EVALUATION OF A SIGNAL FROM REMOTE OBJECT WITH THE USE OF A MATRIX PHOTODETECTOR

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The problem of detection of a signal from stars and meteors with the use of an IR matrix detector to estimate their parameters and the parameters of the atmosphere is discussed. It is shown that the uncertain position of the spot of an object image on the receiving matrix leads to the essentially different statistical characteristics of the signal. The mathematical expectation and variance of the signal are calculated both with and without regard for the signal absorption in the gaps between the sensitive elements. The obtained results are used to explain some experimental data.

In the study of some remote objects (stars, planets, meteors, etc.) with the help of an IR matrix photodetector it is often necessary not only to obtain some information about the object coordinates, but also to evaluate the irradiance of an input pupil produced by the objects. Among these problems are determination of the temperature of objects from their IR radiation and correction for signal distortions due to the atmosphere using several reference radiation sources.[1–4]

The measurement of the IR radiation parameters of distant objects by a matrix photodetector has a number of salient features. Thus due to the uncertainty in the object image position in the frame or the image shift when recording a moving object the statistical characteristics of the signal differ from that obtained during bench testing of a photodetector that requires the perfect matching of the image with one of the matrix elements or implies obtaining of these characteristics with uniform illumination of the photodetector matrix.

In this paper the decrease of the valid signal amplitude and the increase of its variance are calculated as a function of the ratio of the image size and photosensitive matrix element size, transit velocity of the image across the matrix, and method of signal processing. The method of detection of the low–contrast stationary objects with the moving field of view of a detector is discussed. The revealed regularities are confirmed by the experimental data obtained with the use of a detector of stars.

1. ESTIMATE OF THE IRRADIANCE OF INPUT PUPIL FROM REMOTE OBJECT BY MAXIMUM SIGNAL

We consider the problem of detection of distant object with a matrix detector when the size of the blur circle on the objective is matched with the matrix element size. The average sensitivity and dynamic range of the IR radiometer are determined by way of successive focusing of radiation onto several arbitrarily chosen elements located at the centre and on the edges of the matrix. To correct for the nonuniform sensitivity of elements, the data on their responce have been obtained with the uniformly irradiated matrix. The sensitivity of the elements is considered to be calibrated by processing of these indications.

Now we estimate the variation of the basic characteristics of the radiometer during field measurements, that is, for arbitrary initial position of the image spot, as well as for moving image. The radiometer will be considered as an inertialess one. The amplitude of the signal from the element of area with maximum irradiance at the instant of sampling is taken as the irradiance from a point source.

Let the source image move across the matrix of sensitive elements. The behavior of the irradiance of the ith matrix element is described by the expression

$$E(t, x_0, y_0, \nu) = E_0\, f((y_0 - y_i + \nu_y t)/a, (x_0 - x_i + \nu_x t)/a), \quad (1)$$

where E_0 is the irradiance of the ith element whose center exactly coincides with the image center; x_i, y_i are the coordinates of the center of the ith element; x_0, y_0 specify the initial position of the image center; a is the radius of the blur circle; $\nu_{x,y}$ is the transit velocity of the image; t is time from the beginning of the frame; $f(\alpha, \beta)$ describes the law of running of the image spot onto the detector area. Let the spot size be matched to the size of an element of area ($r = a$, where r is half the distance between the neighbouring elements of areas in a row or a column). The concrete form of the law of running of the image of a point object onto the sensitive element of area of the detector depends on the energy distribution through the blur circle of an objective and distribution of the sensitivity over the element of area of the detector. Neglecting the signal absorption in the gaps between the sensitive elements for uniformly sensitive element of area, the law of running of the blur circle, whose size is matched with the size of the element of area, can be quite conveniently and adequately approximated with the square-cosine function[6]

$$f(\alpha, \beta) = \cos^2\left(\frac{\pi}{4}\alpha\right)\cos^2\left(\frac{\pi}{4}\beta\right). \quad (2)$$

For linear inertialess radiometer the signal from the ith matrix element is given by the formula

$$S_i = S_0 \cos^2 \frac{\pi}{4}\left(\frac{y_0 - y_i + \nu_y t}{r}\right) \cos^2 \frac{\pi}{4}\left(\frac{x_0 - x_i + \nu_x t}{r}\right). \quad (3)$$

As indicated above, for the estimate of the irradiance from a point source we take the signal from the element of area with maximum irradiance at the instant of sampling. Let this signal be obtained from the ith element of area at the instant t. This means that the image center must be located within this element of area, that is

$$y_i - r < y_0 + \nu_y t < y_i + r,$$

$$x_i - r < x_0 + \nu_x t < x_i + r. \quad (4)$$

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Averaging Eq. (3) over the possible initial locations of the image spot, we obtain the mathematical expectation of the signal

$$\overline{S}_i = \frac{1}{(2\,r)^2} \int\limits_{y_i - r - v_y t}^{y_i + r - v_y t} \mathrm{d}\,y_0 \int\limits_{x_i - r - v_x t}^{x_i + r - v_x t} \mathrm{d}\,x_0\, S_i(x_0, y_0) =$$

$$= S_0 \left(1 + \frac{2}{\pi}\right)^2 \Big/ 4 = 0.670\, S_0\ . \qquad (5)$$

In this case the signal variance due to the uncertainty in the position of the blur circle reaches the value

$$\sigma^2 = \frac{1}{(2\,r)^2} \int\limits_{y_i - r - v_y t}^{y_i + r - v_y t} \mathrm{d}\,y_0 \int\limits_{x_i - r - v_x t}^{x_i + r - v_x t} \mathrm{d}\,x_0\, (\overline{S}_i - S_i)^2 =$$

$$= S_0^2 \left(\frac{3}{4} + \frac{2}{\pi}\right)^2 \Big/ 4 - S_0^2 \left(1 + \frac{2}{\pi}\right)^4 \Big/ 16 = 0.0323\, S_0^2. \qquad (6)$$

By going to relative units, we obtain $\sigma_{\mathrm{rel}} = 0.27$.

In real matrix of photodetectors the signal absorption in the gaps between the matrix elements reaches significant value. The law of running still remains a smooth curve, which takes the values less than 0.5 while the center of blur circle crossing the boundaries between the two elements. In this case the bell-shaped function is a good approximation. Let us assume that

$$f(\alpha, \beta) = \exp(-\,\alpha^2 - \beta^2)\ , \qquad (7)$$

which corresponds to the absorption of ~26% of signal when the image is formed on two neihbouring elements of area. In this case the signal from the ith element is described by the expression

$$S_i = S_0 \exp\left\{-\left(\frac{y_0 - y_i + v_y\, t}{r}\right)^2 - \left(\frac{x_0 - x_i + v_x\, t}{r}\right)^2\right\}\ . \qquad (8)$$

An averaging over the initial image location (x_0, y_0) yields the mathematical expectation and the variance of the signal

$$\overline{S}_i = \frac{S_0\, \pi\, erf^2 1}{4} = 0.556\, S_0\ , \qquad (9)$$

$$\sigma^2 = \overline{S_i^2} - \overline{S}_i^2 = \frac{S_0^2\, \pi\, erf^2 \sqrt{2}}{8} - \frac{S_0^2\, \pi^2\, erf^4 1}{16} = 0.05\, S_0^2\ ,$$

$$\sigma_{\mathrm{rel}} = \frac{\sigma}{\overline{S}_i} = 0.41\ . \qquad (10)$$

It is seen that during field measurements the average value of the signal from distant object and the signal variance differ significantly from the nominal values and depend on the relation between the image size and the sensitive element size as well as on the gap size since the form of the law of running is determined by all these parameters. It should be mentioned that the transit velocity of image spot across the photosensitive matrix has no effect on the shape of the signal until the time required for the image to cross the element of area is longer than the photodetector time constant. The further increase of the transit velocity leads to the further decrease of the valid signal.

2. ESTIMATE OF THE IRRADIANCE OF THE INPUT PUPIL, PRODUCED BY DISTANT OBJECTS, FROM THE NET SIGNAL

Up to now we have analyzed the estimate of the irradiance from maximum signal. However, for significant excess of the signal above the threshold or when the center of the image is on the edge of the sensitive element, the signals from the neighboring elements become comparable with the maximum signal. Let us consider the processing scheme, which implies the net signal from the central and eight neighboring elements of area being the estimate of the irradiance of the input pupil of radiometer from a distant object. In calculations we use the second form of the law of running given by Eq. (7).

The net signal is calculated according to the formula

$$S_{i\Sigma} = S_0 \sum_{p=-1}^{1} \sum_{q=-1}^{1} \exp\left\{-\left(\frac{x_0 - x_i + v_x\, t}{r} + 2\,p\right)^2 - \left(\frac{y_0 - y_i + v_y\, t}{r} + 2\,q\right)^2\right\}\ . \qquad (11)$$

The maximum signal for this estimate is equal to $1.074\, S_0$. Assuming that the image center can be arbitrary located on the central element of area, the mathematic expectation can be calculated from the formula

$$\overline{S}_{i\Sigma} = \frac{1}{(2\,r)^2} \int\limits_{y_i - r - v_y t}^{y_i + r - v_y t} \mathrm{d}\,y_0 \int\limits_{x_i - r - v_x t}^{x_i + r - v_x t} \mathrm{d}\,x_0\, S_{i\mathrm{R}}(x_0, y_0) =$$

$$= \frac{S_0\, \pi}{4}\left(erf^2 1 + 4\, erf 1 \left(\frac{1 - erf 1}{2}\right) + 4\left(\frac{1 - erf 1}{2}\right)^2\right) =$$

$$= 0.785\, S_0\ . \qquad (12)$$

Rather tedious but simple calculations yield the signal variance

$$\sigma_{i\Sigma} = 0.133\, S_0\ , \qquad \sigma_{\mathrm{rel}} = \frac{\mathrm{s}_{i\Sigma}}{\overline{S}_{i\Sigma}} = 0.17\ . \qquad (13)$$

It is seen that this value is half the signal variance obtained by the first scheme of image operation.

Thus the change of scheme of signal processing leads to the increase of average signal from $0.556\, S_0$ to $0.785\, S_0$.

The results of our calculations allow the conclusion that the dynamic characteristic of the radiometer operating under real conditions differs from the standard characteristic obtained when the image of a point source is centered. In fact, conversion from the signal usually measured in volt, amper, or other relative units to the irradiance of the input pupil according to the standard formula allows obtaining the irradiance averaged over the spot location.

$$\overline{E}_{\mathrm{st}} = \overline{S}_i\, (E_{\mathrm{th}}^0 / S_{\mathrm{th}}^0)\ , \qquad (14)$$

where $E^0_{\rm th}$ and $S^0_{\rm th}$ are the threshold irradiance of the input pupil of a photodetector and the value of the threshold signal, obtained in bench testing. But the true value of the irradiance

$$\overline{E} = \overline{E}_{\rm st}\,(S_0/\overline{S}_i)\ . \tag{15}$$

exceeds $\overline{E}_{\rm st}$.

Substituting Eq. (14) into Eq. (15), we can estimate a new threshold value of the irradiance of input pupil

$$E_{\rm th} = E^0_{\rm th}\,(S_0/\overline{S}_i)\ . \tag{16}$$

For the law of running described by Eq. (2) the real threshold is equal to $1.5\,E^0_{\rm th}$. For the law of running described by Eq. (7) $E_{\rm th}$ is almost twice as large as $E^0_{\rm th}$. In addition, the increase of the signal variance, as the irradiance of the input pupil increases, should be taken into account under real conditions. The signal processing using net signals leads to the substantial decrease of the threshold level and its approach to the nominal value (for example, from $1.8\,E^0_{\rm th}$ to $1.3\,E^0_{\rm th}$ with allowance made for the signal losses in the gaps between the elements of area). In this case the error in estimating the irradiance from the object decreases by more than half (from 0.41 to 0.17).

3. EXPERIMENTAL DATA ANALYSIS

The obtained results were used for an analysis of the data of observation of Betelgeuse star with the use of the cooled matrix IR radiometer placed on board an aircraft laboratory. To facilitate the identification of the signal from the star against the induced images and matrix defects, the method of moving field of view was applied[5]: the aircraft banked turn causes the image moving across the matrix of light sensitive elements at a speed of one matrix element per frame. In individual experiment the star was recorded on 6–8 successive frames. The irradiance from the star in every frame was evaluated as a sum of signals from the central (i.e., having the maximum signal) and neighboring elements.

As a result of these experiments, the average irradiance from the star and its variance were estimated. It should be mentioned that Betelgeuse is the variable star, which is why the average values of irradiance obtained in different experiments may differ but they have to fall within the interval of possible values of irradiance of input pupil ($4\cdot10^{-13}...2\cdot10^{-12}$ W/cm^2). However, from the four experiments the two experiments gave the average irradiance values below the lower limit (see Table I).

TABLE I.

$\overline{E}_{\rm st}$	$3.9\cdot10^{-13}$	$7.95\cdot10^{-13}$	$3.6\cdot10^{-13}$	$8.64\cdot10^{-13}$
$\overline{E}$	$5.3\cdot10^{-13}$	$1.09\cdot10^{-12}$	$4.9\cdot10^{-13}$	$1.18\cdot10^{-12}$
σ	$6.0\cdot10^{-14}$	$1.06\cdot10^{-13}$	$6.5\cdot10^{-14}$	$9.5\cdot10^{-14}$
$\sigma_{\rm rel}$	0.16	0.13	0.18	0.11

Moreover, the experimental estimate of the signal variance significantly exceeds the variance obtained in bench testing ($2.5\cdot10^{-15}$ W/cm^2). This fact can be easy explained by the uncertainty in the image position on the matrix. According to the results obtained in this paper, the standard values must be increased by the factor of $S_{\rm max}/\overline{S}$ from Betelgeuse. Now $\overline{E}$ falls within the interval of possible values of irradiance. The experimentally obtained relative variance $\sigma_{\rm rel}$ agrees fairly well with theoretically predicted variance $\sigma_{\rm rel} = 0.17$.

Thus an account of uncertainty in the location of the image of distant object on the photodetector matrix enables one to correct the obtained quantitative estimates of the irradiance of input pupil.

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