ALGORITHM FOR AUTOMATIC ADJUSTMENT OF A SEGMENTED MIRROR TO AN ARBITRARY RADIATION SOURCE

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This paper describes the iteration method of compensation for wave–front (WF) distortions by means of intensity measurements at each iteration step in two parallel planes located close to the focal one. The algorithm for the above–indicated WF compensation has been constructed for a segmented adaptive mirror based on the physical model of image formation. Its mathematical interpretation is given. For small WF distortions, the algorithm is demonstrated to satisfy the convergence conditions. Examples of numerical simulation of the problem of compensation for WF distortion by the given method are presented.

An optical system with an adaptive mirror consisting of n segments is considered. Let us introduce the vector $\zeta_i = (\beta_i, \gamma_i)$, where β_i and γ_i are the angular deflections of the ith segment from two perpendicular axes of the coordinate system affixed to this segment.

Let us divide the exit pupil aperture into N subapertures corresponding to individual segments of the mirror. If we affix the local coordinates ξ and η to each subaperture corresponding to individual segment of the mirror, within the ith segment the aberration function will be linear and will take the form:

$$W(\xi_i, \eta_i) = \nu_i + \beta_i\, \xi_i + \gamma_i\, \eta_i, \tag{1}$$

where ν_i is the parallel shift of the ith segment. Let us introduce the vector of spatial frequencies $\mathbf{f}_k = (u_k, v_k)$, being so small that the vector $\lambda R\, \mathbf{f}_k$ does not exceed the constructional gap between the segments (here λ is the wavelength, and R is the radius of the Gaussian sphere). In Ref. 1 it has been shown that at these frequencies for linear aberration function (1) the frequency response function of the whole exit pupil $H(\mathbf{f}_k)$ is equal to the sum of frequency responses of individual segments and is independent of parallel shifts of segments ν_i:

$$H(\mathbf{f}_k) = (1/n) \sum_{i=1}^{n} \exp\,[i\, k\, \lambda\, R\, \mathbf{f}_k * \zeta_i], \tag{2}$$

where the symbol $*$ denotes the scalar product of vectors, and i denotes imaginary unity. This makes it possible to construct an algorithm for compensation for angular deflections β_i, γ_i (problem of adjustment) and hence to set off the problem of adjustment from the problem of phasing (compensation for parallel shifts).

Using the frequency response of an optical system with unknown aberration function (1) in the form of Eq. (2) and assuming that the optical system is spatially invariant, we represent the image formation in the frequency plane in the form[2]:

$$J(\mathbf{f}_k) = H(\mathbf{f}_k)\, J_0(\mathbf{f}_k), \tag{3}$$

where $J(\mathbf{f}_k)$ and $J_0(\mathbf{f}_k)$ are the inverse two–dimensional Fourier transforms of the intensity distribution in the image and object planes. Owing to introduction of supplementary known aberrations $\delta W(\xi, \eta)$, we determine the frequency response H^+ different from H. Let us introduce the vector $\Delta\zeta_i = (\Delta\beta_i, \Delta\gamma_i)$, where $\Delta\beta_i$ and $\Delta\gamma_i$ are the supplementary known deflections of the ith segment. Taking into account Eq. (2), we then derive:

$$H^+(\mathbf{f}_k) = (1/n) \sum_{i=1}^{n} \exp\,[i\, k\, \lambda\, R\, \mathbf{f}_k * (\zeta_i + \Delta\, \zeta_i)]. \tag{2$'$}$$

We construct the corresponding inverse transform J^+ from the intensity distribution I^+ in the parallel plane shifted from the initial plane by small distance along the optical axis. This is equivalent to introduction of the known wave aberration $\delta W(\xi, \eta)$ (defocusing). Taking into account Eq. (3), we obtain

$$J^+(\mathbf{f}_k) = H^+(\mathbf{f}_k)\, J_0(\mathbf{f}_k). \tag{3$'$}$$

The relations (3) and (3′) enable us to write the identity

$$H^+(\mathbf{f}_k)/H(\mathbf{f}_k) = J^+(\mathbf{f}_k) \,/\, J(\mathbf{f}_k) = Y(\mathbf{f}_k). \tag{4}$$

At low frequencies Eq. (4) is meaningful and determines the dependence of the frequency response function solely on the results of measurement $Y(\mathbf{f}_k)$.

This paper gives the solution to the problem of reconstruction of $2n$–dimensional vector of unknown aberrations $\zeta = (\zeta_1, \zeta_2, \ldots, \zeta_n)$ by the iteration method from nonlinear identity (4) based on measurements at each iteration step of the right–hand side at n frequencies $\mathbf{f}_k$ ($k = 1, \ldots, n$). To understand the proposed solution algorithm, we address to a simple example. We consider a system with the known input action x and the output measurable parameter $\exp(z - x)$. Here x is referred to as a control. The problem is to find the unknown quantity z from the measurable parameter $\exp(z - x)$. We seek a solution by the method of successive approximations (simple iterations). If the measurable parameter is represented as the expansion

$$\exp\,(z - x) = 1 + (z - x) + \Delta,$$

and the designations $x_n = x$, $x_{n+1} = z + \Delta$ are introduced, the iteration scheme

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$$x_{n+1} = \varphi(x_n) \quad (5)$$

can be obtained, where the function $\varphi(x) = x + \exp(z - x) - 1$ satisfies the convergence condition[3]

$$|\varphi'(x)| < 1 \quad (6)$$

for small z and x. By selecting x_0 as the zero approximation, we obtain $x_n \to z$ according to scheme (5).

It should be noted that in calculation of the function given by Eq. (6) the term $\exp(z - x)$ is the result of measurement (system output) depending on the control x rather than of mathematical calculation. Therefore, we can state that in this method at each iteration step the control x_{n+1} is formed, which transfer the system to a new state when $x \to z$.

In the above methods, the question arises of whether the random measurement errors produce change in the solution z. The simple iteration method has an important advantage, namely, the calculational errors are not accumulated. The calculational error somewhat deteriorates the next approximation.

We use the idea of solution of this model example when solving the problem of adjustment. The $2n$–dimensional vector of angular deflections of WF, caused by controllable deflections of segments, is denoted as $\zeta^y = (\zeta^y_1, \zeta^y_2, \ldots, \zeta^y_n)$. The total WF by compensating control of the segments is determined by the difference between the vectors $\zeta - \zeta^y$. Taking into account control deflections (2) and (2′), we derive

$$H(\mathbf{f}_k, \zeta - \zeta^y) = (1/n) \sum_{i=1}^{n} \exp[i\, k\, \lambda\, R\, \mathbf{f}_k * (\zeta_i - \zeta^y_i)], \quad (7)$$

$$H^+(\mathbf{f}_k, \zeta - \zeta^y + \Delta\zeta) = (1/n) \sum_{i=1}^{n} \exp[i\, k\, \lambda\, R\, \mathbf{f}_k * (\zeta_i - \zeta^y_i + \Delta\zeta_i)]. \quad (7')$$

We first assume that the image source is a point one; then, considering that $J_0(\mathbf{f}_k) = 1$, we derive

$$H^+(\mathbf{f}_k, \zeta - \zeta^y + \Delta\zeta) = J^+(\mathbf{f}_k, \zeta - \zeta^y + \Delta\zeta). \quad (8)$$

Representing the exponent by the Maclaurin formula

$$\exp[i\, k\, \lambda\, R\, \mathbf{f}_k * (\zeta_i - \zeta^y_i)] = 1 + i\, k\, \lambda\, R\, \mathbf{f}_k * (\zeta_i - \zeta^y_i) + \Delta,$$

the function H^+ can be written as

$$H^+(\mathbf{f}_k, \zeta - \zeta^y + \Delta\zeta) = (1/n) \sum_{i=1}^{n} \exp[i\, k\, \lambda\, R\, \mathbf{f}_k * \Delta\zeta_i] \times$$
$$\times (1 + i\, k\, \lambda\, R\, \mathbf{f}_k * (\zeta_i - \zeta^y_i) + \Delta) = H^+(\mathbf{f}_k, \Delta\zeta) +$$
$$+ \sum_{i=1}^{n} a_{ki}(\mathbf{f}_k) * (\zeta_i - \zeta^y_i) + \Delta_1,$$

where

$$a_{ki}(\mathbf{f}_k) = i\, k\, (1/n)\, \mathbf{f}_k \exp[i\, k\, \lambda\, R\, \mathbf{f}_k * \Delta\zeta_i]. \quad (9)$$

It then follows that

$$\sum_{i=1}^{n} a_{ki}(\mathbf{f}_k) * (\zeta_i - \zeta^y_i) = H^+(\mathbf{f}_k, \zeta - \zeta^y + \Delta\zeta) - H^+(\mathbf{f}_k, \Delta\zeta) - \Delta_1. \quad (10)$$

Let us introduce the vector of length $2n$:

$$\mathbf{b}(\mathbf{f}_k, \zeta - \zeta^y) = H^+(\mathbf{f}_k, \zeta - \zeta^y + \Delta\zeta) - H^+(\mathbf{f}_k, \Delta\zeta) =$$
$$= J^+(\mathbf{f}_k, \zeta - \zeta^y + \Delta\zeta) - H^+(\mathbf{f}_k, \Delta\zeta), \quad (11)$$

determined by the measurement J^+ and the known function $H^+(\mathbf{f}_k, \Delta\zeta)$. We also introduce the real matrix A of order $2n \times 2n$ with elements a_{ki}. Then, denoting the matrix inverse to A by A^{-1}, Eq. (10) may be represented in the matrix form

$$A^{-1} * \mathbf{b}(\zeta - \zeta^y) + \zeta^y = \zeta - A^{-1}\Delta_1. \quad (12)$$

Assuming $\zeta_n = \zeta^y$ and $\zeta_{n+1} = \zeta + A^{-1}\Delta_1$, we obtain the iteration scheme $\zeta_{n+1} = \boldsymbol{\varphi}(\zeta_n)$, where the vector

$$\boldsymbol{\varphi}(\zeta_n) = A^{-1}[b(\zeta - \zeta_n) + A\, \zeta_n].$$

The vector $\boldsymbol{\varphi}(\zeta_n)$ has the matrix of the derivatives

$$\varphi'(\zeta_n) = A^{-1}[b'(\zeta - \zeta_n) + A], \quad (13)$$

and the problem reduces to the study of the matrix $[b'(\zeta - \zeta_n) + A]$. A sufficient condition of convergence is that the norm of the matrix of derivatives is less than unity[3]

$$\|\varphi'\| < 1. \quad (14)$$

Considering the definitions of b and A, we find

$$[b'(\zeta - \zeta_n) + A] = \mathbf{f}_k \exp[i\, k\, \lambda\, R\, \mathbf{f}_k * \Delta\zeta] \times$$
$$\times (1 - \exp[i\, k\, \lambda\, R\, \mathbf{f}_k * (\zeta - \zeta_n)]). \quad (15)$$

From comparison of Eq. (15) with Eq. (10) it is evident that the matrix elements of derivatives of the vector $[b'(\zeta - \zeta_n) + A\, \zeta_n]$ are obtained from the elements of the matrix A by multiplying into the factor

$$1 - \exp[i\, k\, \lambda\, R\, \mathbf{f}_k * (\zeta - \zeta_n)]. \quad (16)$$

Imposing the restrictions on the values of frequencies f_k and aberrations ζ, one can make the factors entering into formula (16) rather small so that the matrix of derivatives given by Eq. (15) satisfies inequality (14) for the given matrix A^{-1}, thereby providing convergence of the iterative algorithm $\zeta_{n+1} = \boldsymbol{\varphi}(\zeta_n)$ for automatic adjustment of an optical system to the image of a point source.

Now we consider the case when the radiation source is arbitrary. The image formation in the frequency plane with allowance for control from Eq. (4) is described by the model

$$H^+(\mathbf{f}_k, \zeta - \zeta^y + \Delta\zeta) = H(\mathbf{f}_k, \zeta - \zeta^y) \frac{J^+(\mathbf{f}_k, \zeta - \zeta^y + \Delta\zeta)}{J(\mathbf{f}_k, \zeta - \zeta^y)}. \quad (17)$$

Let us expand $H(\mathbf{f}_k, \zeta - \zeta^y)$ in the Maclaurin series. Omitting the intermediate calculations, we derive

$$H(\mathbf{f}_k, \zeta - \zeta^y) = H(\mathbf{f}_k, 0) + i\, k\, \lambda\, R\, \mathbf{f}_k\, \zeta_{\rm av} + \Delta_2,$$

where $H(\mathbf{f}_k, 0)$ is the response function for zero deflections of WF (optical system without aberrations), and $\zeta_{\rm av} = (1/n) \sum_{i=1}^{n} (\zeta_i - \zeta^y_i)$ is the average tilt of WF determined by the moments of the ith order of the intensity distribution. Assuming the right–hand side of Eq. (13) to be equal to

$$H(J^+/J) \approx (H(\mathbf{f}_k,\ 0) + i\ k\ \lambda\ R\ \mathbf{f}_k\ \zeta_{\text{av}})\ (J^+/J),$$

we introduce the error $\Delta = \Delta_2(J^+/J)$ in measuring the function H^+. For small aberrations, we may introduce such an error into measurement when determining the vector ζ from H^+ by the iteration method.

Numerical simulation at $x_0 = 0$ of different aberrations of WF has indicated that the control x_n reproduces the vector of unknown deflections ζ, and as $n \to \infty$, $x_n \to \zeta$, while the increment $\Delta x_n \to 0$, that is, low spatial frequencies carry information sufficient for solving the problem of compensation for angular aberrations. The algorithm has good convergence. In this case, it should be noted that the measurement vector **Y** and hence the algorithm are invariant to the type of the intensity distribution of the radiation source. Thus, we obtain the system of automatic adjustment of a segmented mirror to an arbitrary radiation source.

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