy flux has an extremum, i.e., insignificant minimum $\downarrow\Delta F \approx 58.20$ Wt/m^2 at $\tau \approx 0.91$. The point corresponds to the value of free energy flux, imported by the Earth, $\downarrow\Delta F = 58.41$ W/m^2 at current $\tau = 0.63$ (according to Eq. (15)) in the model of grey Earth's atmosphere.[24]

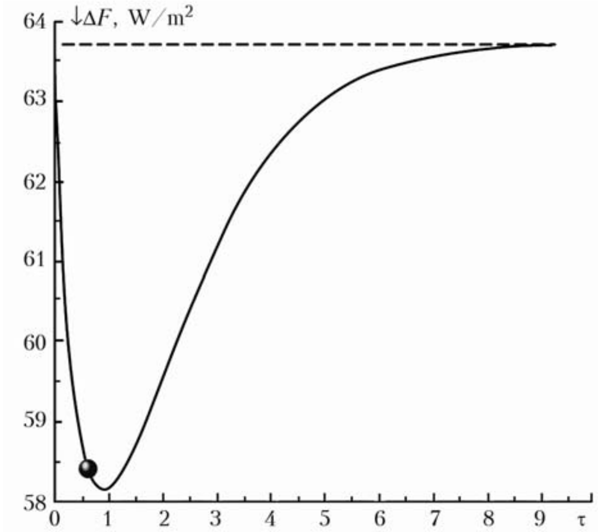

Fig. 1. The planet-imported flux of free energy from space (the distance from the Sun and the planet albedo correspond to the Earth) in the model of grey atmosphere.

In terms of instrumentally measured spectral brightness, the average photon number $<n_\nu> = c^2R_\nu(\theta, \varphi)/2h\nu^3$ [Refs. 6 and 8], where $R_\nu(\theta, \varphi)$ is the spectral brightness of radiation towards the zenith θ and azimuth φ angles, $W/(m^2 \cdot cm^{-1} \cdot sr)$.

The spectral brightness of radiation in the atmosphere R_ν can be modeled with the use of the FIRE-ARMS software package[25] (http://remotesensing.ru) and other similar packages, as well as measured instrumentally. General equations (5) and (7) are convenient both for simulation of free energy fluxes in the atmosphere and for calculations of radiation free energy balance in the atmosphere on the base of experimental data for $R_\nu(\theta, \varphi)$. The applicability of Eqs. (5) and (7) to description of free energy fluxes and Eq. (4) – to the outgoing thermal radiation entropy in the real Earth's atmosphere is confirmed by the behavior of spectral brightness of the outgoing thermal radiation (Fig. 2) and the characteristic behavior of temperature weight functions.

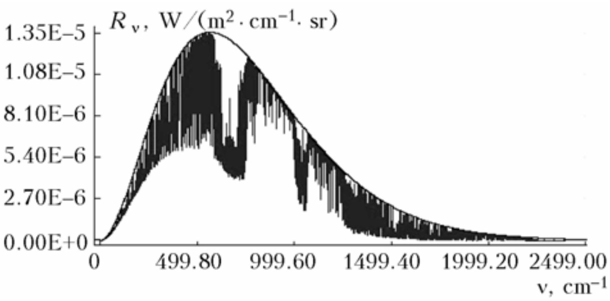

Fig. 2. Spectral brightness of Earth's radiation heat flux in the model of standard atmosphere US Standard. The envelope curve corresponds to the spectral brightness of the surface radiation (in terms of Planck) at $T_s = 288.2$ K.

It is evident that the multi-equilibrium photon statistics adequately reflects the real photon statistics of the Earth's radiation heat flux in all spectral ranges, where its spectral brightness is close to that of the surface radiation, i.e., in atmospheric transparency windows and micro-windows, as well as in the regions of saturation of absorption bands, where the brightness radiation temperature is approximated by the equilibrium temperature of radiating atmospheric layer with a sufficient accuracy.

In spectral regions of CO_2, O_3, and H_2O bands, where the unsaturated absorption is present (absorption line slopes), the level of approximation of the photon statistics to the multi-equilibrium distribution function is determined by the behavior of weight functions relative to temperature, particularly, by the level of accuracy of the outgoing radiation brightness temperature approximation to the temperature of the atmospheric layer, maximally contributing to radiation in given spectral range.

Characteristic behavior of the temperature weight function in the model of standard atmosphere characterizes the R_ν sensitivity (see Fig. 2) beyond atmospheric windows (like, e.g., in the main CO_2 absorption band and[26,27]) to variations of the troposphere temperature at different heights. This implies that the troposphere contribution in the observed parameter R_ν at each frequency is usually determined by one narrow layer (2–3 km) characterized by its average temperature with an acceptable accuracy.

Such behavior of temperature weight functions implies a possibility to consider the formally defined brightness temperature of the outgoing thermal radiation $T_\nu = \dfrac{h\nu}{k\ln(1+2h\nu^3/c^2R_\nu)}$ as close to the equilibrium temperature of the radiating tropospheric layer or the surface (in transparency atmospheric windows), determining the parameter R_ν, almost throughout spectral range 0–2500 cm^{-1}. In this case, the photon number distribution function of real atmosphere thermal radiation can be approximated to multi-equilibrium distribution function (2) with the same accuracy, and equations (5) and (7) can be used in calculation of the free energy balance at the upper boundary of the atmosphere from the satellite measurements of R_ν. Estimate of the free energy flux $\downarrow\Delta F$, imported by the Earth, by Eqs. (10) and (11) within the model of standard atmosphere and with accounting for 50% cloudiness, using the HITRAN[28] data, yields $\downarrow\Delta F = 59.85$ W/m². The above-considered model of grey atmosphere gives the close value: $\downarrow\Delta F \approx 58.41$ W/m² (see Fig. 1).

Measurements of incoming solar radiation with ground-based instruments, sufficiently covering the globe, pose major technical problems, while the planet-reflected solar radiation can be measured all over the globe from satellites. Hence, the incoming flux of solar radiation free energy $(1/4)\downarrow F_s^{in}$ can be defined as the difference between the constant flux of free energy of the solar radiation, reaching the planet, $(1/4)\downarrow F_s$ and the free energy of the reflected solar radiation $(1/4)\uparrow F_s^{reflected}$, supposing $\langle n_\nu \rangle = (r_{sun}^2/L_{es}^2)\langle n_\nu \rangle^{solar}$ in calculations of $(1/4)\downarrow F_s$ and $\langle n_\nu \rangle = (r_{sun}^2/L_{es}^2)\langle n_\nu \rangle^{solar} A$ – for the flow $(1/4)\uparrow F_s^{reflected}$. However, in general case, the fulfillment of the equality

$$(1/4)\downarrow F_s^{in} = (1/4)(\downarrow F_s - \uparrow F_s^{reflected}) \quad (20)$$

is not evident.

Figure 3 shows the comparison of values for $(1/4)\downarrow F_s^{in}$ and $(1/4)(\downarrow F_s - \uparrow F_s^{reflected})$, calculated for the Earth at different albedos A and a constant flux of the free energy from the Sun $\downarrow F_s = -95.48$ W/m^2.

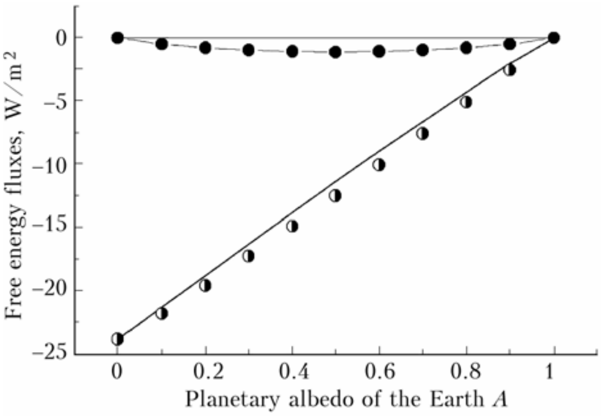

Fig. 3. Free energy fluxes at the upper boundary of the atmosphere. The line designates the free energy flux of solar radiation $(1/4)\downarrow F_s^{in}$, having passed the upper boundary. Semi-dark circles correspond to the difference between fluxes of solar radiation coming to the upper boundary and reflected from it $(1/4)(\downarrow F_s - \uparrow F_s^{reflected})$. and the dark circles correspond to the difference between them $(1/4)\downarrow F_s^{in} - (1/4)(\downarrow F_s - \uparrow F_s^{reflected})$.

It follows from Fig. 3, that the incoming flux of the solar radiation free energy for the Earth $(1/4)\downarrow F_s^{in}$ can be calculated with sufficient accuracy (the error is less than 1 W/m^2 for the Earth albedo between 0.25 and 0.3) from the space-measured R_ν of planet-reflected sunlight using Eqs. (7) and (20) and supposing $\langle n \rangle_\nu = c^2 R_\nu/(2h\nu^3)$ and constant $(1/4)\downarrow F_s = -23.87$ W/m^2. Since the overlapping region of spectral brightness of the reflected solar and outgoing thermal radiations is negligibly small (Fig. 4), their fluxes of free energy can be calculated by Eq. (7) independently.

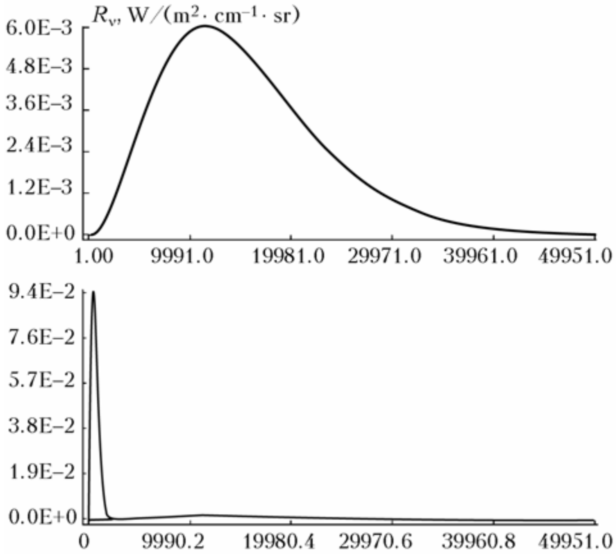

Fig. 4. Spectral brightness of the Earth-reflected solar radiation (upper figure). Solar radiation is approximated by the blackbody radiation with the temperature $T_s = 5785$ K, the Earth albedo is considered as frequency independent and equal to 0.3. Bottom figure shows the spectral brightness of cumulated Earth irradiation into the outer space, including reflected solar radiation (weak right peak) and proper thermal radiation of the Earth in the blackbody model at $T_e = 255.2$ K (sharp left peak).

Conclusion

It is known, that the model of grey atmosphere sufficiently well describes qualitative characteristics of thermal condition of real atmosphere and partially – its quantitative characteristics.[24] In addition, radiation propagating through the grey atmosphere is accurately described by the multi-equilibrium photon statistics.[5,8] The behavior of temperature weight functions of R_ν for outgoing thermal radiation in models of standard atmospheres confirms the assumption on the closeness of the photon statistics of outgoing thermal radiation in the real atmosphere to the multi-equilibrium statistics. This justifies the use of the method, suggested in this work, in estimating the radiation free energy balance at the upper boundary of the Earth atmosphere.

The flux of free energy, imported by the Earth, is assessed in the model of grey atmosphere as $\downarrow\Delta F \approx 58.41$ W/m^2; the model assessment of standard atmosphere with the use of HITRAN data under 50% cloudiness gives a close result: $\downarrow\Delta F = 59.85$ W/m^2. The two-model-averaged assessment for the Earth equals to 59.13 W/m^2. As a result, the integral power of the Earth heat engine (a 6400 km radius), which is approximately equal to $3\cdot 10^{16}$ W, is mainly consumed on the maintenance of processes of general atmospheric and ocean circulation; photosynthetic growing of biomass; as well, the power is partially stored up in thermal reservoirs of the atmosphere and ocean in the form of chemical energy in biota. The flux of the coming to the Earth total energy of solar radiation is about 240 W/m^2 [Refs. 23 and 24], where the fraction of free energy (59.13 W/m^2) is about 25%.

The model of grey atmosphere, considered in this work, is quite simple but nontrivial, because it takes into account main physical laws of the Earth radiation heat exchange. As is shown above, accounting for the frequency dependence of spectral brightness of the outgoing thermal radiation within the model of standard atmosphere gives a little higher assessment of the imported free energy flux as compared to the model of the equivalent grey atmosphere (the difference is 1.44 W/m^2). Naturally, there arise questions: how the value of free energy flux, imported by the Earth, will change, if the spectral brightness of planetary albedo will be taken into account as well; and how the parameters of free energy minimum will behave when going from the model of grey atmosphere to the model of standard atmosphere? The analysis, carried out with different spectral dependences for the same radiation energy flux, shows that the accounting for the spectral dependence results in some decrease of absolute value of free energy flux of this radiation in contrast to radiation of the equivalent grey atmosphere. As a result, a difference, greater than 1.44 W/m^2, is to be expected between the free energy flux, assessed in the model of grey atmosphere and with the flux assessed with accounting for the real spectral dependence of both the outgoing thermal radiation and the reflected solar one.

Thus, the problem of modeling the free energy balance at the upper boundary of the atmosphere with realistic annually-average spectral parameters of the Earth planetary albedo and its comparison with the flux of imported free energy, obtained from satellite measurements of brightness in the whole spectral range of outgoing thermal and reflected solar radiation is a subject of future investigations. From the gnoseological point of view, the model of grey atmosphere includes an important parameter – the optical depth of equivalent grey atmosphere, in contrast to the model of standard atmosphere. This parameter is easily calculated for any real atmosphere and can serve an integral indicator of optimality of the regime of free energy import to a planet from the outer space through the upper boundary of its atmosphere at any spectral characteristics of both outgoing thermal radiation and the planetary albedo.

The intensity of general atmospheric and ocean circulation is two times higher that those of photosynthesis[7] and can be neglected in this case; hence, the ratio of the obtained free energy flux imported to the Earth can be considered as the efficiency (about 25%) of the heat engine of our planet. This magnitude can be obtained from satellite-recorded spectra of the Earth radiation heat flux and the reflected solar radiation in the whole spectral range from microwave to UV and within the range of observation angles from –90 to +90° with respect to nadir.

It should be also noted that simultaneous satellite monitoring of the radiation balance and free energy balance at the upper boundary of the Earth atmosphere can be a useful instrument for detecting possible changes in integral power of the Earth heat engine and its efficiency in the global warming.

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References

1. G.S. Golitsyn and I.I. Mokhov, Izv. Akad. Nauk SSSR, Fiz. Atmos. Okeana, No. 14, 271–277 (1978).
2. G. Nicolis and C. Nicolis, Quart. J. Roy. Meteorol. Soc. **106**, No. 450, 691–706 (1980).
3. G.W. Paltridge, Quart. J. Roy. Meteorol. Soc. **107**, No. 451, 531–547 (1981).
4. C. Essex, Astrophys. J. **285**, No. 41, 279–293 (1984).
5. G.B. Lensins, J. Atmos. Sci. **47**, No. 6, 795–803 (1990).
6. G.L. Stephens and D.M. O'Brien, Quart. J. Roy. Meteorol. Soc. **119**, No. 121, 121–152 (1993).
7. V.G. Gorshkov, *Physical and Biological Grounds for Life Stability*, K.S. Losev, ed. (Moscow, 1995), 470 pp.
8. R. Goody and W. Abdou, Quart. J. Roy. Meteorol. Soc. **122**, No. 123, 483–494 (1996).
9. M.N. Izakov, Usp. Fiz. Nauk **167**, No. 10, 1087–1094 (1997).
10. R. Goody, Quart. J. Roy. Meteorol. Soc. **126**, No. 565, 1953–1970 (2000).
11. C. Warner, J. Atmos. Sci. **62**, No. 2, 1588–1600 (2005).
12. L.D. Landau and E.M. Lifshitz, *Statistical Physics,* 2 Vols, 3rd, ed. (Pergamon Press, Oxford, 1980).
13. I.P. Bazarov, *Thermodynamics* (Vysshaya Shkola, Moscow, 1991), 376 pp.
14. D. Kondepudi and I. Prigogine, *Modern Thermodynamics. From Heat Engines to Dissipative Structures*, O. Jacob, ed. (John Wiley & Sons, 1999), 489 pp.
15. R.L. Stratonovich, *Theory of Information* (Sov. Radio, Moscow, 1975), 424 pp.
16. R. Loudon, *The Quantum Theory of Light* (Clarendon Press, Oxford, 1973).
17. M.O. Skulli and M.S. Zubairy, *Quantum Optics* (Cambridge University Press, Cambridge, 1997), 624 pp.
18. M. Planck, *Heat Radiation* (Translated in Dover Ed., 1959), 496 pp.
19. V.I. Zakharov and Vl.G. Tyuterev, Laser and Particle Beams, No. 5, 27–42 (1987).
20. V.P. Shlyaikh, *Quantum Optics in Phase Space* (Fizmatlit, Moscow, 2005), 361 pp.

21. P. Rosen, Phys. Rev. **96**, No. 3, 555–556 (1954).
22. A. Ore, Phys. Rev. **98**, No. 4, 887–888 (1955).
23. Yu.M. Timofeyev and A.V. Vasil'iev, *Theoretical Foundations for Atmospheric Optics* (Nauka, St. Petersburg, 2003), 474 pp.
24. L.T. Matveev, *The Theory General Atmospheric and Climat Circulation* (Gidrometeoizdat, Leningrad, 1991), 149 pp.
25. K.G. Gribanov, V.I. Zakharov, S.A. Tashkun, and Vl.G. Tyuterev, J. Quant. Spectrosc. and Radiat. Transfer **68**, No. 4, 435–451 (2001) (http://remotesensing.ru).
26. K.G. Gribanov and V.I. Zakharov, Atmos. Sci. Lett. **5**, Is. 1–4, 1–11 (2004).
27. H. Kobayashi, ed. *Interferometric Monitor for Greenhouse Gases: IMG Project Technical Report/IMG Mission Operation & Verification Committee* (CRIEPI, Tokyo, Japan, 1999), 52 pp.
28. L.S. Rothmann, D. Jacquemart, A. Barbe, D.C. Benner, M. Birk, L.R. Brown, M.R. Carleer, C. Chackerian, Jr., K. Chance, L.H. Coudert, V. Dana, V.M. Devi, J.M. Flaud, R.R. Gamache, A. Goldman, J.M. Hartmann, K.W. Jucks, A.G. Maki, J.Y. Mandin, S.T. Massie, J. Orphal, A. Perrin, C.P. Rinsland, M.A.H. Smith, J. Tennyson, R.N. Tolchenov, R.A. Toth, J.V. Auwera, P. Varanasi, and G. Wagner, J. Quant. Spectrosc. and Radiat. Transfer **82**, No. 5, 5–44 (2003).