SIGNAL IDENTIFICATION METHOD FOR AIRBORNE HYDROOPTICAL LIDAR

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An original method for the solution of the inverse problem of laser sensing of the upper ocean layer which has a multilayered light scattering structure is considered. It is suggested that a parametric model of the lidar return be used instead of a discrete representation. A modified version of the Gauss–Newton method is proposed for the estimation of the model parameters which describe the depth of occurrence and reflectivity of the underwater layers. The advantages of this modification are high stability and small errors of the solution.

When dealing with problems of laser sensing of atmospheric formations one is often faced with a wide range of variation of the optical parameters. A number of approaches to the inversion of the lidar equation have been developed to solve this problem. Things are quite different for laser sensing of the ocean, especially from airborne platforms. A formulation of the lidar equation suitable for practical application is impeded by the high optical thickness of ocean water and the greater number of light scattering parameters compared to case of the atmosphere. Attempts made so far have addressed specific cases (e.g., Ref. 1). At the same time, the range of variation of the optical properties of water in the open ocean is smaller than the range of variation of the parameters of the atmosphere. Therefore, in many cases of scientific interest advantageous use is made of a simple model of the lidar return.[2] From a physical point of view, this model includes three components: the reflection from the air—water boundary; rapidly decaying signals scattered by marine hydrosols and water molecules within a nearly uniform water depth; pulses with a moderately high rate of rise due to scattering by underwater anomalies. This model can describe a layer of enhanced particle number density at the depth of the temperature discontinuity, a "cloud" of phytoplankton, a scool of fish, and, finally, the sea bottom. The depth of occurrence of such an anomaly detected by a lidar can vary from a few meters up to several tens of meters. These factors simplify the modeling of the signal and facilitate further computation using the parameters of the model rather than the signal itself.

Two more problems arise in the sensing of the ocean which complicate the matter. The depth of penetration of a laser pulse is generally rather small and, given the currently available hardware for signal recording and processing, we have but only a small number of counts. Moreover, the data often fall out due to physical reasons. There are micropatches on the rough ocean surface, which result in glint reflections onto the receiving telescope. As a consequence, the photodetector is overloaded or the ADC operates outside its input dynamic range, which leads to fallouts of valid data.

The procedure for constructing a lidar return (identification) model in hydrooptical sensing of the upper layer of the ocean proposed in Ref. 2 is based on a model of the form

$$S_t = \alpha_1/(1 + \alpha_3(t - \alpha_5)^2) + \alpha_2/(1 + \alpha_4(t - \alpha_6)^2) . \quad (1)$$

Here the origin of the coordinate system is at the air—water interface and the time t is counted from this boundary down to the water depth, α_1 and α_2 are the amplitudes of the first and second pulses, α_5 and α_6 are the positions of their centers, and α_3 and α_4 characterize the slopes of the two pulses. Of special interest are the parameters α_2, α_4, and α_6, which characterize anomalies (the hydrosol layer whith enhanced water turbidity, the ocean bottom, the school of fish, etc.). The parameters $\alpha = (\alpha_1, \alpha_2,..., \alpha_6)$ are determined by the Gauss—Newton method,[2] using individual realizations $S_{t,\,k}$, where k is the number of the lidar return.

For small samplings and data fallouts caused by the above mentioned physical and technical reasons, the Gauss—Newton method frequently results in unstable estimates of the parameters and gross errors. Well known methods, that provide stable statistical inferences and parameter estimates, in particular, directly or indirectly take account of supplemental data on the parameters α (Ref. 3). Two approaches are relevant here: deterministic and probabilistic statistical. The former assumes that the supplemental data are known *a priori* in terms of certain estimates β^* of the parameters of model (1), which have been derived from the results of processing of individual realizations of echo signals similar to the signal being studied. It should be noted that the form of the dependence β^* on the parameter vector α is assigned *a priori*. As rule, a linear dependence is assumed.

The probabililistic–statistical approach assumes that the parameters **a** are random variables whose distribution is well known. The normal distribution with average value β and covariance matrix W is often taken as the *a priori* distribution of the parameter . Under real conditions of airborne lidar sensing, the use of these methods would entail serious difficulties associated with the selection of analogs to the signal being processed, determination of the form of the dependence between β^* and , and ascertainment of the form of the distribution of , etc.

We suggest here a method which is free of the above limitations and allows one to solve the given problem as formulated in a fairly general way.

To estimate the parameters of the model (1), let us examine the two interrelated systems

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$$\tilde{S}_i = \tilde{J}_i = f(t_i, \alpha) + \varepsilon_i,\ i = \overline{1, n}, \tag{2}$$

$$\beta^*_{jk} = \varphi_j(_j) + \eta_{jk}\ ,\ j = \overline{1, m},\ k = \overline{1, l}. \tag{3}$$

System (2) is a discrete model of the lidar return of form (1), where $\boldsymbol{\alpha}$ is the vector of parameters $\boldsymbol{\alpha}_1, \boldsymbol{\alpha}_2, \ldots, \boldsymbol{\alpha}_m$.

System (3) is a discrete model of the supplemental data given in terms of the estimates of the parameters $\boldsymbol{\alpha}_j$, $j = \overline{1, l}$, related to the unknown parameters α by the functions φ_j, $j = \overline{1, m}$. The unknown functions $\varphi_j \in D$, $j = \overline{1, m}$, where $\tilde{D}$ is the space of single–valued functions. The variables ε_i and η_{jk} are random quantities with zero means and finite variances.

The estimates in Eq. (3) can be either values of the parameters of lidar return model (1) obtained for certain types of anomalies under controlled conditions, or the values of these parameters for the backscattered pulse train $\tilde{S}_{tk}$, $k = \overline{1, l}$, calculated during the course of the experiment.

Formally, using models (2) and (3) the estimates $\boldsymbol{\alpha}^*$ of the parameters α at a fixed point $\tilde{\boldsymbol{\alpha}}$ can be represented in the form[4]

$$\boldsymbol{\alpha}^* = \arg\min_{\alpha}(\Phi_) = J + Q_\alpha), \tag{4}$$

where $J_\alpha = \sum_{i=1}^{n}(\tilde{J}_i - f(t_i, \alpha))^2 = ||-F_\alpha||^2$ is the Gaussian error functional, $||X||$ is the norm of x, and

$$Q_\alpha = \sum_{j=1}^{m}\sum_{k=1}^{l} K\left[\frac{\tilde{\alpha}_j - \tilde{\beta}^*_{jk}}{h}\right]\left(\beta^*_{jk} - \varphi_j(\alpha_j)\right)^2$$

is the functional of the total weighted mean square error in the assignments of the supplemented data in Eq. (3). Here, $K(\cdot) > 0$ is a weighting function which takes into account the uncertainty of functions φ_j, $j = \overline{1, m}$ in describing the supplemental data (3) (in analogy with the concept of fuzzy sets). The parameter h indicates the degree of discrepancy between the supplemental data on the parameters and the estimated parameters α_j, $j = \overline{1, m}$, $k = \overline{1, l}$, i.e., the uncertainty in the equality $= \boldsymbol{\alpha}_j$. If a bell–shaped function centered at is chosen as the weighting function $K(\cdot)$, the role of the parameter h is obvious from a physical point of view. The larger the value of h the greater is the discrepancy between and the parameters α_j, and the more inaccurate are the supplemental data. At $h = 0$, the supplemental data are exact.

Problem (4) reduces to the solution of systems of nonlinear equations of the form

$$\frac{\partial \Phi_\alpha}{\partial \alpha_j} = 0,\ \ j = \overline{1, m} \tag{5}$$

and is a rather laborious procedure from the computational point of view. Therefore, as in the Gauss–Newton method[2] it is expedient to reduce nonlinear problem (5) to a set of linear problems by linearizing systems (2) and (3) in the vicinity of the point $\boldsymbol{\alpha}^0$, which is the starting approximation for the parameters α.

The recursion formula for calculating the next approximation of the parameters α is

$$\boldsymbol{\alpha}^{p+1} = \boldsymbol{\alpha}^p + \boldsymbol{\gamma}^p \cdot \Delta\boldsymbol{\alpha}^p,\ \ p = 0, 1, 2, \ldots,$$

$$\Delta\boldsymbol{\alpha}^p = \boldsymbol{\alpha} - \boldsymbol{\alpha}^p,\ \ \gamma^p \equiv 1. \tag{6}$$

Here, the increment ^{p} in each step is calculated by solving of a system of linear equations (SLE) of the form

$$(F^{\mathrm{T}}F + K)^p \Delta\boldsymbol{\alpha}^p = (F^{\mathrm{T}}\varepsilon + \sum_{j+1}^{l} K_j\, \beta^*_j)^p, \tag{7}$$

where $p = 0, 1, 2, \ldots$. In formula (7)

$$(F^{\mathrm{T}}F)^p = \sum_{k=1}^{l}\left(\frac{\partial f(t_k, \alpha)}{\partial \alpha_i}\right)^p\left(\frac{\partial f(t_k, \alpha)}{\partial \alpha_j}\right)^p,\ i, j = \overline{1, m}$$

is the product of the matrices of the partial derivatives of the model lidar return (2) with respect to the parameters α. The subscript p is the step number and indicates that the derivatives are calculated at point $\alpha = \alpha^p$, $p = 0, 1, 2, \ldots$,

$$F^{\mathrm{T}}\varepsilon = \left(\sum_{k=1}^{l}\left(\frac{\partial f(t_k, \boldsymbol{\alpha})}{\partial \alpha_i}\right)^p (Y_k - f(t_k, \alpha)^p,\ i = \overline{1, m})\right)$$

is the column–vector of the free terms.

The functional

$$\sum_{j=1}^{l} K_j \tilde{\beta}^*_j = \left(\begin{matrix}\sum_{j=1}^{l}\sum_{i=1}^{m} K\left[\frac{\tilde{\alpha}_j - \tilde{\beta}^*_{ij}}{h}\right]\left(\beta^*_{ij} - \alpha^p_i\right)\\ i = \overline{1, m}\end{matrix}\right),$$

is the reference column–vector of the free terms which result from taking account of the supplemental data (3).

$$\tilde{K} = \mathrm{diag}\,(\omega_{11}, \omega_{22}, \ldots, \omega_{mm}),$$

$$\omega_{ii} = \sum_{j=1}^{l} K\left[\frac{\tilde{\alpha}^p_j - \tilde{\beta}^*_{ij}}{h}\right],\ i = \overline{1, m}$$

is the diagonal matrix of weights ω_{ii} obtained in the same way as the vector $\sum_{j=1}^{l} K_j \tilde{\beta}^*_j$ by taking Eq. (3) into account

To prove statement (7), it is sufficient to substitute the linearized systems (2) and (3) into the functional (4), to take the derivatives with respect to the vector of parameters $\Delta\boldsymbol{\alpha}$, and to set them equal to zero.

It should be noted that the proposed method (6) for determining the parameters $\boldsymbol{\alpha}$ (Eq. (1)) is more stable than the Gauss—Newton method in that a solution of the obtained

SLE (7) also exists (owing to matrix $\tilde{K}$) in the case of a singular or ill–conditioned matrix. Under certain conditions, the matrix $F^T F + \tilde{K}$ is close to the matrix $F^T F + \beta E$ which has been proposed by A.N. Tikhonov[5] for the regularization of the solution of the SLE ($\alpha > 0$, E is the identity matrix).

Indeed, using as weighting factor K an approximation of the form $K(u_i) = \exp\{-u_i^2\}$ for $(\alpha_i^p - \beta_{jk}^*)/h \gg 1$ and $l = 1$ we obtain $K(u_i) \approx \beta$, $i = \overline{1, m}$, and $K \approx \beta E$, where β is small.

It should also be noted that the problem of choosing the control parameter h in the kernel $K(\cdot)$ is similar to the problem of choosing the regularization parameter β and can be solved by well–known methods[5] both in each step p and once at the start of the iteration (6).

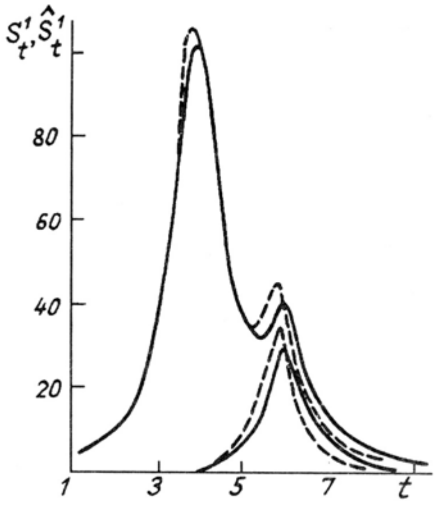

FIG. 1. Reconstruction of lidar returns by the Gauss—Newton method in the presence of underwater light scattering formations. Discrete time t is plotted along the abscissa with a step of 25 ns (2.8 m *depth resolution). The signal intensity is plotted in relative units. The solid line represents the actual lidar signal, the dashed line – the reconstructed signal. There is no cut–off limit on the input signal.*

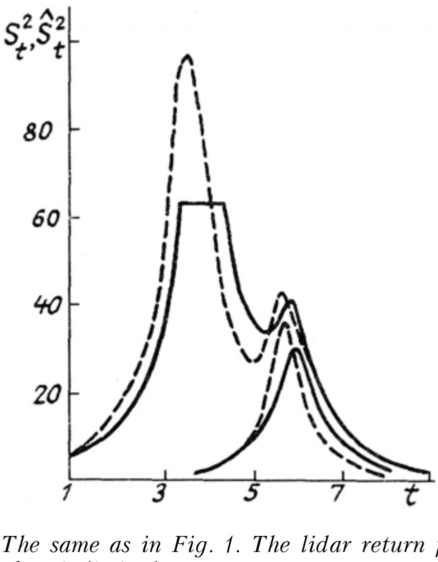

FIG. 2. The same as in Fig. 1. The lidar return from the ocean surface is limited.

Results illustrating the efficiency of the proposed algorithm (6) are shown in Figs. 1–4 and in Table I. They are based on experimental data obtained in the course of different flights, including those described in Ref. 6.

Shown in Fig. 1 and 2 are actual lidar returns (solid line) described by Eq. (1) and their reconstructed values (dashed line) obtained by the Gauss—Newton method. The second spike on the curves for S_t^2 and S_t^1, i.e., the echo pulse just from under the water, characterizes an anomaly. The true behavior of the anomaly described by the right side of Eq. (1) centered at $t_i = 7$ is represented by a solid line (the single spike). The discrete model (*1*) is given in the form of Eq. (2) with $\varepsilon_i = 0$, $t_i = i$, $i = \overline{1, 9}$).

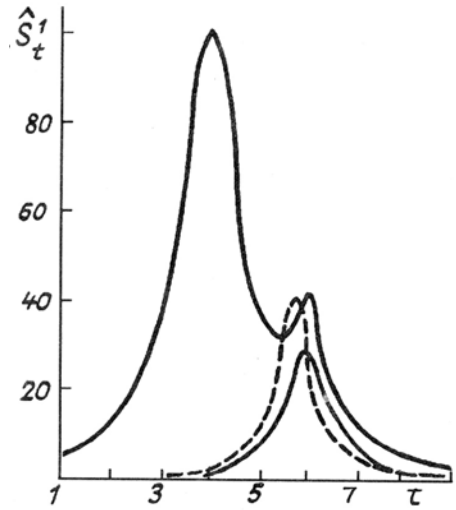

FIG. 3. Reconstruction of lidar returns by the modified Gauss—Newton method taking into account supplemental data Eq. (3). Notation is the same as in Fig. 1. There is no cut–off limit on the input signal.

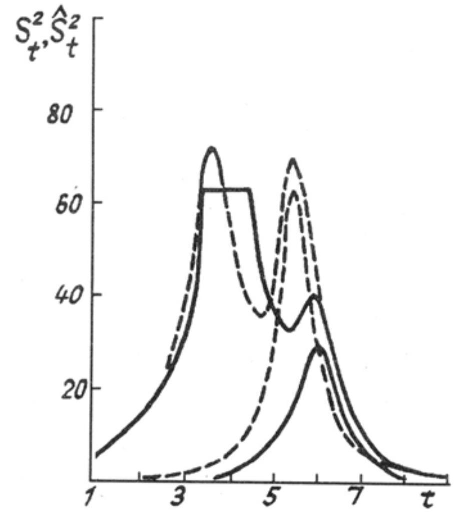

FIG. 4. The same as in Fig. 3. The lidar return from the ocean surface is limited.

Figures 3 and 4 show the values of $\tilde{S}_t^1$ and $\tilde{S}_t^2$ reconstructed by the modified Gauss—Newton method (6) (dashed line) and the actual lidar returns S_t^1 and S_t^1 (solid line).

The approximate values β^*_{jk} of the parameters α_j from Eq. (1) with a 20–30% relative error were employed as the supplemental data (3), i.e.,

$$\beta^*_{jk} = \alpha_j + \alpha_j c\, \eta_k,\ k = \overline{1, 5},\ j = \overline{1, 6}.$$

Here η_k are normally distributed random quantities with zero mean and unit variance $\sigma_k = 1$, $K = \overline{1, 5}$. The parameter c characterizes the noise level, $c \in (0.2–0.3)$.

The second signal $\tilde{S}^2_t$ shown in Figs. 2 and 4 represents the case in which the useful signal is outside the input dynamic range of the ADC and the signal is cut off at the level $S(t) = 63$ units of the ADC code. The cutoff is depicted by a step in the solid line, and in discrete model (2) the cutoff corresponds to an erroneous value of the signal at the point $t_4 = 4$, $S(t_4) = 63$.

It can be seen from Fig. 2 that the fallout of one correct value from the lidar return causes a significant error, especially in the estimation of the amplitude of the lidar return from an anomaly by the Gauss—Newton method. The relative error is greater than 100%.

Fig. 4 shows similar results for the modified Gauss—Newton method (6), using the supplemental data. Here, the error in the estimated amplitude of the lidar return from the anomaly is about 14%.

Relative errors in the reconstructed values of the parameters α_1, α_2, α_5, and α_6, which characterize the pulse amplitudes and the positions of their centers are given in Table I. It can be seen from Table I that the accuracy of the parameters reconstructed by the Gauss—Newton method for a signal with a value of one count is much worse than for the signal S^2_t. The proposed method (6) provides more stable estimates.

TABLE I. Relative errors in the reconstructed amplitudes α_1 and α_2 of the signals S^1_t and S^2_t and the positions of their centers α_5 and α_6 obtained using the Gauss—Newton (GN) method and our modified version of the Gauss—Newton method (GNM).

Method	α_1		α_2		α_5		α_6	
	S^1_t	S^2_t	S^1_t	S^2_t	S^1_t	S^2_t	S^1_t	S^2_t
GN	$4.4\cdot10^{-4}$	0.3	0.37	1.14	$1.6\cdot10^{-4}$	$7\cdot10^{-2}$	$4.2\cdot10^{-2}$	$7.0\cdot10^{-2}$
GNM	$3.9\cdot10^{-2}$	$4\cdot10^{-2}$	0.15	0.14	$1.0\cdot10^{-2}$	0.01	$2.5\cdot10^{-2}$	$2.5\cdot10^{-2}$

Finally, it should be noted that our method of estimating the lidar return parameters makes it possible to take into account supplemental data provides more stable results as compared to the Gauss—Newton method, thus leading to a considerable improvement in the accuracy of the estimated parameters of underwater light scattering formations.

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