# ANALOG METHOD FOR PHASE RETRIEVAL WITH THE USE OF THE HARDY RESONATOR

**M.A. Kazaryan, T.I. Kuznetsova, and Yu.P. Vasil'ev**

*P.N. Lebedev Physical Institute of the Russian Academy of Sciences, Moscow*
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*It is shown that with the help of the Hardy resonator in the absence of an object, one can obtain its three-dimensional image with the use of two intraresonator amplitude transparencies.*

This paper is based on the following properties of the Hardy resonator (Ref. 1): as known, the high-Q free oscillations that have quite complex transverse structure capable of image transfer are inherent in this resonator. It is essential that the field distributions across the mirrors are either identical or one distribution represents a magnified copy of another for different curvature radii of the mirrors; in this case, not only the field amplitude distribution, but also the phase distribution is reproduced. This resonator was tested by us together with an amplifying unit containing strontium vapor for obtaining magnified images of small-scale objects. We believe that with the help of the Hardy resonator in the absence of the object, its three-dimensional image can be obtained using two amplitude transparencies fabricated in the following way.

Let the initial amplitude-phase object be placed on a spherical surface with radius $R_1$. Let it be illuminated with the monochromatic radiation and the field reflected from the object takes the form $E(x, y, z)\exp(-i\omega t)$. Let us place the origin of coordinates at the center of the sphere and designate the spatial field distribution over the sphere as $f(x, y)$:

$$f(x, y) = E(x, y, \sqrt{R_1^2 - x^2 - y^2}\ ).$$

For simplicity, we consider further that $f(x,y) = f(-x, -y)$. The field in the plane $z = 0$ is

$$E(x,y,0) = \frac{ik}{R_1} \exp\left(ik\, R_1 + ik\, \frac{x^2 + y^2}{2R_1}\right) \tilde{f}\left(\frac{kx}{R_1}, \frac{ky}{R_1}\right).$$

Here, $\tilde{f}$ is the Fourier transform of the function $f$:

$$\tilde{f}\left(\frac{kx}{R_1}, \frac{ky}{R_1}\right) = \frac{1}{2\pi} \int f(x_1, y_1) \exp\left(-ik\frac{xx_1 + yy_1}{R_1}\right) \mathrm{d}x_1\, \mathrm{d}y_1$$

The transparencies with the transmissions

$$a(x, y) = |f(x, y)|,\ A(x, y) = \frac{k}{R_1} \left|\tilde{f}\left(\frac{kx}{R_1}, \frac{ky}{R_1}\right)\right|.$$

can be obtained by taking photographs.

If the transparencies $a(x, y)$ and $A(x, y)$ are placed in the Hardy resonator near the mirror with radius $R_1$ and the lens, respectively, and the amplifying element is placed between this mirror and lens, the transverse structure of the field established when lasing builds up from the spontaneous noise in such a laser will be described by the equation

$$f_{n+1}(x, y) = \hat{F}\,[A^2\, \hat{F}\,(a^2\, f_n(x, y))],$$

where $f_n$ is the field after $n$th passage, coming to the mirror with radius $R_1$ and $\hat{F}$ is the Fourier transform operator.

As follows from numerical analysis (Ref. 2), the solution of the equation converges to the function close to the initial function $f(x, y)$. This means that for sufficient number of passages in this resonator the complex field is established that differs insignificantly from the field produced by the initial amplitude-phase object.

Essential peculiarity of this scheme is that if we have the initial field on the mirror $R_1$, on the mirror with radius $R_2$ we will have the field $.f\left(\frac{R_1}{R_2}x, \frac{R_1}{R_2}y\right)$.

## REFERENCES

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