THERMAL DEFOCUSING OF A FOCUSED GAUSSIAN BEAM OF ELECTROMAGNETIC RADIATION

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In the context of the wave theory the axial intensity of a focused Gaussian beam has been calculated to the first order of the perturbation theory in the nonlinearity parameter for stationary and nonstationary regimes of thermal defocusing. An approximation formula has been derived that allows the generalization of the obtained results to be made, including strong nonlinearity.

1. INTRODUCTION

Attempts to consider analytically the problem of thermal defocusing of a Gaussian beam of optical radiation in the context of the wave theory led to success in solving several particular problems.[1–3] The present paper is devoted to analytical consideration of the most important practical problem on stationary regime of thermal defocusing of a focused Gaussian beam and derivation of a unified generalized expression describing thermal defocusing of a focused Gaussian beam for the case of long radiation pulse.

To calculate a Gaussian beam intensity in a nonlinear medium, the technique proposed in Ref. 4 for solving the nonlinear parabolic equation was used. This technique leads to the following general expression for the normalized intensity of a beam with arbitrary initial amplitude–phase distribution in the first order of the perturbation theory:

$$I_n = I(x, y, z)/I_{lin}(x, y, z) = 1 - N, \qquad (1)$$

$$N = -\frac{(k r_0)^2}{2\pi n_0} \mathrm{Re}\left\{ W^{-1}_{lin} \int\limits_0^z \frac{dz_1}{z - z_1} \iint\limits_{-\infty}^{+\infty} dx_1\, dy_1 \times \exp\left(-\frac{(x - x_1)^2 + (y - y_1)^2}{2i(z - z_1)}\right) n_1(x_1 y_1 z_1 W_{lin})\, W_{lin}(x_1 y_1 z_1) \right\},$$

where W_{lin} is the solution of the parabolic equation for a wave beam propagating through the unperturbed medium with the refractive index n_0, I_{lin} is the intensity of this beam, $n_1 = n - n_0$ is the refractive index deviation from its unperturbed value, $k = 2\pi n_0/\lambda$ is the wave vector in the unperturbed medium, r_0 is the transverse radius of the beam, z is the coordinate along the beam propagation direction, and Re denotes the real part of the complex expression. The dimensionless variables are used (primes are omitted below)

$$x' = x/r_0,\ y' = y/r_0,\ z' = z/k r_0^2,\ \ W = A/A_0,\ \ \alpha' = \alpha k r_0^2,$$

where $\alpha = 4\pi\kappa/\lambda$ is the radiation absorption coefficient in the medium, κ is the imaginary part of the refractive index of the medium, and A_0 is the characteristic initial value (for $z = 0$) of the complex amplitude of the beam field. The intensity of radiation $I(x, y, z)$ is expressed in terms of the dimensionless function $W(x, y, z)$ in the following way:

$$I(x, y, z) = (P_0/\pi r_0^2)\ |W|^2 \exp(-\alpha z),$$

where P_0 is the total beam power and the function W is normalized by the ratio

$$\iint\limits_{-\infty}^{+\infty} |W(x, y, 0)|^2 dx\, dy = \pi.$$

For a focused Gaussian beam the linear solution is given by the following expression:

$$W_{lin} = \frac{1}{1 + iz(1 + iF)} \exp\left(-\frac{(x^2 + y^2)(1 + iF)}{2(1 + iz(1 + iF))}\right).$$

The present paper deals with the long radiation pulse for which the condition $t \gg r_0/c_s F$ is satisfied. Here t is the radiation pulse duration, c_s is the sound speed in the medium, and $F = k r_0^2/z_F$ is the Fresnel number. The main mechanism of heat elimination from the region occupied by the beam is the removal of heated medium from the propagation channel (with the velocity $v_\perp$ along the x axis). Therefore, nonstationary (when $t \ll t_1 \approx r_0/v_\perp \sqrt{1 + 3F^2}$) and stationary (when $t \gg t_1$) regimes of beam propagation are established.

2. NONSTATIONARY REGIME OF THERMAL DEFOCUSING

In the nonstationary regime variations in the refractive index of the medium are determined by the expression

$$n_1 = -(n_0 - 1)(\gamma - 1)\,\alpha\, I_{lin}\, t/\gamma p,$$

where γ is the adiabatic exponent and p is the surrounding pressure. For weakly absorbing medium ($\tau = \alpha z \ll 1$) the following expression for the normalized intensity on the beam axis is obtained with the use of Eq. (1):

$$I_n(z) = 1 - \frac{t}{2\sqrt{\pi}\, t_{nl}\, z^2} \ln\frac{(1 - zF)^2 + 9z^2}{(1 - zF)^2 + z^2}, \qquad (2)$$

where t_{nl} is the nonlinearity parameter (in the dimension units)

$$\frac{1}{t_{nl}} = \frac{(n_0 - 1)(\gamma - 1)\,\alpha P_0 z^2}{2\sqrt{\pi}\, n_0\, \gamma\, p\, r_0^4}.$$

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In the limiting cases of collimated ($F = 0$) and focused ($z = 1/F$) beams formula (2) gives the results presented in Refs. 2 and 3.

For a focused Gaussian beam the radiation intensity in the center of the focal plane can be calculated ignoring the fact that the parameter $\tau = \alpha z_F$ is small. In this case we obtain

$$I_{\text{n}} = 1 - \frac{t F^2 \ln 3}{\sqrt{\pi}\, t_{\text{nl}}} \Phi_{\text{nst}}(F, \tau), \quad (3)$$

where

$$\Phi_{\text{nst}}(F, \tau) =$$

$$= \frac{1}{\ln 9} \exp\left(-\frac{9F\tau}{9F-1}\right)\left[Ei\left(\frac{9F\tau}{9F-1}\right) - Ei\left(\frac{\tau}{9F-1}\right)\right] -$$

$$- \frac{1}{\ln 9} \exp\left(-\frac{F\tau}{F-1}\right)\left[Ei\left(\frac{F\tau}{F-1}\right) - Ei\left(\frac{\tau}{F-1}\right)\right],$$

and $Ei(x)$ is the integral exponent.

The function $\Phi_{\text{nst}}(F, \tau)$ satisfies the condition $\Phi_{\text{nst}} \lesssim 1$ in the ranges of the variables F and τ. At the boundaries of the domain of definition the function takes the values $\Phi_{\text{nst}}(F, \tau) = 1$ and $\Phi_{\text{nst}}(F \to \infty, \tau) \to \exp(-\tau)$.

3. STATIONARY REGIME OF THERMAL DEFOCUSING

In the stationary regime variations in the refractive index of the medium are determined by the expression

$$n_1 = -\frac{(n_0 - 1)(\gamma - 1)}{\gamma\, p\, v_\perp} \int\limits_{-\infty}^{x} I_{\text{lin}}(x_1, y, z)\,\text{d}x_1.$$

The normalized intensity of Gaussian beam in the stationary regime calculated on the basis of Eq. (1) for collimated beam is known

$$I_{\text{n}} = 1 - N_0\, \Phi(\tau), \quad (4)$$

for $z \ll 1$, where $N_0 = r_0/v_\perp t_{\text{nl}}$ and $\Phi(\tau) = 2(\tau - 1 + \exp(-\tau))/\tau^2$. For focused beam in the focal point we derive

$$I_{\text{n}}(z_F) = 1 - N_0 G_0 = 1 - \frac{N_0\, \pi F^2}{3\sqrt{1 + 3F^2}} \Phi_{\text{st}}(F, \tau), \quad (5)$$

where

$$\Phi_{\text{st}}(F, \tau) = \left(1 - \frac{6F}{\pi(1 + 3F^2)} \ln \frac{1 + 3F^2}{eF}\right) \exp\left(-\frac{\tau F}{0.6 + F}\right)$$

and e is the logarithmic base. The relation for $\Phi_{\text{st}}(F, t)$ was analytically derived at $\tau = 0$ in the approximation allowing for the dominant terms of expansion in the parameter $2F/(1 + 3F^2) \le 1/\sqrt{3}$, while for $\tau > 0$ – by way of fitting the calculated results by the relation $\exp[-\tau F/(\alpha + F)]$, where α is the adjustable parameter.

4. GENERALIZED FORMULAS

The formulas (3) and (5) can be written in the form of unified expressions combining the nonstationary and stationary regimes, which for short ($t \ll t_1$) and long ($t \gg t_1$) times are reduced to corresponding limiting cases. At the same time these formulas can be generalized to strong nonlinearity by way of substituting the expression $\exp(-N)$ for $(1 - N)$. Such generalization holds the validity of the results to the first order of the perturbation theory in the nonlinearity parameter and describes qualitatively realistically the behavior of defocusing beam for strong nonlinearity.

The generalized expression for the normalized intensity in the focal plane of a defocusing Gaussian beam in the entire domain $F \ge 0$, $\tau \ge 0$ has the form

$$I_{\text{n}} = \exp\left[-N_0 G_0 \left(1 + \frac{r_0}{t\, v_\perp \sqrt{1 + 3F^2}} \frac{\pi\sqrt{\pi}\, F_{\text{st}}}{3 \ln 3\, F_{\text{nst}}}\right)^{-1}\right].$$

For $\tau \ll 1$, allowing for the term of the lowest order in the expansion of $G_0(F, 0)$ in the parameter $2F/(1 + 3F^2)$, the obtained formula simplifies and assumes the form

$$I_{\text{n}} = \exp\left[-\frac{\pi N_0 F^2}{3\sqrt{1 + 3F^2}} \left(1 + \frac{\pi\sqrt{\pi}\, r_0}{t\, v_\perp\, 3 \ln 3 \sqrt{1 + 3F^2}}\right)^{-1}\right].$$

The obtained approximation expressions for the normalized intensity of a Gaussian focused beam in the case of long pulse (i.e., for pulse whose duration satisfies the condition $t \gg r_0/c_{\text{s}} F$) describe the transition from the nonstationary regime of defocusing to the stationary one removing the fictitious singularity on $v_\perp$ appearing in formulas (4) and (5).

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