SEMIEMPIRICAL MODELS OF THE AEROSOL COMPOSITION OF THE UPPER ATMOSPHERE. I. SEDIMENTAL MODEL

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A general formulation of the problem of calculating the altitude behavior of the concentration of primary cosmic-dust particles (micrometeorites) with different masses in the diffusion-sedimentation model, i.e., taking into account the magnitudes of the particle influx, sedimentation, and turbulent mixing of the atmosphere, is presented. Numerical calculations for the purely sedimentation model were performed. It is shown that particles with masses of 10^{-16} ... 10^{-14} g *make the largest contribution to aerosol scattering of radiation with wavelengths of* 0.1 ... 0.5 μm. *However the relative atmospheric turbidity produced by them is small: it does not exceed 2% of the Rayleigh molecular scattering.*

INTRODUCTION

In Refs. 5 and 8 it was shown that at altitudes above 30 km the source of aerosols are solid particles of the interplanetary medium, which are constantly entering the earth's atmosphere. The character of the interaction with the atmosphere depends on the initial mass m, the initial velocity ν, the density ρ_a, and other characteristics of the particles. The minimum recorded particle masses are $m \approx 10^{-17}$ g.[12,15,16] There is virtually no upper limit of the particles masses, since the probability for large particles to encounter the earth decreases rapidly as their mass increases. For example, bodies with masses $m > 10^3$ metric tons strike the earth once in several hundreds of years.

Small dust particles with masses m greater than some limiting value m' virtually completely evaporate as they decelerate in the atmosphere. For particles whose composition and density are close to those of the most widespread types of rocky meteorites — chondrites (or to quartz), with an average exoatmospheric velocity of the particles $v = 30$ km/sec $m' \approx 10^{-8}$ g,[7] and the dependence of m' on v has the form $m' \sim v^{-9}$. Particles with masses $m < m'$ give up their initial kinetic energy almost exclusively into thermal radiation from the surface and, being stopped at altitudes above 100 km, they slowly settle straight through the entire thickness of the atmosphere in the form of micrometeorites.

The concentration of vapors of the meteor matter in the upper atmosphere is determined not only by the processes leading to evaporation of the particles of cosmic dust, but also diffusion, aeronomic reactions with participation of atmospheric atoms, molecules. Ions, and radicals, and condensation of vapors of drops of sulfuric acid in the stratospheric Junge aerosol layer, etc.

The problem of determining the aerosol composition of the upper atmosphere taking into account all these factors is extremely difficult to solve completely, and a solution can be obtained only by the method of successive construction of simpler models, which permit estimating the contribution of different processes at different altitudes and In different Intervals of the aerosol masses.

Data on several models of aerosols in the upper atmosphere — both purely empirical data, based on generalization of the measurements of the aerosol content in the atmosphere,[3,4,9,18] and semiempirical data, which take Into account one or another of the processes indicated above,[5,11,17,19] have now been published. B. N. Lebedinets's sedimentation model of primary cosmic-dust particles,[5] which was based on the then existing data on the influx of cosmic dust into the atmosphere. Is the closest model to the one studied in this paper. This model can now be greatly improved and made much more detailed owing to the detailed model, developed in Ref. 8, of the influx of cosmic-dust particles in a wide interval of masses 10^{-17} ... 10^4 g.

1. THE DIFFUSION-SEDIMENTATION MODEL

We shall express the number of aerosol particles $dC(r)$ having radii from r to $r + dr$ per unit volume in terms of the spectral particle size function $n(r, z)$ with the help of the equation

$$dC(r) = n(r, z)dr. \quad (1)$$

The spectral function $n(r, z)$ depends on the altitude z. The weaker dependence on the horizontal coordinates and time cam be neglected, to a first approximation, in developing some average stationary model.

In this case the change in $n(r, z)$ with altitude is described by the equation

$$d\Phi/dz = 0, \tag{2}$$

where

$$\Phi(r, z) = -D_t(z)\left[\frac{dn}{dz} + \frac{n}{H(z)}\right] - U_s(r, z)n(r, z) \tag{3}$$

is the aerosol flux density, D_t is the turbulent diffusion coefficient, $H(z)$ is the scale height of the atmosphere, and $U_s(r, z)$ is the aerosol sedimentation rate.

We give the boundary conditions in the form

$$\left.\begin{aligned} \Phi(r, z)\big|_{z=z_{max}} &= -\Phi_0(r) \\ n(r, z)\big|_{z=z_{min}} &= n_0(r) \end{aligned}\right\}, \tag{4}$$

where $\Phi_0(r)$ is the flux density of particles of cosmic dust at the boundary of the atmosphere and $n_0(r)$ is the aerosol concentration on the bottom boundary of the altitude interval under study.

The general solution of the problem (2)–(4) has the form

$$n(r, z) = \exp\left\{-\int_{z_{min}}^{z}\left[\frac{1}{H(z')} + \frac{U_s(r, z')}{D_t(z')}\right]dz'\right\} \times \left[n_0(r) + \Phi_0(r)\int_{z_{min}}^{z}\exp\left\{+\int_{z_{min}}^{z'}\left[\frac{1}{H(z'')} + \frac{U_s(r, z'')}{D_T(z'')}\right]dz''\right\}\frac{dz'}{D_t(z')}\right]. \tag{5}$$

Equation (5) has a complicated nonlocal (integral) dependence on the parameters of the atmosphere. The solution is greatly simplified for altitudes and particle sizes such that the characteristic sedimentation time

$$t_s = \frac{H}{U_s}, \tag{6}$$

is much shorter than the characteristic diffusion time

$$t_D = \frac{H^2}{D_t}, \tag{6.1}$$

i.e.,

$$D_t(z) \ll U_s(r, z)H(z). \tag{7}$$

In this case diffusion can be neglected and only sedimentation need be taken Into account, i.e.,

$$\Phi(r, z) = -U_s(r, z)n(r, z). \tag{8}$$

Then Eq. (5) is replaced by the very simple equation

$$n(r, z) = \frac{\Phi_0(r)}{U_s(r, z)}, \tag{9}$$

in which the dependence of $n(r, z)$ on the parameters of the atmosphere is of a local character.

The condition (γ) for the solution (9) to be applicable is of a qualitative character. To obtain a quantitative condition the solutions (5) and (9) must be compared with one another. To do so specific dependences must be given for $D_T(z)$, $H(z)$, $U_s(r, z)$, $\Phi_0(r)$, and $n_0(r)$. Since the dependence $D_T(z)$ can be taken only for different empirical models, the obtained model $n(r, z)$ becomes semiempirical.

We shall decrease the sedimentation rate by Stokes equation with the correction factor[1,10,14]

$$U_s(r,z) = \frac{2}{9}\frac{\rho_a g r^2}{\eta}\left\{1 + \frac{l}{r}\left[1.257 + 0.4\exp\left(-1.1\frac{r}{l}\right)\right]\right\}, \tag{10}$$

where g is the acceleration of gravity, ρ_a is the density of the aerosol matter, l is the mean free path of molecules in the atmosphere, and η is the dynamic viscosity of air. The parameters l and η can be calculated with the help of the equations

$$l = \frac{1}{\sqrt{2}\,C_a(z)\pi Q_d^2}, \tag{11}$$

$$\eta = \frac{5\sqrt{\pi m_a kT}}{16\pi Q_d^2}, \tag{12}$$

where Q_d is the effective diffusion cross section of atmospheric molecules, m_{air} is the average mass of the molecules, $C_{air}(z)$ is the concentration of air molecules, T is the temperature of the atmosphere, and k is Boltzmann's constant.

It is very difficult to give *a priori* the boundary condition at the bottom boundary of the atmospheric layer under study. We shall choose $n_0(r) = 0$ at some altitude z_{min} that is much lower than the bottom boundary of the atmospheric layer under study; in this case a boundary condition of this form has virtually no effect on the solution in the altitude range in which we are interested 30–100 km. We shall choose $z_{min} = 20$ km.

The quantity $\Phi_0(r)$ is given by the model describing the Influx of cosmic dust.

2. MODEL OF THE INFLUX OF COSMIC DUST

Figure 1 gives the model integral mass distribution, taken from Ref. 8, of the average flux density of particles of cosmic dust at the boundary of the earth's atmosphere. This distribution has a complicated structure, which can be approximated in the mass interval of interest to us $10^{-17} \ldots 10^{-8}$ g by four equations of the type

$$N(m) = N_{0i}\left[\frac{m}{m_{0i}}\right]^{1-S_i}, \text{ for } m_{0i} \le m \le m_{1i},$$
$$i = 1,\ 2,\ 3,\ 4. \qquad (13)$$

Here $m_{01} = 10^{-17}$, $m_{11} = m_{02} = 10^{-16}$, $m_{12} = m_{03} = 10^{-15}$, $m_{13} = 10^{-11}$, $m_{04} = 10^{-11}$, $m_{14} = 10^{-8}$ g; $N_{01} = 1$, $N_{02} = 10^{-1}$, $N_{03} = 10^{-3}$, $N_{04} = 10^{-7,6}$ $cm^{-2}s^{-1}(2\pi \text{ step})^{-1}$; $S_1 = 2$, $S_2 = 3$, $S_3 = 5$, $S_4 = 1.6$.

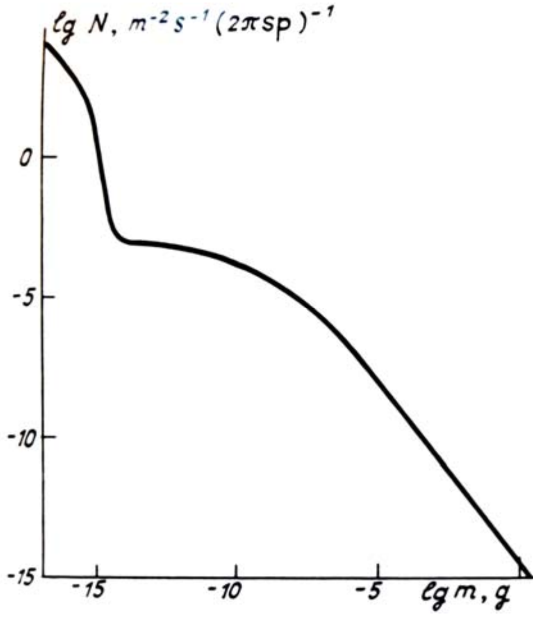

FIG. 1. Model of the influx of particles of cosmic dust at the boundary of the atmosphere.[8]

TABLE I

The flux density of meteor matter in different intervals of the masses m and radii of the particles.

i	1	2	3	4	Remainder
m, g	$10^{-17} \ldots 10^{-16}$	$10^{-17} \ldots 10^{-16}$	$10^{-17} \ldots 10^{-16}$	$10^{-17} \ldots 10^{-16}$	10^{-8}
r, μm	0.0098...0.021	0.021...0.045	0.045...0.097	0.98...9.8	9.8
Φ_M, $g\cdot cm^{-2}s^{-1}\cdot(2\pi st)^{-1}$	$2.3\cdot10^{-17}$	$2\cdot10^{-17}$	$0.13\cdot10^{-17}$	$1.2\cdot10^{-17}$	$5\cdot10^{-17}$

As pointed out above, particles with masses $m > 10^{-8}$ g virtually completely evaporate in the atmosphere. The Influx of particles with masses in the interval $10^{-14} \ldots 10^{-11}$ g (which are pushed out of the solar system by the pressure of the solar radiation) can be neglected.

We shall calculate the influx of mass Φ_{M_i} in each interval of particle masses

$$\Phi_{M_i} = \int_{m_{0i}}^{m_{1i}} m\left(-\frac{dN}{dm}\right)dm = N_{0i}(S_i - 1)m_{0i}\int_{m_{0i}}^{m}\left(\frac{m}{m_{0i}}\right)^{1-S_i}\frac{dm}{m_{0i}} =$$

$$= \begin{cases} \Phi_{M_i} = N_{0i}m_0 \lg\left[\frac{m_{1i}}{m_{0i}}\right], \quad \text{for } i = 1, & (14) \\ \Phi_{M_i} = N_{0i}m_{0i}\frac{S_i - 1}{S_i - 2}\left[\left[\frac{m_{1i}}{m_{0i}}\right]^{2-S_i} - 1\right], & (14.1) \\ \text{for } i = 2,\ 3,\ 4 & \end{cases}$$

Table I gives the values of Φ_{M_i} for the four intervals of particle masses indicated above and the corresponding end-point radii of the particles (assuming that the particles are spherical and $\rho_a = 2.5\ g\cdot cm^{-3}$).

3. SEDIMENTATION MODEL

In the upper atmosphere for the interval of particle radii of Interest to us $r < 10$ μm Knudsen's number

$$Kn = \frac{l}{r} \gg 1. \qquad (15)$$

Then we find from Eq. (10) the approximate form of the dependence of the sedimentation rate of the particles on the mass m in each of the mass intervals indicated above

$$U_s(m) = U_s\left(m_{S_i}\right)\left(\frac{m}{m_{01}}\right)^{1/3}. \qquad (16)$$

From Eqs. (13) and (16) we obtain

$$n(m) = -\frac{1}{U_a(m)}\cdot\frac{dN}{dm} = n_{01}\left(\frac{m}{m_{01}}\right)^{1/3-S_1}, \tag{17}$$

where

$$n_{01} = n(m_{01}) = \frac{N_{01}(S_1 - 1)}{m_{01}U_a(m_{01})}. \tag{18}$$

We shall find the spectral distribution function of the particle radii from Eqs. (17)–(18), taking into account the fact that

$$rn(r) = 3mn(m), \tag{19}$$

$$n(r) = \frac{3N_{01}(S_1 - 1)}{r_{01}U_a(r_{01})}\left[\frac{r}{r_{01}}\right]^{3(1-S_1)},$$

$$\text{for } r_{01} \le r \le r_{11},\ 1 = \overline{1,4}, \tag{20}$$

where r_{01} and r_{11} are the radii of particles with masses m_{01} and m_{11}, respectively.

For $r \ll l$ the dependence of the sedimentation rate of the particles on r and z assumes the form

$$U_a(r,z) = U_0(z)r,$$

where

$$U_0(z) = 0.37\,\frac{\rho_a g l}{\eta} = \frac{0.84\rho_a g}{C_a(z)\sqrt{\pi m_a kT}}. \tag{21}$$

4. THE AEROSOL LIGHT-SCATTERING COEFFICIENT

For an isolated spherical particle of radius r with the refractive index k_a the effective cross section for the scattering of light can be written in the form[2,13]

$$\sigma_1(r,\ k_a,\ \lambda) = K(r,\ k_a,\ \lambda)\pi r^2, \tag{22}$$

where k_a is the refractive index of the particle matter, λ is the wavelength of light, and $K(r, k_a, \lambda)$ is the 1ight-scattering efficiency factor. $K(r, k_a, \lambda)$ can be calculated on the basis of Mie's theory[13] with the help of very complicated equations. In our case $K(r, k_a, \lambda)$ can be approximated by two quite simple equations: for $x = 2\pi r/\lambda \ll 1$ (Rayleigh scattering)

$$K(r,\ k_a,\ \lambda) = K_1(x,\ k_a) = \frac{8}{3}x^4\cdot\left[\frac{k_a^2 - 1}{k_a^2 + 2}\right]^2, \tag{23}$$

and for $x \gg 1$ (large particles)

$$K(r,\ k_a,\ \lambda) = K_2(x,\ k_a) = 2. \tag{24}$$

For quartz particles with $k = 1.5$ Eq. (23) is applicable for $x < 1$ and Eq. (24) is applicable for $x > 10$, i.e., for the wavelength $\lambda = 0.5$ μm Eq. (23) is applicable for particle size intervals $i = 1$, 2, and 3, and Eq. (24) is applicable for $i = 4$.

For polydispersed systems we shall find the volume light-scattering coefficient of aerosols with particle radii ranging from r_0 to r_1 with the help of the equation

$$\sigma_a = \int_{r_0}^{r_1}\sigma_1(r,k_a,\lambda)n(r)\,dr = \int_{r_0}^{r_1}K(x,k_a)\pi r^2 n(r)\,dr. \tag{25}$$

If $n(r) = n_0\left(\frac{r}{r_0}\right)^{-s}$ and the scattering is of the Rayleigh type, then we obtain from Eqs. (23) and (25)

$$\sigma_a = \frac{\sigma_1(r_0)nr_0}{7 - S}\left[\left[\frac{r_1}{r_0}\right]^{7-S} - 1\right], \tag{26}$$

where $\sigma_1(r_0)$ is the effective light-scattering cross section of an isolated spherical particle with radius r_0

$$\sigma_1(r_0) = \frac{8}{3}\left[\frac{2\pi r_0}{\lambda}\right]^4\left[\frac{k_a^2 - 1}{k_a^2 + 2}\right]\pi r_0. \tag{27}$$

One can see from Eq. (26) that for $s < 7$ large particles make the main contribution to the volume light-scattering coefficient; in this case we cam write approximately

$$\sigma_a \approx \frac{\sigma_1(r_0)n_0r_0}{7 - S}\left[\frac{r_1}{r_0}\right]^{7-S} = \frac{\sigma_1(r_1)n_1r_1}{7 - S}. \tag{28}$$

For $s > 7$ we obtain from Eq. (26)

$$\sigma_a \approx \frac{\sigma_1(r_0)n_0r_0}{S - 7}. \tag{29}$$

We shall study the contribution of each of the four particle size intervals which we have separated to the volume aerosol 1ight-scattering coefficient. For $i = 1$, 2, and 3, $x < 1$ and $s < 7$. We rewrite Eq. (20) in the form

$$n(r) = n(r_{01})\left[\frac{r}{r_{01}}\right]^{3(1-S_1)}, \tag{30}$$

where

$$n(r_{01}) = \frac{3N_{01}(S_1 - 1)}{r_{01}U_a(r_{01})}. \tag{31}$$

Then instead of Eq. (26) for $n(r) \sim \left(\frac{r}{r_1}\right)^1$, i.e., for $S = 3(1 - S_1)$ we obtain

$$\sigma_{a1} = \frac{\sigma_1(r_{01})n(r_{01})r_{01}}{10 - 3S_1}\left[\left[\frac{r_{11}}{r_{01}}\right]^{10-3S_1} - 1\right], \tag{32}$$

where σ_{a1} is the contribution of particles from the ith interval of radii to the volume light-scattering coefficient.

Substituting the numerical values $S_1 = 2$, $S_2 = 3$, $S_3 = 5$, $\frac{r_{11}}{r_{01}} = 10$, we obtain from Eq. (32)

$$\sigma_{a1} = \frac{\sigma_1(r_{11})n(r_{11})r_{11}}{4}, \tag{33}$$

$$\sigma_{a2} = \frac{\sigma_1(r_{12})n(r_{12})r_{12}}{1}, \tag{34}$$

$$\sigma_{a3} = \frac{\sigma_1(r_{13})n(r_{13})r_{13}}{5} \tag{35}$$

For the fourth interval of particle sizes $i = 4$ we obtain from Eqs. (24) and (25)

$$\sigma_{a4} = \int_{r_{04}}^{r_{14}} 2\pi r^2 n(r_{04})\left(\frac{r}{r_{04}}\right)^{3(1-S_4)} dr = \frac{\sigma_1(r_{14})n(r_{14})r_{14}}{3(2 - S_4)}, \tag{36}$$

where

$$\sigma_1(r_{14}) = 2\pi r_{14}^2, \tag{37}$$

and, taking into account only the sedimentation of such large particles.

$$n(r_{14}) = \frac{3N_{04}(S_4 - 1)}{r_{04}U_s(r_{04})}. \tag{38}$$

5. RESULTS OF NUMERICAL CALCULATIONS OF THE SEDIMENTATION MODEL

Figures 2 and 3 shows the absolute concentration of the primary particles of cosmic dust (micrometeorites) with different masses at different altitudes in the upper atmosphere.

Figure 4 show the contribution of the particles of different masses to the aerosol scattering coefficient for three different wavelengths at an altitude $z = 100$ km (in the sedimentation model the relative distribution of the contribution of particles of different masses at altitudes of 30 ... 100 km does not depend on the altitude). Figure 5 shows the altitude behavior of the turbidity of the atmosphere (i.e., the ratio of the volume aerosol scattering coefficient σ_a to the Rayleigh scattering coefficient σ_{air}) for three wavelengths.

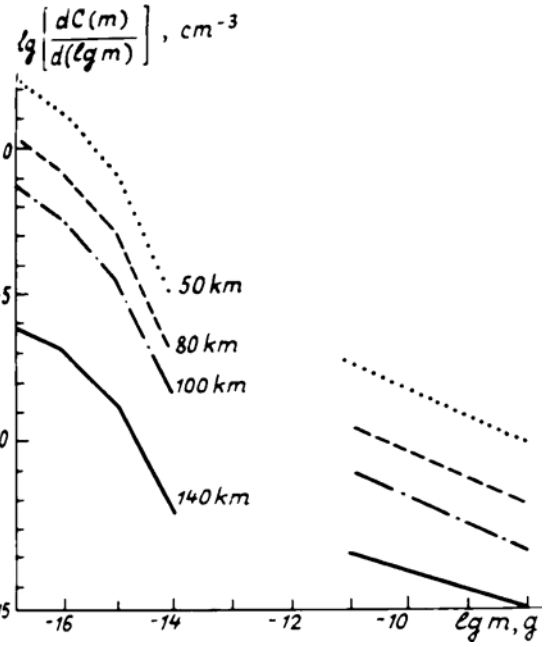

FIG. 2. The concentration of micrometeorites of different masses at altitudes of 50, 80, 100, and 140 km.

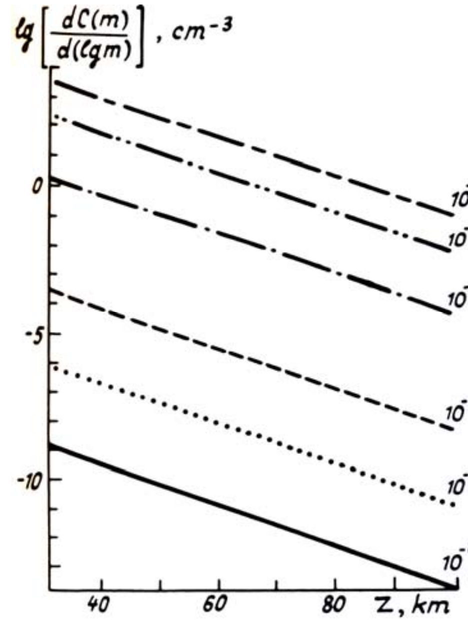

FIG. 3. The altitude dependence of the concentration of micrometeorites with masses of 10^{-17}, 10^{-16}, 10^{-15}, 10^{-14}, 10^{-11}, *and* 10^{-8} g.

As one can see from Fig. 4 very small micrometeorites with masses of 10^{-16} ... 10^{-14} g make the main contribution to the aerosol scattering coefficient for wavelengths from 0.1 to 0.5 μm; the contribution of larger micrometeorites with masses exceeding 10^{-11} g is relatively small and decreases rapidly as the wavelength of the light decreases. In Refs. 6 and 8 it was shown that this what explains the ultraviolet excess brightness of the zodiacal light observed In a number of cosmic experiments.

The primary particles create only a small relative atmospheric turbidity. This is natural, since among such particles there are virtually no particles with masses 10^{-14} ... 10^{-11} g that scatter light most efficiently. Such aerosols can only be the products of condensation and coagulation of the meteoric matter in the atmosphere.

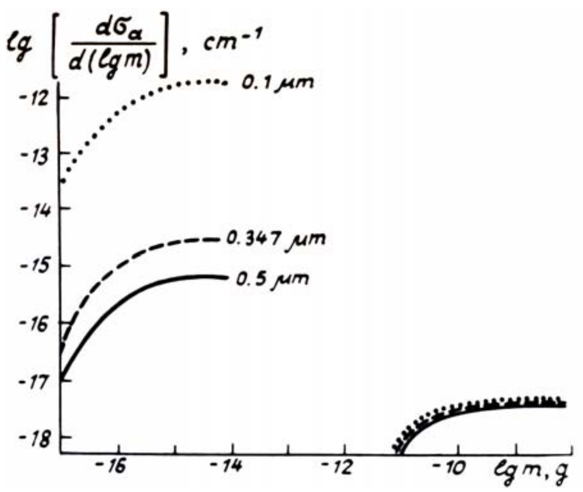

FIG. 4. The contribution of micrometeorites of different masses to the aerosol scattering coefficients for wavelength of 0.1 μm *(dots),* 0.347 μm *(dashed line), and* 0.5 μт *(solid line) at an altitude of z = 100 km.*

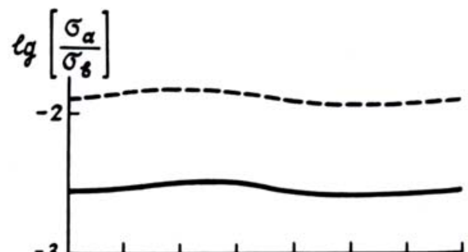

FIG. 5. The altitude behavior of the relative turbidity of the atmosphere for wavelengths of 0.1 μт *(dashed line) and* 0.34 *B and* 0.5 μm *(solid line).*

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