EFFECT OF ATMOSPHERIC TURBULENCE ON THE REFRACTION-INDUCED OF THE IMAGE OF AN OPTICAL SOURCE

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The effect of atmospheric turbulence on the refraction-induced displacement of an image of an optical source is calculated. The calculation is qualitatively confirmed by experimental data obtained in the real atmosphere. Data from a model experiment, which confirm quantitatively the dependence of the refraction-induced displacement of an optical image on the conditions of diffraction by the radiating aperture (V. A. Banakh et al., Opt. Spektrosk. 62 (5), 1136 (1987) and V.A. Banakh and B.N. Chen, Opt. Atmos. 1 (2), 106 (1988)), are presented.

When an optical beam propagates in an inhomogeneous medium refraction causes the beam to be deflected from the starting direction. It is important to take this deflection into account, for example, when designing optical ranging and detection systems as well as in other areas of science and technology.[1] In Refs. 2 and 3 it was shown that the magnitude of the refract ion-induced displacement of the image of an optical beam propagating in a regularly nonuniform medium depends on the conditions of diffraction at the transmitting aperture of the optical source and the spatial coherence of the radiation from the source. The dependence found cam be used as a basis for developing fundamentally new devices for measuring the angle of atmospheric refraction, whose sensitivity will be much greater than that of the devices based of the use of the dispersion properties of the atmosphere. However the real atmosphere is, as a rule, a randomly inhomogeneous medium. Atmospheric turbulence is manifested as a negative factor which reduces the effectiveness of using the dependence found to determine the angle of atmospheric refraction based on the displacement of the image of an optical source.

In this paper we present the results of theoretical and experimental investigations of the effect of atmospheric turbulence on the magnitude of the refraction-induced displacement of an optical image.

Let an optical source be located in the plane $z' = z_0$. We shall give the mutual coherence function of the source in the form

$$\Gamma_{20}(\vec{R}, \vec{\rho}) = U_0\left[\vec{R} + \frac{1}{2}\vec{\rho}\right]\cdot U_0^*\left[\vec{R} - \frac{1}{2}\vec{\rho}\right] = U_0^2 \exp\left\{ -\frac{R^2}{a^2} - \frac{\rho^2}{4a^2} - \frac{ik}{F}\vec{R}\cdot\vec{\rho} - \frac{\rho^2}{4a_k^2} \right\}, \tag{1}$$

where a, F, and a_c are the radius of the beam, the radius of curvature of the wave front, and the spatial coherence radius in the plane of the transmitting aperture.

The receiving lens with an amplitude transmittance $T(\vec{\rho})$, given by the expression

$$T(\vec{\rho}) = T_0 \exp\left\{ -\frac{\rho^2}{2a_t^2} \right\}, \tag{2}$$

where a_t is the effective radius of a lens with the focal length F_t, is located in the plane $z' = z$.

We shall represent the dielectric constant of the medium $\varepsilon(z', \rho)$ between the planes $z' = z_0$ and $z' = z$ in the form[4]

$$\varepsilon(z', \vec{\rho}) = 1 + \mu(z')\cdot\vec{\rho}\cdot\vec{x}_0 + \tilde{\varepsilon}(z', \vec{\rho}). \tag{3}$$

where $\tilde{\varepsilon}(z', \vec{\rho})$ is the fluctuation component, $\rho\{\vec{x}, y\}$, $\mu(\vec{z}')$ is the transverse regular gradient of the dielectric constant of the medium, and $\vec{x}_0$ is a unit vector perpendicular to the oz' axis.

The propagation of an optical wave in the medium (3) is described by a parabolic equation of the form[4,5]

$$2ik\frac{\partial U}{\partial z'} + \Delta_\perp U + k^2\left[\mu(z')\,\vec{\rho}\,\vec{x}_0 + \tilde{\varepsilon}(z', \vec{\rho})\right] \times U(z', \vec{\rho}) = 0 \tag{4}$$

with the boundary condition $U(z_0, \vec{\rho}) = U_0(\vec{\rho})$ ($\Delta_\perp$ is the transverse Laplacian).

We shall determine the refract ion-induced displacement of the optical image of the source as follows[2]

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$$\sigma = \frac{\vec{\rho}_t \vec{x}_0}{l}, \tag{5}$$

where $\vec{\rho}_t = \int d^2\rho\vec{\rho}I_t(l,\vec{\rho})\Big/\int d^2\rho I_t(l,\vec{\rho})$ is the vector of the coordinates of the energy center of gravity of the image, $I_t(l,\vec{\rho})$ is the distribution of the average intensity of the field of the optical wave after the receiving lens in a plane located at a distance l from the lens.

The intensity $I_t(l,\vec{\rho})$ can be determined with the help of Debye's relation,[6] which relates the complex amplitude of the field behind the lens with the complex amplitude of the field of the wave incident on the lens. In this case the magnitude of the refract ion-induced displacement of the optical image σ is expressed in terms of the mutual coherence function of the complex amplitude of the field of the optical wave Incident on the lens. Then Eq. (4) can be used to construct am equation for the mutual coherence function of the complex amplitude of the field of the wave incident on the lens. From the solution of this equation,[4] using Eqs. (5), (1), and (2), we obtain in the plane of a sharp image of the lens

$$\sigma = \frac{L}{2}\left[\nu_0(L) + P\nu_1(L)\right], \tag{6}$$

where

$$\nu_0(L) = \int_0^1 d\xi\ \xi\mu(z_0 + \xi L);$$

$$\nu_1(L) = \int_0^1 d\xi\ (1-\xi)\mu(z_0 + \xi L);$$

$$P = \frac{\Omega^2\cdot\left(1 - \frac{L}{F}\right) - \frac{2}{3}\,\theta\Omega\beta_0^{12/5}}{1 + \Omega^2\cdot\left(1-\frac{L}{F}\right)^2 + \frac{4}{3}\,\theta\Omega\beta_0^{12/5} + \frac{a^2}{a_k^2}};$$

$$\theta = \left(\frac{0.365}{0.31}\right)^{6/5};$$

$$\Theta = \frac{ka^2}{L}, \qquad \beta_0^2 = 0.31C_\varepsilon^2k^{7/6}L^{11/6},$$

C_ε^2 is the structure factor of the fluctuations of the dielectric constant of the medium, $k = 2\pi/\lambda$ is the wave number, λ is the wavelength, and $L = z - z_0$ is the path length.

It follows from the expression (6) that as $\Omega \to 0$ (the source of a spherical wave) or $a_c \to 0$ (incoherent light source)

$$\sigma = \sigma_{sph} = \frac{L}{2}\,\nu_0(z),$$

which is identical to the angle of refraction calculated in the geometric-optics approximation.[1] In addition, the atmospheric turbulence (the parameter β_0^2) has no effect on the magnitude of the refract ion-induced displacement of the optical image of such sources. The effect of atmospheric turbulence, as follows from Eq. (6), is first observed when $\Omega \neq 0$ (source of a non-spherical wave) or $a_c \neq 0$ (partially coherent light source), and is maximum in the case of a coherent ($a_c \to \infty$) light source. Figure 1 shows the computed dependences of the quantity $1 + P$ on the degree of divergence of a coherent ($a_c \to \infty$) optical beam L/F for different values of the parameter β_0^2, characterizing the turbulent conditions of propagation under the assumption that the path in the longitudinal direction is uniform ($\mu(z') = \text{const}$).

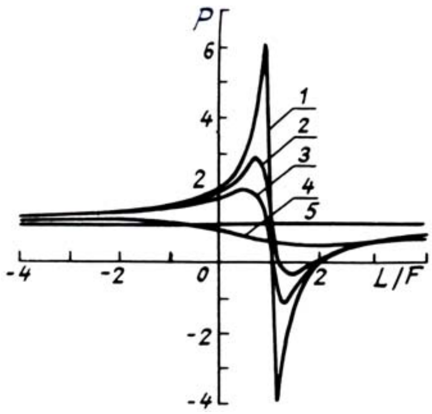

FIG. 1. The computed dependences of the quantity 1 + P on the degree of divergence of a coherent optical beam L/F ($\Omega = 5$) for different values of the parameter β_0^2: $\beta_0^2 = 0$ (1); $\beta_0^2 = 0.36$ (2); $\beta_0^2 = 1$ (3); $\beta_0^2 = 16$ (4); *the curve (5) corresponds to a spherical wave.*

It is obvious from the figure that atmospheric turbulence has a strong effect on the magnitude of the refraction-induced displacement of the image, but even under the most turbulent conditions of propagation ($\beta^2 \to \infty$) for a collimated beam $\left(\frac{L}{F} = 0\right)$ $\sigma = \frac{1}{2}\sigma_{sph}$.

The maxima of the refraction-induced displacement are reached in beams with the parameter

$$\frac{L}{F} = 1 - \frac{2\theta\Omega\beta_0^{12/5}}{3\Omega^2} \pm \frac{\left[4(\theta\Omega\beta_0)^{24/5} + 9\Omega^2\left[1 + \frac{4}{3}\,\theta\Omega\beta_0^{1/5} + \frac{a^2}{a_k^2}\right]\right]^{1/2}}{3\Omega^2},$$

while for a coherent beam ($a_k \to \infty$) in the absence of turbulence ($\beta_0^2 \to 0$] the maxima are reached at $\frac{L}{F} = 1 \pm \frac{1}{\Omega}$.

The experimental check of the computed data was made in two stages. At the first stage the effect of refraction on the regular displacement of the image of an optical source was studied experimentally as a function of the conditions of diffraction at the transmitting aperture of the source[2] on a model setup. An optical system based on a He-Ne laser was used as the source; the system produced an optical beam with the Fresnel number $\Omega = 50$ and permitting varying the focal parameter L/F from –3 to 5. The path length was equal to 3 m. The receiving lens consisted of an objective with a focal length $F_t = 50$ cm. The displacement of the optical image was measured with the help of a searching-tracking apparatus, built based on a dissector,[7] which measured the position of the energy center of the image with an accuracy of 2.5 μm (the apparatus had a sensitivity of 400 readings/mm). The gradient of the dielectric constant of the medium was modeled by a pair of optical wedges, which were displaced relative to one another with adequate accuracy using a step motor, which provided good reproduceability of the experiment. The local character of the optical nonuniformity introduced by the wedges was taken into account in the calculation of the coefficients $\nu_0(L)$ and $\nu_1(L)$ under the assumption that over the length of the wedges $\mu(z') \neq 0$, and on the rest of the path $\mu(z') = 0$. For each value of L/F the wedges were placed into the zero position and in this manner the optical nonuniformity on the path was eliminated. Then the wedges were moved with a step from 32″ to 288″ and back, and in each step the position of the energy center of the image was measured. The ratio of the measured angular displacement of the optical image (the ratio of the linear displacement to the distance from the objective to the plane of the image) to the angular displacement of the image of the source of a spherical wave (strongly diverging beam) $-\sigma\left(\frac{L}{F}\right)\Big/\sigma_{sph}$ was determined for each value of L/F. The results of this comparison are presented in Table I.

TABLE I.

$\frac{L}{F}$	5.5	3.95	3.41	2.5	2.1	1.65	1.45	0.909	0.75	0.66
$(1+P)_{cal}$	0.138	-0.325	-0.61	-1.59	-3.27	-4.97	-7.61	41.67	16.4	12.37
$(1+P)_{meas}$	0.15	-0.351	-0.66	-1.7	-3.4	-5.41	-7.74	43.9	15.6	12.48

$\frac{L}{F}$	0.5	0.33	0.25	0.204	0	-0.15	-0.35	-0.54	-1.3	-2.07	-2.89
$(1+P)_{cal}$	8.74	6.81	6.3	5.81	4.6	4.43	0.138	-0.325	-0.61	-1.59	-3.27
$(1+P)_{meas}$	8.75	7.16	6.2	5.84	4.88	4.36	3.87	3.52	2.68	2.26	2.01

As one cam see from Table I the experimental data reproduce well (with an error of not more than 10%) the calculations based on the formula of Ref. 2.

At the second stage the effect of atmospheric turbulence on the refraction-induced displacement of the optical image (the formula (6)) in a real atmosphere was studied experimentally. An optical system based on a He-Ne laser was used as the optical source; the system formed two beams: a strongly diverging beam (source of a spherical wave) and a beam with the Fresnel number $\Omega = 3$; the second beam was tunable over the range $L/F = -1.67$–3.66. The measuring path passed above an even surface and was equal to 132 m.

The receiving system consisted of an optical system with a long focal length. This system made it possible to hold the plan of the photo cathode of the dissector in a fixed position when the system was adjusted for a sharp image. The positions of the centers of the image of sources of the spherical wave and the beam with a definite ratio L/F were measured successively from 10 to 16 h) on one or two days. The values of the displacements of the images of the sources in each realization were integrated over an interval of 5 min, and then the average values of the displacements were calculated based on 15–20 realizations, obtained with the same value of the ratio L/F. The measurements of the temperature were performed in parallel at the same end of the path for transmitters for beams propagating at a height of 2 m above the underlying surface. The results of the experiments in the real atmosphere are presented in Fig. 2.

In Fig. 3 the same results are presented in coordinates that are convenient for making comparisons with theoretical calculations based on the formula (6). The values of $\Delta\sigma = \sigma - \sigma_{sp}$, equal to the difference of the coordinates of the images of the optical beam with a fixed radius of curvature of the wave front and the spherical wave, are plotted along the vertical axis. The vertical lines denote the spread in the experimental values of the displacement. The theoretical curves were calculated for- beams with $\Omega = 3$ and degrees of turbulence most likely to be realized during the measurements. To compare the experimental data the scale of the values for P along the vertical axis was chosen so that the maximum value $P = 1.47$ in the absence of turbulence would be close for a beam with the parameter $L/F = 0.73$, measured during rain, when the degree of turbulence and refraction are minimum.

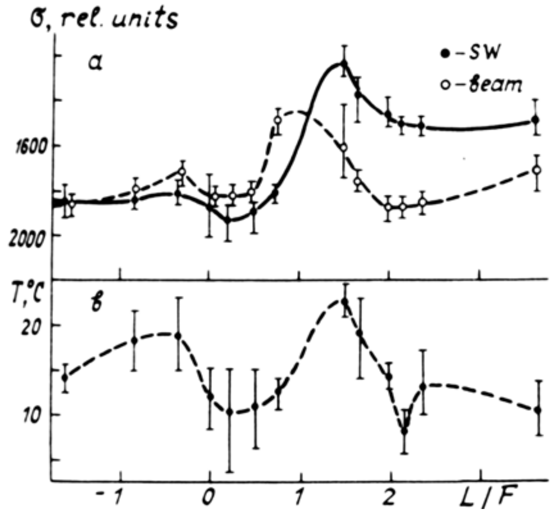

FIG. 2. The results of measurements of the displacement of an image of sources of a spherical wave and a beam with $\Omega = 3$ (a) and the air temperature (b).

It is obvious from Fig. 3 that the experimental data qualitatively confirm the computed dependences of the effect of atmospheric turbulence on the refraction-induced displacement of the image of an optical source.

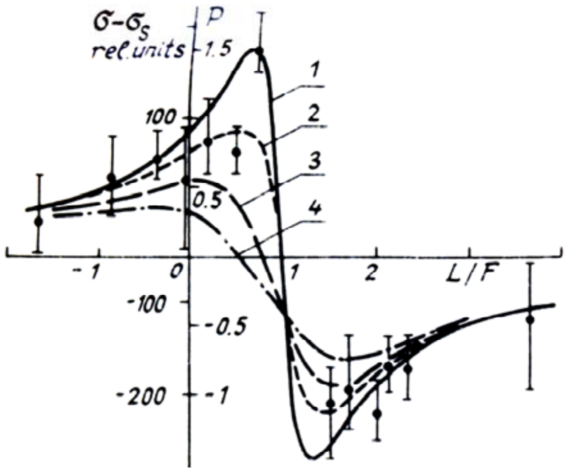

FIG. 3. $\sigma - \sigma_{sp}$ versus L/F based on the experimental results and the theoretical curves for P with the values $\beta_0^2 = 0$ (1); $\beta_0^2 = 0.23$ (2); $\beta_0^2 = 0.7$ (3); $\beta_0^2 = 1.6$ (4); *The parameter* $\Omega = 3$.

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