INTENSITY FLUCTUATIONS IN EXPERIMENTS OF LOCATION AN ARRAY OF CORNER-CUBE REFLECTORS THROUGH THE TURBULENT ATMOSPHERE

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In this paper an analysis of the variance and probability density of the intensity fluctuations of a spherical optical wave reflected from a two-dimensional array of corner-cube reflectors is presented. Depending on the intensity of the turbulence and the pathlength, the relative fluctuations of the wave intensity tend to saturate at a level which corresponds to that of a focused light beam whose initial diameter equals the diameter of the reflector array. In the region of weak intensity fluctuations their probability distribution in the fading range strongly differs from the lognormal, being in much better agreement with the universal law. In the case of saturated fluctuations the experimentally obtained histograms are well approximated by the universal and K-distributions. The experimental values of the variance differ substantially from the calculated ones[7].

Various statistical characteristics of the intensity fluctuations 'of an optical wave reflected from a corner-cube reflector have been investigated both theoretically and experimentally in many studies (see, e.g., 1–4). In practice, the case of optical wave reflection from an array of corner-cube reflectors is of greater interest, since such array are used, for example, in the ranging of space objects[5]. In Ref. 6 the spatial structure of the intensity distribution was analyzed not taking into account the influence of atmospheric turbulence, in application to laser satellite ranging. Characteristics the fluctuations of the of radiation intensity and flux under the influence of atmospheric turbulence and interference of the waves reflected from the individual reflectors of the corner-cube reflector array were investigated in[7–8]. Comparison of the model calculations[7] and the experimental results obtained in Ref. 8 using corner-cube reflectors installed on the discrepancy, which GEOS-4 revealed significant discrepancies. These results were explained by the authors of Ref. 8 as being due to a smoothing effect of the recording- device on the magnitude of the fluctuations due to insufficient width of the frequency band of the receiver electronics.

In this paper we present an analysis is presented of experimentally obtained intensity fluctuations of a directed spherical wave reflected from a two dimensional array of corner-cube reflectors at different levels of atmospheric turbulence. The experimental data are compared with calculated data from Ref. 7. It is suggested in Ref. 7 that the structure function of the phase fluctuations across the diameter of the reflector be used to describe the dependence of the magnitude of the intensity fluctuations on the path-length and the level of atmospheric turbulence. The analysis has shown that, depending on the structure function of the phase fluctuations, the intensity fluctuations saturate at a level, which corresponds to that for a focused beam. The probability density of the intensity fluctuations as a function of the atmospheric turbulence level is also analyzed in the paper and the comparison of the K and universal distribution laws is made, it is shown that saturated intensity fluctuations can be well approximated by these laws.

The experimental study has been carried out using the measurement technique and equipment described in Refs. 9–10. The transmitter emitted a directed spherical wave of the parameter greater than 100. The reflector was installed at distances L = 500 and 1250 m from the transmitter. The reflector was assembled of 12 individual corner-cube reflectors in the form of almost hexagonal in shape two dimensional array. The diameter of the whole array $2R$ = 12.5 cm, the diameter of the individual reflector being about 2.6 cm. The arrangement of the corner-cubes in the array was close to hexagonal. The corner-cube reflectors were of high quality and characterized by small deviations of the dihedral angle at their apex from a right angle which did not exceed 8 for two of them and 2″ for the other four. Since the reflecting surfaces of the reflectors were metalized, one can consider their reflecting properties to be close to those of a hypothetical plane mirror[11] directing incident radiation back to the source. The size of an individual corner-cube reflector is smaller than the diameter of the first Fresnel zone already at a distance of 500 m. Polarizational selection of received radiation was not used in our experiments. The diameter of the diaphragm installed in front of the receiver was approximately 0.5 mm.

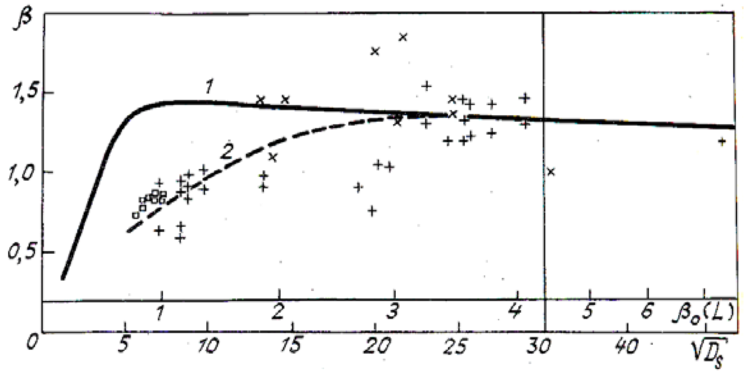

FIG. 1. Normalized rms fluctuations β of the Intensity as a function of the phase difference structure function $D_s(2R,\ L)$; × represents the date obtained in 1986 at the path of 1250 m length, + in 1983, L = 1250 m; □ in 1983, L = 500 m.

Figure 1 presents the normalized rms values of the intensity fluctuations $\beta^2[\langle I^2\rangle / \langle I\rangle^2] - 1$ (here $\langle\ldots\rangle$ denotes temporal averaging) as a function of the phase-difference structure function of the spherical wave $D_s(2R,\ L)$ for the spacing equal to the array diameter.

$$D_s(2R, L) = 1.1\ C_n^2\ k^2(2R)^{5/3},$$

where k is the wave number and C_n^2 is the structural characteristic of the refractive index. Also presented in Fig. 1 are the values of the parameter $\beta_0^2 = 1.21 C_n^2 k^{7/6} L^{11/6}$, which characterizes, the propagation conditions. Solid curve 1 in the Figure represents the dependence of β on the value of $D_s(2R,\ L)$ obtained in[12] for the focused beam on a direct propagation path. Dashed curve 2 represents an empirical (by eye) approximation of the experimental data. The choice of the parameter $D_s(2R,\ L)$ as a value on which the flicker index β depends is based on following considerations. Let a spherical wave source be placed at the point 0 (Fig. 2) and a reflector (plane mirror or a corner-cube reflector array) at a distance L from it. In the case of the plane mirror the structure of the reflected wave is not changed, compared with that of the incident one (the rays 1 and 1′ in Fig. 2), whole in the case of the corner-cube reflector array it will be substantially modified. Indeed, each corner-cube reflector transforms the phase of the reflected beam according to the formula $\psi_{refl}(r_n) = -\psi_{ins}(r_n)$, where r_n is the radius vector of the optical apex of the n-th corner-cube reflector (rays 2 and 2′ in Fig. 2). This results in the division of the wave into several beams each of which is reflected back to the source. The size d of a single beam at the plane of the source O is determined by the diffraction at an individual reflector. In other words, in the case of the reflection of spherical waves from a -compact array of corner-cube reflector array this array can be considered as a self-focusing system with the focal length f coinciding with the pathlength L, the size d of the spot at the focus being determined by the diffraction on an individual reflector and the level of atmospheric turbulence along the path. In fact, in the experiments the visual size of the spot in the receiver plane was approximately twice the size of an individual reflector for a path of 500 m in length and 2.5 times this size for a path of 1250 m. As can be seen from the data presented in Fig. 1. the intensity fluctuations are saturated at β = 1.3 to 1.4, practically the same as in the case of the focused beam[12] but with substantially larger values of the parameter $D_s(2R,\ L)$. This is probably due to the fact that the corner-cube reflector partially inverts the wavefront of the incident wave. That, in turn, leads to a slower increase of the fluctuations with increase of the parameter $D_s(2R,\ L)$ parameter than in the case of direct propagation. Note that in this case the fluctuations are weaker than in the case when the spherical wave is reflected from a specular disc, and, in contrast to the case of direct propagation, the saturation of fluctuations is monotonic, i.e., the saturation curve has no maximum followed by a decrease off of the fluctuation intensity.

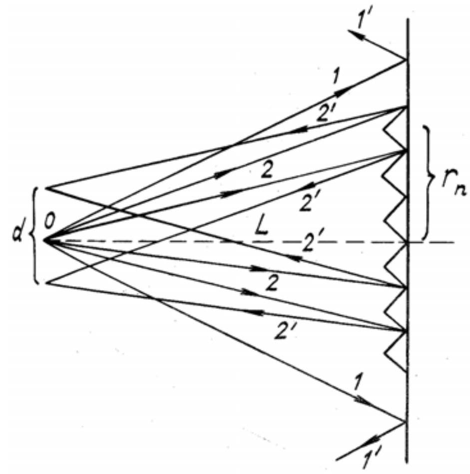

FIG. 2. Illustration of some peculiarities in reflection of a spherical wave from an array of corner-cube reflectors compared to the case of reflection from a place mirror.

The coincidence of the β values observed for paths differing in lengths by a factor of 2.5 confirms the usefulness of choosing the parameter $D_s(2R,\ L)$ as the value determining the level of the intensity fluctuations. The large scatter of the experimental data for the long path, which substantially exceeds the corridor of possible measurement errors, is probably caused by the intermittence of the atmosphere[17] as well as by variation of the inner scale of atmospheric turbulence along the path. It should be- rioted at the same time that thirty points obtained for the path length of 500 m occupy only a narrow region in the figure, for which reason we could show only some of them.

The spatial distribution of the flicker index β within the limits of a diffraction-limited beam reflected from an individual corner-cube reflector is practically constant. This means that random wanderings of the beam reflected from the corner-cube reflectors do not materially contribute to the intensity fluctuations in the saturation region.

In Ref. 7 (Eq. (21)) one can find a formula for β^2 based on qualitative considerations, which has the form

$$\beta^2 = 6\ \beta_A^2 + 2\ (\beta_A^2)^2, \tag{1}$$

where $\beta_A^2 = \exp\,[4 \cdot 0.4\ \beta_0(L)] - 1$ is the relative variance of the intensity fluctuations of spherical wave, calculated using the smooth perturbation technique. One can see from Fig. 1 that calculations made using this expression overestimate the fluctuation level. This overestimation by formula (1) with respect to experimental data was discussed in Ref. 8, but in this paper it was "explained" by a smoothing effect of the recording system on the values of β due to limited frequency bandwidth of the electronics.

More information on the fluctuation process can be obtained from an analysis of probability density of the instantaneous magnitudes of the fluctuations. Figures 3-5 show typical histograms of the instantaneous intensity values of I for different values of the flicker parameter and the phase structure function D (2R, L). For comparison, the model probability densities are also shown in this Figure, i.e., the lognormal distribution[12]:

$$P(I) = (2\sqrt{2\pi\sigma}\,I)^{-1} \exp\,[1/2\ \sigma^2(\ln I - \xi)^2]; \tag{2}$$
$$\sigma = \ln\,(1 + \beta^2);\ \xi = \ln \langle I\rangle\ /\ (1+\beta^2)^{1/2};$$

the K-distribution[13]:

$$P(I)\langle I\rangle = \frac{2}{\Gamma(y)}\ y^{(y+1)/2} \times I^{(y-1)/2}\ K_{y-1}[2(Iy)^{1/2}]; \tag{3}$$
$$y = 2\ (\beta^2-1),\ y > 0,$$

where $K_\nu(z)$ is the McDonald function[16], and the universal law[14]

$$P(I) = 1/2 \int_0^\infty z\ J_0(z\sqrt{I})\ {}_1F_1(M,\ 1;\ -\frac{cz^2}{4M})$$
$$\times\ {}_1F_1(m,\ 1;\ -\frac{bz^2}{4m})\ dz, \tag{4}$$

where ${}_1F_1(\alpha,\ \beta;\ z)$ is the confluent hypergeometric function[16]. The K distribution satisfactory describes the experimental data in the region of saturated fluctuations[9], while the distribution (4), according to Ref. 14 describes, probability density of the intensity over the entire range of β values. It should be emphasized, however, that these conclusions were based entirely on an analysis of the highest normalized moments (up to fifth order) not taking into account for their shifts estimated from the experimental data[15]. The shape of distribution (4) depends on the ratio r of the amplitude of the direct component of the wave $\langle A^2\rangle = c$ to that of the multiply scattered component $\langle R^2\rangle = b$ ($r = c/b$).

In the experimental study just discussed the parameters M and m of the distribution and the ratio r were determined by a technique presented in Ref. 14, in which one solves three equations derived from the expression for the moments of the intensity:

$$\langle I^n\rangle = (b/m)^n \sum_{k=0}^{n} C_n^k\ \frac{\Gamma(M+K)\ \Gamma(m+n-k)}{\Gamma(M)\ \Gamma(m)}\ (\frac{rm}{M})^k,$$
$$n = 1,\ 2,\ 3,\ \ldots\ . \tag{5}$$

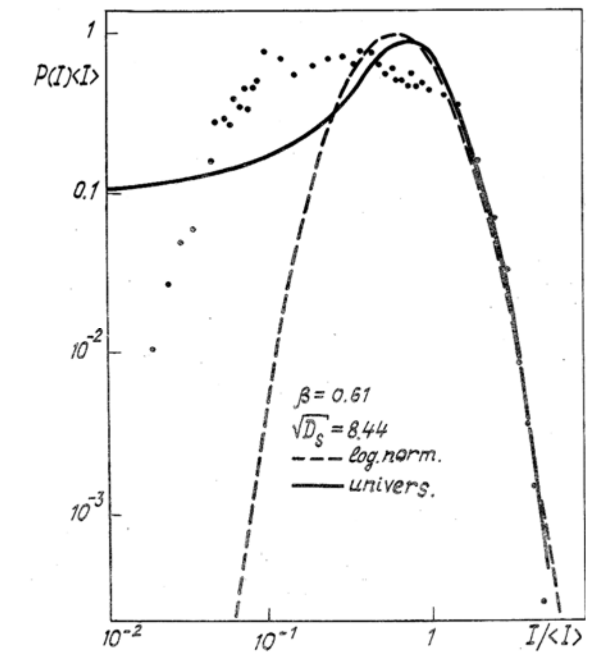

FIG. 3. Comparison of the histogram P(I) of the normalized intensity I/<I> values with the lognormal (2) and universal (4) laws for weak fluctuations (β = 0.61).

We has analyzed the histograms of more than 50 realizations obtained for 500 m and 1250 m long paths and the most typical of them were compared with distributions (2)–(4). As can be seen from Fig. 3 already for weak fluctuations (β < 1) which for reflection from a plane mirror are well approximated by the lognormal distribution[9], a much greater probability of deep fadings is observed in our case. In the case of spikes

($I/\langle I\rangle > 1$) the experimental data are in good agreement with distributions (2)–(4). In the region of fadings the universal law approximates the experimental results better than does the lognormal law although the discrepancies are large, which renders this model inapplicable for the approximation of experimental data in application where the statistics of fadings is important. In the region of saturated fluctuations the experimental data are well approximated by distributions (3) and (4) throughout the intensity range observed in the measurements. Here the K distribution is preferable compared to the universal, being simpler and depending on smaller number of parameters.

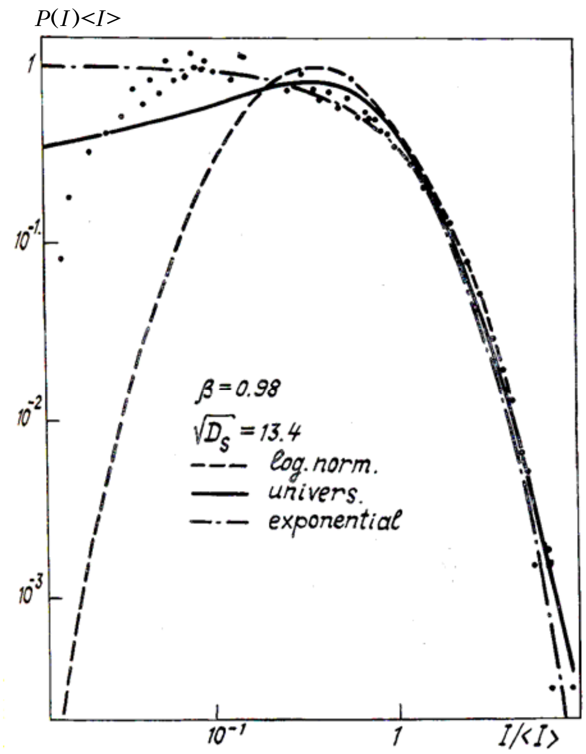

FIG. 4. Comparison of the histogram $P(I)$ of the normalized intensity values $I/\langle I\rangle$ with the distributions (2), (4) and with the exponential one at $\beta = 1$.

As was already noted, the field of spherical wave reflected from a corner-cube reflector array is a superposition of partial waves from individual reflectors. Since large enough number of corner-cube ref lectors used in the experiment is rather large (twelve) one may consider the conditions of the central limit theorem to be fulfilled. The application of this theorem results in the Rayleigh distribution of the total intensity and hence in the exponential intensity distribution. But, in fact, such considerations are not valid in the case. Actually variations of the interference picture in the observation plane are first of all caused by the phase fluctuations, which are still strongly correlated at a distance compared to the size of array[12]. As a consequence, the components of the reflected wave cannot be considered to be independent in this case. This explains why the fluctuations saturate at a level considerably higher than unity, and the fact that the histogram of experimental data strongly differs from the exponential distribution in the region of fadings ($I \ll \langle I\rangle$) at $\beta = 1$.

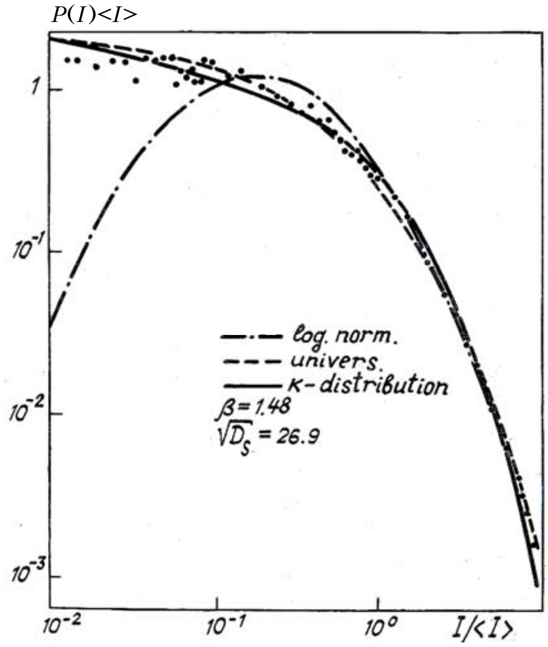

FIG. 5. Comparison of the histogram $P(I)$ with the distributions (2) to (4) in the region of saturated fluctuations ($\beta = 1.48$).

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