ON THE DEFOCUSING OF A LIGHT BEAM UNDER CONDITIONS OF STRONG NONLINEAR RESPONSE OF THE MEDIUM

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Received October 26, 1989

The self-action of a Gaussian beam in a cubic medium under conditions of strong nonlinear response of the medium is modeled numerically. It is shown that in the propagation process the beam acquires a close to hyper Gaussian profile, and as the nonlinearity parameter a increases the intensity on the axis of the beam becomes equal to the maximum intensity and for $\alpha > 5000$ *the maximum intensity and the intensity on the beam axis do not differ by more than* 2%. *The efficiency of the calculations performed by the method of splitting and nonlinear difference schemes are compared. It is shown that the self-action of powerful beams is best calculated by using nonlinear schemes combined with Newton's method as implemented in the paper.*

In works concerning the self-action of optical radiation the characteristics of the defocusing of light beams under conditions of strong nonlinear response of the medium have been studied the least, in spite of the fact this situation is realized in a number of practical problems, such as, for example, the transfer of light energy. The case when the characteristic nonlinearity parameter a was of the order of 200–1000 has been studied in only several works (see Refs. 1–3). This situation is explained by the fact that the numerical modeling of the propagation of such beams presents significant difficulties, for example, large amounts of computer time are required. In this connection, to calculate the self-action of optical radiation in strongly nonlinear media it is necessary to develop efficient numerical methods. In particular, it is necessary to compare the nonlinear difference schemes[4] and the splitting methods[5] currently widely employed in the modeling of nonlinear optical problems.

Such a comparison must be performed, since numerical modeling is increasingly used in nonlinear optics, and more and more investigators are using this technique for the first time. For some problems in nonlinear optics this comparison was performed in Ref. 6, where, in particular, the method of splitting, which differs from the traditionally employed method by the fact that it uses an iterative process and makes it possible to expand substantially its range of application, was proposed and justified. In Ref. 7 different nonconservative schemes for calculating the self-focusing of a beam in a curve Kerr medium were compared and it was concluded that among the methods studied the method of splitting is best. Thus the logical continuation of these works, for the purpose of determining the most efficient method of numerical modeling, in particular, in problems of nonlinear self-action, is to compare the method of splitting and nonlinear schemes. We shall do this for the example of defocusing of a beam in a curve Kerr medium, taking into account the relative simplicity of the organization of the iterative processes and the existence of several invariants of the propagation of optical radiation in such a medium. Therefore the purpose of this work is, first of all, to determine the characteristics of defocusing of a Gaussian beam in a Kerr medium under conditions of strong nonlinear response of the medium. Second, to compare the results of calculations performed using different methods of numerical modeling; and, third, to describe the difficulties arising in the modeling of the propagation of optical radiation under such conditions and possible methods for overcoming them.

The nonlinear propagation of a light beam in a Kerr medium is described by the quasioptic equation

$$\frac{dA}{dz} + i\Delta_{\perp}A + i\alpha|A|^2A = 0 \quad (1)$$

with the boundary condition in the case of a collimated Gaussian beam

$$A\Big|_{z=0} = \exp(-r^2), \quad (2)$$

where A is the complex amplitude of the beam normalized to the peak value; z is the coordinate along which the optical radiation propagats and which is measured in units of the diffraction length $l_d = 2ka^2$, where k is the wave number and a is the initial radius of the beam; Δ is the transverse Laplacian; r is the transverse coordinate normalized to a; and, α is equal to 50, 500, and 5000 (the first two values were employed to compare the different methods of numerical modeling).

0235-6880/90/03 247–04 $02.00

The methods mentioned above were used in the numerical experiments. In addition, for the nonlinear schemes both the method of simple iteration and Newton's method, which has not been used previously for problems in this class, were employed. A variable cubic grid with an initial step size h_{r_0} (near $r = 0$) and N_r points was introduced for the transverse coordinate.

Comparison of the computational results for moderate values of the nonlinearity (for example, $\alpha = 50$) shows that the method of splitting with the same number of points N_r (for example, $N_r = 100$ and $h_{r_0} = 0.02$) requires a smaller (than the nonlinear scheme) step size h_z for integration along z already for paths $z > 0.12$. Thus $h_z = 0.01$ in the section $z = 0.2$ the intensities on the axis and in the aberration ring, determined by both methods, differ by 10%, while the position of the maximum intensity differs correspondingly by 0.11 a. The discrepancy in the values of the beam radii is insignificant.

The advantage of nonlinear schemes is most clearly evident for self-action of optical radiation with $\alpha = 500$. Thus the method of splitting with $h_{r_0} = 0.02$, $N_r = 100$, and $h_z \geq 10^{-3}$ gives qualitatively correct results only on the path $z \leq 0.05$ (nonlinear schemes give correct results on path $z \leq 0.1$). As the path length is increased the previously formed aberration ring disappears and the intensity on the axis starts to increase. It is important to underscore the fact that the integral characteristics, for example, the beam radius, change insignificantly as h_z is decreased, while the intensity on the beam axis depends strongly on h_z. Thus, for example, in the section $z = 2$ for $h_z = 10^{-3}$, 10^{-4}, and 10^{-5} the beam radius is equal to 2.83, 2.98, and 2.99, respectively (the correct value is equal to 4.4), while the peak intensity is equal to 2.36, 0.347, and 0.346. We note that errors appear in the numerical integration owing to the pore approximation of the transverse structure of the beam beyond the section $z = 0.05$: the region of the drop in intensity near the geometric shadow of the aberration ring contains only a few points of the approximation, which results in an overestimation of the intensity and the formation of a tubular beam profile, as well as further focusing of the beam. As n_r is increased up to 300, $h_{r_0} = 0.0066$, the intensity distribution changes right up to the section $z = 0.25$ (calculations were not performed beyond this section): the radius of the ring increases monotonically as z increases and intensity in it exceeds its value on the axis by approximately 10% (see Fig. 1). The distribution of $|A|^2$ within the ring is practically uniform (this effect is observed on path $z \sim 0.05$), and the beam propagates as a hyper Gaussian beam.

Analogous dependence of the solution on the integration steps is also obtained in calculations using the nonlinear scheme. In contrast to the method of splitting, however, the nonlinear scheme gives the correct result on the path $z \leq 0.12$ already for $N_1 = 200$.

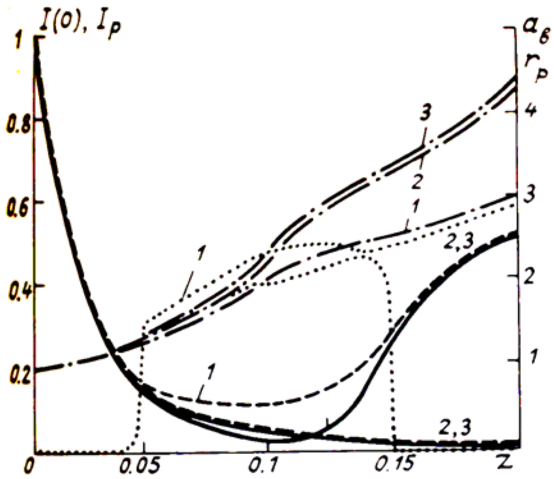

FIG. 1. The intensity on the axis I(0) (solid curves), the peak intensity I_p (dashed curves), the beam radius a_b (dot-dashed curves), and the coordinates of the peak intensity r_p (dotted curves) versus z in calculations by the nonlinear scheme for a = 500 *with* $h_z = 10^{-3}$; h_{r_0} = 0.02, 0.06, *and* 0.005; N_r = 100, 300, and 400 *(curves* 1, 2, *and* 3, *respectively).*

We shall study the case of strongly nonlinear response of the medium $\alpha = 5000$. The calculations were performed up to the section $z = 0.1$. We note that the main characteristics of the propagation of the beam are identical to the preceding case (an aberration ring forms, the radius of the ring increases as z increases, and the distribution within the ring is close to uniform: the difference in the values of the intensity in the section $z = 0.1$ is equal to 2%). However the uniform profile of the beam is formed on shorter paths ($z < 0.015$) and the intensity differential in the aberration ring is much smaller than for $\alpha = 500$. It is important to note that after the path $z = 0.015$ the profile of the beam remains unchanged.

Comparing the calculations by the two methods gives the following results. The calculation performed by the nonlinear scheme up on the path $z = 0.05$ are already correct for $N_r = 600$. The method of splitting, however, gives in this case a much larger aberration ring (1.7 times larger; the correct value is equal to 2.87) and it overestimates by a factor of 1.3 the intensity in it. The method of splitting gives close results (see Fig. 2) only for $N_r = 750$. As the path length is further increased the difference in the ratio of the steps in the transverse coordinate, for which correct results are obtained for the nonlinear scheme and for the method of splitting, increases rapidly.

From the foregoing comparison of the effectiveness of the numerical modeling based on the different methods it follows that the calculations performed by the nonlinear schemes can be performed with a smaller number of points along the transverse coordinate. For the nonlinear scheme, however, an iteration process must be organized. Therefore there arises the question of the relative computer time required (it may be more advantageous to perform calculations with a larger number of points by method of splitting than by the nonlinear scheme). For this reason we compared two iterative methods: simple iteration and Newton's method. As an illustration Fig. 2 (dot-dashed curves) shows the dependence of the number of iterations on z for $\alpha = 500$, $h_z = 10^{-3}$, $h_{r_0} = 10^{-2}$, $N = 200$, and $\varepsilon = 5 \cdot 10^{-3}$ (relative accuracy). It shows that Newton's method at the initial stage of propagation requires 5 times fewer iterations than the simple iteration alone. The number of iterations subsequently decreases to 2–3. Thus the computer time required in the nonlinear scheme combined with Newton's method remains virtually constant compared with the use of the method of splitting for the calculating self-action.

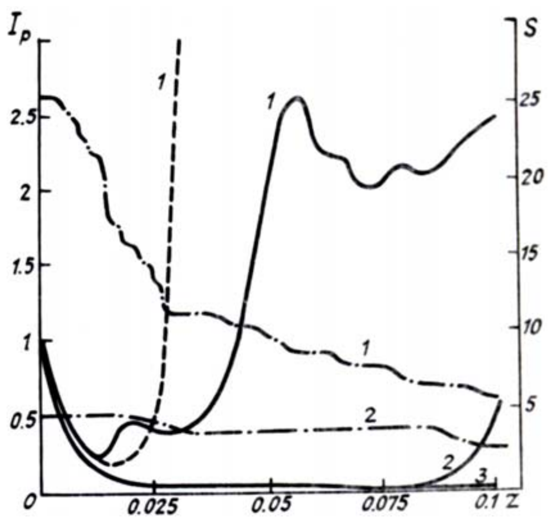

FIG. 2. The peak intensity I_p versus z in calculations by the nonlinear scheme (solid curves), by the method of splitting (dashed cures) for $\alpha = 5000$, $h_z = 10^{-3}$, $h_{r_0} = 10^{-2}$, $4 \cdot 10^{-3}$, and $2 \cdot 10^{-3}$; $N_r = 200$, 500, and 100 (curves 1, 2, and 3, respectively) and the number of iterations S (dot-dashed curves) for the method of simple iteration (1) and Newton's method (2) with $\alpha = 500$.

In conclusion we shall briefly formulate the basic results of this work.

1. The self-action of light beams in a medium with a cubic nonlinearity for a $\alpha \geq 200$ results in the formation of a close to uniform intensity distribution in the aberration ring for path $z \sim 50/\alpha$.

2. The intensity differential in the ring decreases rapidly as a increases, and for $\alpha > 10^4$ does not exceed 1%, i.e., the beam profile become practically uniform.

3. The method of splitting requires a larger (compared with the nonlinear difference scheme) number of points along the transverse coordinate, and it increases as z and α increases. To reduce the required computer time significantly the calculation of self-action of the light beams should be performed on grids that can be adapted (restructed) to the solution combined with the nonlinear scheme and Newton's method.

We also note that this paper is the first paper in a series of works devoted to the investigation of the self-action of light beams and pulses under conditions of strong nonlinear response (this situation can also be encountered for propagation in semiconductors, since in semiconductors, as is well known the non linearity constant are large). In which il was possible to test a method of calculation with the help of Newton's method and to determine the existing difficulty in the numerical modeling.

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