A FLEXIBLE MIRROR FOR ADAPTIVE COMPENSATION OF STATIONARY THERMAL BLOOMING

F.Yu. Kanev and S.S. Chesnokov

Moscow State University
Received August 3, 1988

In this paper we examine a mathematical model of a typical system of adaptive optics, whose active element is a flexible mirror. The influence of the constructive elements of the system on the process of focusing of radiation is determined by means of numerical experiment and recommendations for its practical development are worked out. On the basis of these conjugation and multidither algorithms an analysis is made of the limits of adaptive control of the wavefronts of light rays which are propagating under conditions of stationary thermal blooming.

One of the main factors limiting the efficiency of atmospheric optics systems is the thermal blooming of high power laser beams. As both theoretical[1] and experimental[2] investigations have shown, thermal blooming strongly distorts light beams, thereby decreasing the power density of light beam at the observation point.

At present great attention has been given to theoretical studies aimed at the development of techniques for compensating the influence of thermal distortions. Amplitude-phase correction[4,5] and phase correction[3,6] algorithms have been widely discussed in the literature as well as methods of real-time dynamic beam control[7,8]. At the same time, the practically important question of the influence of the mechanical properties of the elastic corrector on the maximum achievable beam focusing under stationary conditions has been insufficiently explored.

This paper deals with a theoretical analysis of the limiting capabilities of the phase monitoring of beams propagating under conditions of stationary thermal blooming. Two basic algorithms are compared in the paper, viz. the methods of phase conjugation and the method of aperture sensing. The comparison is made by means of the basic model of an adaptive system[9] the final control element of which is a flexible mirror.

NUMERICAL MODEL OF AN ADAPTIVE SYSTEM

A typical adaptive system is composed of a phase corrector, optical channel, feedback system and a unit of control signal generation.

In this paper the phase corrector is assumed to be a thin flexible mirror of octagonal shape, hinged at its center. This mirror is deformed by a system of transverse forces. The bending $W(x, y)$ of the middle surface at this mirror, which coincides approximately with the reflecting surface can be described by the following equation[10]

$$D(\partial^4 W/\partial x^4 + 2\partial^4 W/\partial x^2\partial y^2 + \partial^4 W/\partial y^4) = f(x,y) \quad (1)$$

with third-order boundary conditions on the free portions of the contour and second-order boundary conditions at the hinge point. Here D is the cylindrical rigidity, $f(x, y)$ is the transverse distributed load.

Propagation of the quasicontinuous radiation through a moving weakly absorbing medium is described by a system of equations in the complex amplitude of the electric field $E(x, y, z)$ and the temperature perturbations $T(x, y, z)$:

$$2ik\partial E/\partial z = \partial^2 E/\partial x^2 + \partial^2 E/\partial y^2 + 2k^2/n_0(\partial n/\partial T)TE; \quad (2)$$

$$\rho C_p V\partial T/\partial x = \alpha n_0 c/(8\pi)|E|^2, \quad (3)$$

where ρC_p is the heat capacity per unit volume, V is the velocity of the medium movement along the x axis, n_0 is the unperturbed value of the refractive index. The information on the refractive index inhomogeneities along the path is contained in the wave reflected from the target $\Psi(x, y)$. This wave propagates through the thermal field induced by the direct wave,

$$2ik\partial\Psi/\partial z = \partial^2\Psi/\partial x^2 + \partial^2\Psi/\partial y^2 + 2k/n_0(\partial n/\partial T)T\Psi. \quad (4)$$

The distribution of the field Ψ in the plane $z = 0$ is used to form the wavefront of the transmitted wave at the next iteration. Thus, in particular, for the case of the phase conjugation algorithm

$$U_{n+1} = -\varphi_n, \quad (5)$$

where $\varphi(x, y) = \arg\Psi(x, y)$ is the phase of the reflected wave, n is the iteration number. In order to improve the stability of the correction procedure, one is also advised to use the modified phase conjugation algorithm[6].

$$U_{n+1} = U_{max} - \beta_n (U_{max} + \varphi_n), \qquad (6)$$

where U_{max} is the phase profile which provides the best compensation of the distortions introduced at all the preceding iterations, β_n is a positive quantity which is decreased by a factor of two at each correction failure.

In the case of cross-aperture sensing

$$U_{n+1} = U_n + \beta_n \mathrm{grad} j_n, \qquad (7)$$

where the components of the quality criterion gradient (the object function j_n of the control) is calculated following the test variation procedure. In this paper we used the focusing criterion as the object function j_n of the control

$$j = cn_0/(8\pi) \iint \exp(-(x^2+y^2)/S_t^2) |E|^2_{z=z_n} , \qquad (8)$$

This criterion is proportional to the power of the radiation incident upon the output aperture of radius S_t.

Equation (1), which describes the mirror bending, was solved in this work using a finite element technique[11], while Eqs. (2) and (3) were solved by the method of splitting into physical factors using the Fast Fourier Transform[12]. Beam propagation along short paths ($z_0 \le 0{,}15 z_d$) was described using a lens coordinate system[13].

LIMITS OF APPLICABILITY OF PHASE CONTROL

The limiting capabilities of control of the characteristics of light beams propagating in a nonlinear medium can be properly determined on the basis of the model of an ideal corrector which reproduces the required wavefront without any limitations. The results of our numerical simulations, obtained for a variety of pathlengths and beam powers, are presented in Table 1.

Table 1.

Phase compensation of blooming by an ideal corrector

Distance to the focus	0.1			0.3		0.8
Nonlinearity Parameter $\|R\|$	50	60	70	20	40	20
j_0	0.51	0.39	0.33	0.29	0.06	0.01
j_{opt}	0.74	0.55	0.51	0.48	0.10	0.05
η, %	45	41	60	56	71	500

This Table gives the values of the criterion j_0 obtained for nonadaptive focusing on the target and the values j_{opt} obtained using the modified phase conjugation method (6). The Table gives normalized values, i.e., the pathlength is normalized by the diffraction length of the beam z_d and the focusing criterion is normalized by the diffraction-limited value j. The radius of the output aperture is taken close to the radius of the focal spot in a vacuum. The degree of nonlinear distortion of a beam propagating through a moving medium with stationary refraction is characterized by the dimensionless nonlinearity parameter

$$R = 2I_0 k^2 \alpha a_0^3 (\partial n/\partial T) / (\rho C_p V n_0). \qquad (9)$$

which is proportional to the beam power $I_0 a_0$ and to the interaction time $\tau = a/V$ (I_0 is the peak value of the beam incident on the medium).

The efficiency of the phase correction of the nonlinear distortions can be clearly demonstrated using the following criterion

$$\eta = ((j_{opt} - j_0)/j_0)\ 100\% \qquad (10)$$

The values of this criterion are also presented in Table 1. The data presented in Table 1 allow one to construct isopleths of j_0 and j_{opt} on the plane of the variables z_0 and R (Figs. 1a, 1b).

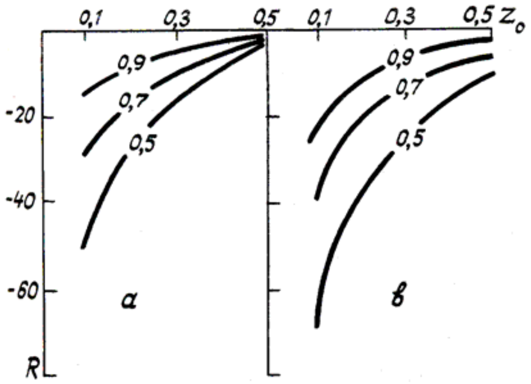

Fig. 1. Equal value curves of normalized focusing criterion (8) on the plane of the variables R, z_0; a — nonadaptive focusing at the observation plane; b — modified phase conjugation.

The analysis of the results presented enables one to determine the region of pathlengths and beam powers in which phase control increases the power density at the object up to the preset level. In particular, it can be seen that the relative increase of the criterion j (the efficiency of focusing) grows with increase of the nonlinear distortions of the beam. At the same time the absolute values of the criterion j under conditions of strong distortions (long paths, strong nonlinearity) remain constant. One can see from Fig. 1 that the region where the correction Is efficient is much wider for shorter paths.

OPTIMIZATION OF THE FLEXIBLE MIRROR

The calculational model developed in this paper of a phase corrector in the form of a flexible mirror allows one to solve some problems of practical importance. In first place is the problem of the determination of the number and configuration of the servodrives which are needed to provide the required beam parameters in the observation plane with the required accuracy.

Consider now the problem of focusing a Gaussian beam in a linear medium using a flexible mirror hinged at the center and deformed with a system of transverse forces as shown in Fig. 2. Let the beam focusing be performed in two ways. The first one consists of matching the mirror bending at the control points to the ideal phase surface $\varphi_{opt} = k(x^2 + y^2)/2x_0$ which provides for optimal focusing of the Gaussian beam at the distance z_0. The second way consists of maximizing the peak intensity I_{max} in the observation plane.

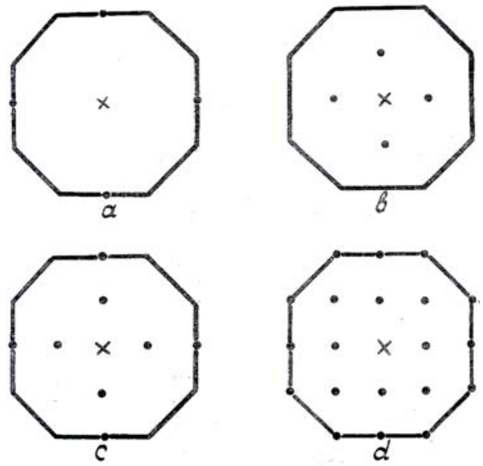

Fig. 2. Configuration of fastening of the adaptive mirror servodrives.

Calculated results are presented in Table 2, and in Fig. 3, where the mirror profiles (solid lines) obtained by matching to the parabolic surface (dashed lines) are shown. The size of the mirror is $L_R = 10\ a_0$.

Table 2.

Peak intensity I_{max}/I_{opt} in the local plane with focusing by a flexible mirror in a linear medium

Distance to the focus	Drive configuration	Drive number	MIRROR CONTROL	
			Coincidence at points N	Aperture sensing
0.25	a	4	—	0.72
	b	4	0.30	0.66
	c	8	0.90	—
	d	20	0.91	—
0.5	a	4	0.56	0.91
	b	4	0.80	—
	c	8	0.94	—
	d	20	0.95	—

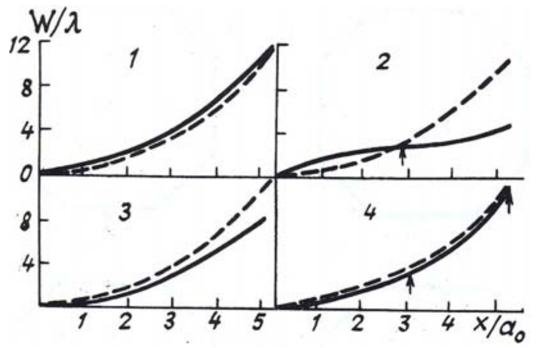

Fig. 3. Focusing a beam in a linear medium. Solid lines — the mirror profile; dashed lines — the parabolic phase φ_{opt}; arrows are points of fastening servo- drives. 1, 2, 4 — mirrors matching at the controlled points with the parabolic phase φ_{opt}; 3 — "going uphill"; 1, 3 — configuration (a) in Fig. 2; 2, 4 — configuration (b) and (c).

As can be seen from Fig. 3 and Table 2, in the control by surface matching the profile of the mirror with four drives differs considerably from the parabolic surface, which results in decrease of the focusing quality for short paths. Similar dependence on pathlength is also observed for the case of the cross-aperture sensing technique for 4, although, on the whole, the latter algorithm enables one to substantially increase the field density in the local plane. Finally, control at eight points provides the practical possibility of achieving the diffraction-limited quality independent of pathlength.

FOCUSING IN A NONLINEAR MEDIUM

In the analysis of the focusing quality in a nonlinear medium based on the phase conjugation method, the shifts of the controlled points of the mirror were assumed to be equal to the values of the reflected-wave phase ip(x,y) taken at the same points but with the opposite signs. In the cross-aperture sensing algorithm the of focusing criterion j was maximized using the technique of "going uphill".

The calculated values of j obtained using these methods are given in Table 3, where the results of modeling the ideal corrector[3], which does not distort the formed wavefront, are also presented for comparison.

Analysis of the data in this Table shows that when the number of points under control is N ≥ 8 the use of a flexible mirror based on the phase conjugation algorithm yields only a weak decrease of the focusing quality in a nonlinear medium compared with that provided by the ideal corrector. Increasing of the servodrive number from 8 to 20 leaves the characteristics of the field of the target practically unchanged.

Comparison of the data obtained using the phase conjugation method and the method of cross-aperture sensing, in which the slave mechanism is a flexible mirror, shows that the characteristics of the field at the target are approximately the same. It should be noted, at the same time, that the phase conjugation technique has a much faster response. On the other hand the use of the cross aperture sensing technique allows one to decrease the number of points under control (servodrives) by almost a factor of two with practically the same quality of correction as in the phase conjugation method.

Table 3.

Phase compensation of blooming by a flexible mirror and ideal corrector

Distance to the focus z_0/z_d	$\|R\|$	Phase conjugation			Aperture sensing	Optimal control
		Mirror N=4	Mirror N=20	Ideal corrector	Mirror N=4	Ideal corrector
0.1	50	0.69	0.71	0.78	0.66	—
	70	0.49	0.51	0.52	0.46	—
0.5	10	0.57	0.60	0.71	0.57	0.74
	20	0.42	0.43	0.46	0.48	0.58

REFERENCES

1. K.D. Egorov, V.P. Kandidov, and S.S. Chesnokov, Izv. Vyssh. Uchebn. Zaved., SSSR; Ser. Fizika, No. 2, 66 (1983).

2. J.W. Strohbehn (ed.), *Laser Beam Propagation in the Atmosphere* Springer—Verlag, New York (1978).

3. S.A. Akhmanov, et al., Izv. Vyssh. Uchebn. Zaved., SSSR, Ser. Radiofizika, **23**, No. 1, 1 (1980).

4. V.A. Vysloukh, K.D. Yegorov, and V.P. Kandidov, Vestnik MGU, Fizika, Astronomiya, No. 2, 16 (1980).

5. V.A. Trofimov, Vestnik MGU, Fizika, Astronomiya, No. 5, 36 (1987).

6. F.Yu. Kanev and S.S. Chesnokov, Kvant. Elektronika, **14**, No. 10, 2125 (1987).

7. P.A. Konyaev and V.P. Lukin, Izv. Vyssh. Uchebn. Zaved. , SSSR, Ser. Fizika, No. 2, 79 (1983).

8. V.A. Visloukh, V.P. Kandidov, S.S. Chesnokov, and S.A. Shlenov, Izv. Vyssh. Uchebn. Zaved., SSSR, Ser. Fizika, No. 11, 30 (1985).

9. S.S. Chesnokov, Izv. Vyssh. Uchebn. Zaved., SSSR; Ser. Radiofizika, **30**, No. 10, 1213 (1987).

10. P.M. Ogibalov, *Bending, Stability and Vibrations of Plates* (Izdat. MGU, Moscow, 1958).

11. V.P. Kandidov, S.S. Chesnokov and V.A. Visloukh, *Finite Element Methods in Dynamic Problems* (Izdat. MGU, Moscow, 1980).

12. S.S. Chesnokov, Vestnik MGU, Fizika, Astronomiya, No. 6, 27 (1980).

13. V.I. Talanov, Pis'ma v Zh. Eksp. Teor. Fiz. No. 4, 303.