Modulator of laser radiation based on nonlinear ring interferometer: model and analysis of characteristics

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We propose an optical arrangement of a modulator of laser radiation based on nonlinear ring interferometer (NRI). In this instrument the information microwave signal affects the initially isotropic nonlinear (Kerr) medium changing its optical length. A dynamic model of the nonlinear phase change in the nonlinear medium is developed under certain approximations. Series of bifurcation diagrams and modulation characteristics, demonstrating the capabilities of the modulator, are presented.

Introduction

With the development of nonlinear optics, laser physics, and information technologies, the interest has grown to the problem of optical information processing. The use of phenomena of nonlinear dynamics allows one to develop new methods of information protection and its secure transmission.[1,2] Some problems of opto-informatics employ the spatial and spatiotemporal light modulators.[3,4] However, specialists developing such modulators, in particular, to improve the efficiency of control over laser radiation, almost ignore the methods of nonlinear dynamics. The experience of using these methods is gained in the study of radio[5] and optical[6–8] generators with delayed feedback.

Taking the previously mentioned into account, it is worth revealing the possibilities of creating the new types (optical in both the operating principle and in the components) of optical modulators (in particular, self-modulators) and generators of chaotic electromagnetic fields. A nonlinear ring interferometer (NRI) is one of the devices suitable for this purpose.[9] In developing a modulator based on the NRI, it is possible to make use of the ideas of the processes of field interaction in NRI and the corresponding mathematical models, as well as known techniques of description and calculation of characteristics of light modulators.[10,11] The main goals of this paper are the following:

– to describe the optical arrangement of the modulator based on the NRI and to construct the model of processes in it;

– to construct the series of bifurcation diagrams, to study the peculiarities of their structure, and to select the ranges for parameters to be modulated within;

– to construct some modulator characteristics in thus found ranges, and to analyze them.

1. Optical arrangement of the modulator and its mathematical model

To analyze and describe the processes in the NRI-based modulator, it is logical to consider it as a resonator in the non-autonomous regime, that is, when the information signal affects the initially isotropic nonlinear medium. The arrangement is shown in Fig. 1, where E_{in} and E_{out} are the amplitudes of the field at the NRI input and output; NM is the nonlinear medium of the length L; G is an element of the large-scale transformation of the light field (field elongation, shift, rotation); M_1 and M_2 are semitransparent mirrors with the intensity reflection coefficient R; M_3 and M_4 are the mirrors with the reflection coefficient $R = 1$; V_{mw} is the source of the external modulating microwave signal with the amplitude E_{mw}.

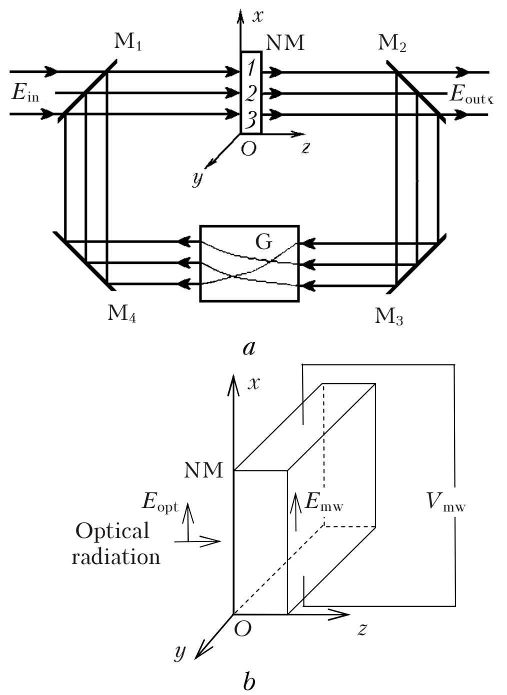

Fig. 1. Optical arrangement of the NRI. Orientation of the electric field strength vectors of the optical radiation E_{opt} (*a*) and the external modulating microwave signal E_{mw} (*b*); light rays *1*, *2*, *3*.

The processes in the ring resonator are governed by the NM influence and, given the level of medium nonlinearity, by the rate of diffusion of polarized

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molecules in the liquid-crystal medium. The refractive index $n(\mathbf{r}, t)$ obeys the equation

$$\tau_n(\mathbf{r})\frac{dn(\mathbf{r},t)}{dt} = n_2(\mathbf{r})\mathbf{E}^2(\mathbf{r},t) - -[n(\mathbf{r},t) - n_0(\mathbf{r})] + D_e(\mathbf{r})\Delta_{xy}[n(\mathbf{r},t) - n_0(\mathbf{r})], \quad (1)$$

where $\tau_n(\mathbf{r})$ is the relaxation time of the nonlinear part of the NM refractive index; $\mathbf{r}$ is the radius vector of the plane xOy, whose absolute value (length) is normalized to the characteristic dimension $\mathbf{r}_0$ (beam radius); n_2 is the NM nonlinear refraction coefficient; n_0 is the linear part of the NM refractive index; $\mathbf{E}(\mathbf{r}, t)$ is the strength of the nonoptical field applied to NM; D_e is the equivalent diffusion coefficient, taking into account the diffusion of both NM molecules and charge carriers of the photoconductor; Δ_{xy} is the Laplace operator in the transverse coordinates.

Let the electric field of the optical radiation be polarized along the Ox axis and the light propagates along the axis Oz (Fig. 1*b*). According to the theory of the Kerr effect, under this action upon the electrooptical crystal NM, it becomes uniaxial. Its (induced) optical axis is directed along the Ox axis. Let the field strength E_{mw} of the external modulating microwave signal, acting on the crystal, be also oriented along the Ox axis. Then the effect caused by the modulating microwave signal also favors the formation of the induced optical axis (and the change in the optical length of the crystal), not changing its orientation.

From Eq. (1) we can obtain the model of processes in the modulator (in the approximation of slowly changing (for the period $T = 2\pi/\omega$) amplitudes a_{nL} and b_{nL} of the input fields and the amplitude E_{mw} of the external microwave signal; the phases $\varphi(\mathbf{r}, t)$; positions of the planes of polarization $\psi(\mathbf{r}, t)$ of the input fields; the delay time τ of the field in the resonator; slow relaxation of the nonlinear part of the refractive index), which has the following form in the approximation of single passage of the feedback loop (FL):

$$\tau_n(\mathbf{r})\frac{\partial U(\mathbf{r},t)}{\partial t} = f(\mathbf{r},t) - U(\mathbf{r},t) + D_e(\mathbf{r})\Delta_{xy}U(\mathbf{r},t),$$

$$f(\mathbf{r},t) = n_2(\mathbf{r})Lk[a_{nL}^2(\mathbf{r},t) + b_{nL}^2(\mathbf{r},t) + E_{mw}^2(\mathbf{r},t)] =$$

$$= K_{ab}(\mathbf{r},t,\mathbf{r}_n) + pK_{ab}(\mathbf{r}',t-\tau,\mathbf{r}) + \left[\frac{\gamma(\mathbf{r}',t)}{\sigma}\right]\{K_a(\mathbf{r},t,\mathbf{r}',t-\tau)\times$$

$$\times\cos[(1+q)\omega\tau + \varphi(\mathbf{r},t) - \varphi(\mathbf{r}',t-\tau) + \psi(\mathbf{r},t) - \psi(\mathbf{r}',t-\tau)] +$$

$$+K_b(\mathbf{r},t,\mathbf{r}',t-\tau)\cos[(1-q)\omega\tau + \varphi(\mathbf{r},t) - \varphi(\mathbf{r}',t-\tau) -$$

$$-\psi(\mathbf{r},t) + \psi(\mathbf{r}',t-\tau)]\} + K_{mw}(\mathbf{r},t), \quad (2)$$

where $U(\mathbf{r}, t)$ is the nonlinear phase change of the light wave in NM; $k = \omega/c$ is the wave number (ω is the frequency of the optical radiation); $\tau \equiv \tau(\mathbf{r}', t) = = t_e(\mathbf{r}', t) + U(\mathbf{r}', t - t_e(\mathbf{r}', t))/\omega$ is the time of propagation of the light field, coming through FL by the time t at the point $\mathbf{r}$ of the NM entrance plane from the point $\mathbf{r}'$ of the same plane, i.e. the full time of the light field travel through the interferometer (the full delay time of the light field); $t_e(\mathbf{r}', t)$ is the fraction of the light field propagation time τ, caused by the linear part $n_0(\mathbf{r})$ of the NM refractive index $n(\mathbf{r}', t)$ and by the propagation time $t_0(\mathbf{r}', t)$ of the field passed through the feedback loop ($t_e(\mathbf{r}', t)$ is the equivalent delay time in the resonator); q is the parameter of bichromaticity of the input light field; $\gamma(\mathbf{r}', t)$ is the doubled loss factor (doubled transmission coefficient) for a passage ($\gamma \in [0; 2]$), slowly varying for the time $T = 2\pi/\omega$; $p = 0$ for the high-loss approximation, but $p = [\gamma(\mathbf{r}', t)/\sigma/2]^2$ for the single-passage approximation;

$$K_{ab}(\mathbf{r}, t, \mathbf{r}_n) \equiv (1 - R)n_2(\mathbf{r}_n)Lk[a^2(\mathbf{r}, t) + b^2(\mathbf{r}, t)],$$

$$K_a(\mathbf{r}, t, \mathbf{r}', t - \tau) \equiv (1 - R)n_2(\mathbf{r})Lka(\mathbf{r}, t)a(\mathbf{r}', t - \tau)$$

and

$$K_b(\mathbf{r}, t, \mathbf{r}', t - \tau) \equiv (1 - R)n_2(\mathbf{r})Lkb(\mathbf{r}, t)b(\mathbf{r}', t - \tau)$$

are the nonlinearity coefficients; σ is the stretch coefficient of the cross section of the light beam;

$$K_{mw}(\mathbf{r}, t) \equiv n_2(\mathbf{r})LkE_{mw}^2(\mathbf{r}, t)$$

is the parameter, taking into account the effect of the microwave field on NM. Neglecting the diffusion of the NM molecules, by analogy with Ref. 12, we can obtain the description of the dynamics $U(\mathbf{r}, t)$ in the resonator NM in the point approximation.

When constructing the bifurcation diagrams (BDs), as well as the modulation characteristics, it is assumed that the field in the cross plane of the laser beam in FL is turned by an angle $\Delta = 2\pi/m$, where m is a prime number. The number m indicates how many passages in NRI are needed for the light beam, entering the nonlinear beam at an arbitrary point of the beam cross section, to come back to the same point. In our case, the approximation of single passage of the field through the FL is used, in which $p = (\gamma\eta(\mathbf{r}', t)/2)^2$, $q = 0$, $\varphi(\mathbf{r}, t) = \text{const}$, and $\psi(\mathbf{r}, t) = \text{const}$. The parameter ωt_e is treated as a phase shift, rather than a measure of delay $t_e(t)$.

2. Results of simulation

A bifurcation diagram of the static states is a graphical interpretation of the behavior of the solutions (2), corresponding to the static regimes of the system. The variety of BD shapes (Fig. 2), depending on the model parameters, allows us to expect that different form of modulated oscillations can be obtained. In this case, BDs themselves can be interpreted as static "transmission functions."

2.1. Bifurcation behavior in the model without field delay modulation

Consider the limiting case of no field delay in the FL ($t_e = 0$) at $K_{mw} = 0$, when the nonlinearity coefficient is $K = 5$, the loss factor is $\gamma = 0.5$, and

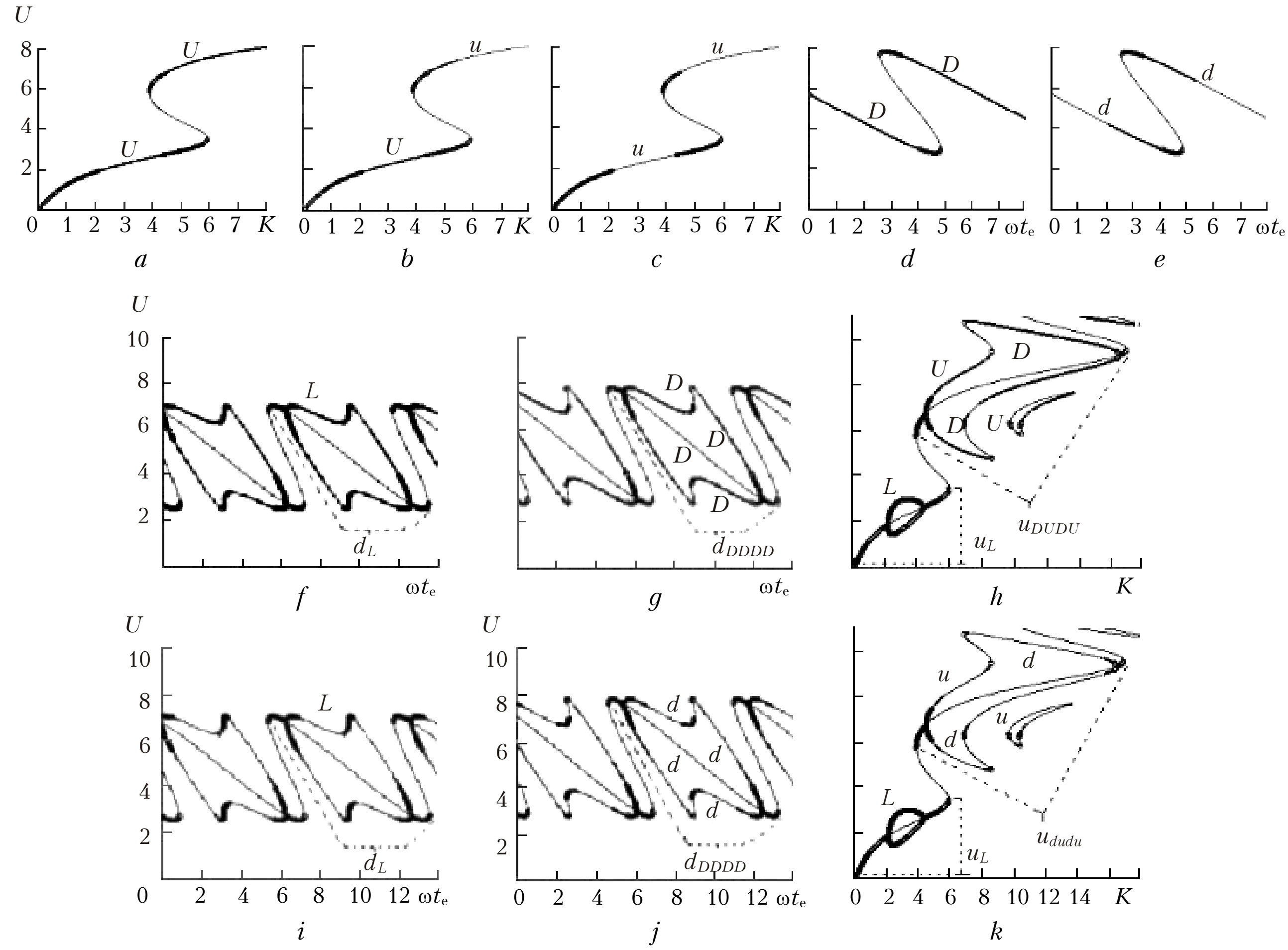

Fig. 2. BD forms, examples of hysteresis types (*a*–*k*) in BD and BD branches, containing bistability loops (*f*–*k*). Unstable states are shown by thin lines. Roman letters indicate the characteristic morphological elements of the BD structure. If areas, which can become unstable, are present in the structure of branches (bistability loops), then small letters are used in notations.

the field rotation is either absent ($m = 1$) or the rotation angle is 180° ($m = 2$). At $m = 1$ BD in the plane U–ωt_e consists of hystereses of the DD type (Fig. 2*d*), repeating with the period of 2π (at $q = 0$: if $q \neq 0$, then the period may differ from 2π [Ref. 13]). With this (moderate) degree of nonlinearity, the NRI stability can be lost in a narrow range of small delay values $\omega t_e \approx 0.4$.

As the field rotates in the transverse plane by the angle π (that is, at $m = 2$), bistability loops of the d_{DDDD} type (Fig. 2*g*) arise in the BD structure on descending branches (stable at $m = 1$). As the nonlinearity coefficient decreases down to $K \approx 4.45$ the loop type alternates and becomes d_L (Fig. 2*f*) [Ref. 13]. Thus, the field rotation through 180° causes the transformation of BD of the $DDDD$ type (taking place in the absence of the field rotation in FL) into BD of the $d_{DDDD}\,d_{DDDD}$ type.

In the absence of the field rotation ($m = 1$) and phase delay in FL ($\omega t_e = 0$), BD of static states in the plane U–K contain hystereses of the UU type (similarly to Fig. 2*a*). As the phase delay ωt_e varies with the step of 2π, the structure of the BD branches $U(K)$ repeats. If the variation of ωt_e is smaller than 2π, then only bifurcation points shift along the axis K.

If the field in FL rotates by the angle π ($m = 2$), the branch u_L is formed in the BD structure $U(K)$ at $2.5 < K < 4.5$ (Fig. 2*h*). The upper branches have the type u_{DUDU}. In addition, the BD structure below the part U of the branch u_{DUDU} at $9.5 < K < 14$ includes the "closed" hysteresis DU. The calculations show also the "closed" hysteresis UD (between the upper part U of the loop and the upper part D of the loop of the second branch u_{DUDU}, lying to the right). At $K > 13$, two closed hysteresis contours DU lie between the lower part D of the loop and the lower part U of the loop of the second branch u_{DUDU}. This tendency develops with the increase of K.

When the phase delay ωt_e varies in the considered range of K values, the BD structure repeats with the period of 2π. At $m = 2$, this change of the structure is accompanied by the appearance of different types of hysteresis on the branches, for example, $u_L u_{DUDU}$, uu_{DUDU} [Ref. 13]. But at some values of ωt_e, hysteretic parts, including closed ones, may disappear.

It is obvious that in the absence of microwave modulation in the NRI the total phase shift after the passage through the NRI is $\omega\tau \equiv \omega\tau(\mathbf{r}', t) = \omega t_e(\mathbf{r}', t) + U_0(\mathbf{r}', t - t_e(\mathbf{r}', t))$. As the microwave signal with the power characterized by the coefficient K_{mw} is applied, an additional phase shift (K_{mw}) occurs in NM, and therefore the dynamics of the processes in NRI and its bifurcation behavior change. In this case, the total phase shift in the system is

$$\omega\tau_\Sigma \equiv \omega\tau_\Sigma(\mathbf{r}', t) = \omega t_e(\mathbf{r}', t) + U_0(\mathbf{r}', t - t_e(\mathbf{r}', t)) + K_{mw}.$$

The influence of the additional phase shift K_{mw} can be compensated for down to the previous value $\omega\tau$ by changing the phase delay ωt_e in the FL. It can be shown that in the static regime the influence of the parameters K_{mw} and ωt_e on the state of the modulator, for example, on the phase change U, is *equivalent* in the sense defined by the validity of the system of equalities:

$$\omega t_{e1} + K_{mw1} = \omega t_{e2} + K_{mw2}; \quad U_1 + K_{mw2} = U_2 + K_{mw1}, \tag{3}$$

where the subscripts 1 and 2 correspond to the numbers of the modulators, differing by the parameters ωt_e and K_{mw} [Ref. 14]. In other words, when K_{mw} is the bifurcation parameter, BD is similar to BD in the plane U–ωt_e, but the points in the latter are shifted along the axis OU by K_{mw}. It follows from Eqs. (3) that in the presence of the external modulating action (K_{mw}) BD shifts along the axis ωt_e by K_{mw}.

It can be concluded that to perform the modulation, it is worth selecting the following ranges of the bifurcation parameter based on the BD structure. At $m = 1$ and 2 for switching purposes (obtaining the rectangular laser pulses) it is advisable using the hysteretic parts. At the same time, long stable branches are attractable, for their linearity, for a linear control over the laser beam intensity. These branches may have both the positive and negative derivatives, as well as different steepness. The BD parts, at which the branches loss stability, are interesting in view of obtaining the laser beam with the intensity changing in a complicated way.

2.2. Influence of the modulation regime on modulation characteristics

Let the coefficient K_{mw} follow the law $K_{mw} = K_{mw0}\cos^2(2\pi t/T_{mw})$, where $K_{mw0} = 3$, and T_{mw} varies within $10^3\tau_n \geq T_{mw} \geq \tau_n$. Then the average value $\langle K_{mw}\rangle_{T_{mw}} = K_{mw0}/2$ has the meaning of the working point in terms of the parameter K_{mw}. For an NRI to operate in the hysteretic area during the modulation, we select the working point in terms of the parameter ωt_e (the value ωt_{e0}) according to the equation $\omega t_{e0} = \omega t_{ecg} - K_{mw0}/2$, where ωt_{ecg} is the position of the center of the hysteresis loop in BD $U(\omega t_e)$, constructed for $K_{mw} = 0$ (Fig. 3).

The location of the working point in the upper or lower part of the hysteresis loop can be provided for by the proper choice of the initial conditions U_0.

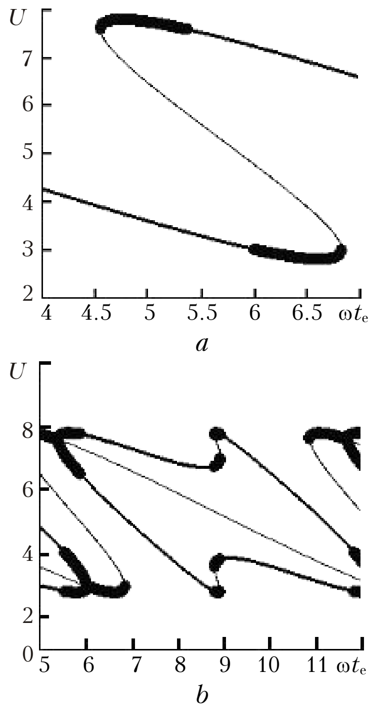

Fig. 3. BD in the plane (U–ωt_e) for $K = 5$, $K_{mw} = 0$, $m = 1$ (*a*), $m = 2$ (*b*).

Thus, $\omega t_{e0} = 4$ for BD in Fig. 3*a* and $\omega t_{e0} = 882–3/2 = 7.32$ in Fig. 3*b*. For these cases, Fig. 4 shows the series of modulation characteristics $I(K_{mw})$, calculated using the above model, where I is the intensity of the output optical field. At $m = 2$ (that is, at the field rotation by 180°, Figs. 4*b*, *d*, *f*, *h*, *j*), the modulation characteristics for the intensities I_1 and I_2 at two diametrically opposite points of the laser beam cross section and the average intensity I_{av} are shown by the black, gray, and light-gray lines.

At small (as compared to τ_n) modulation periods T_{mw}, the optical system has not enough time to respond to the action, and the shape of the modulation characteristic does not reflect the BD structure features (Figs. 4 *a*–*f*, *h*).

To the contrary, at large modulation periods T_{mw}, from BD and the chosen working point it is possible to predict the shape of the modulation characteristic. To obtain rectangular laser pulses, it is worth selecting the regime corresponding to Figs. 4*g*, *i*, *j*. Our computer experiments demonstrate the expected possibility of obtaining the laser beam with the intensity following a complicated law.

Conclusion

The arrangement of the modulator based on a nonlinear ring interferometer is proposed. In this modulator, the information signal acts on the initially isotropic Kerr medium, changing its optical length. The model of dynamics of the nonlinear phase shift $U(\mathbf{r}, t)$ in the nonlinear medium is developed in the approximation of the slowly changing amplitudes,

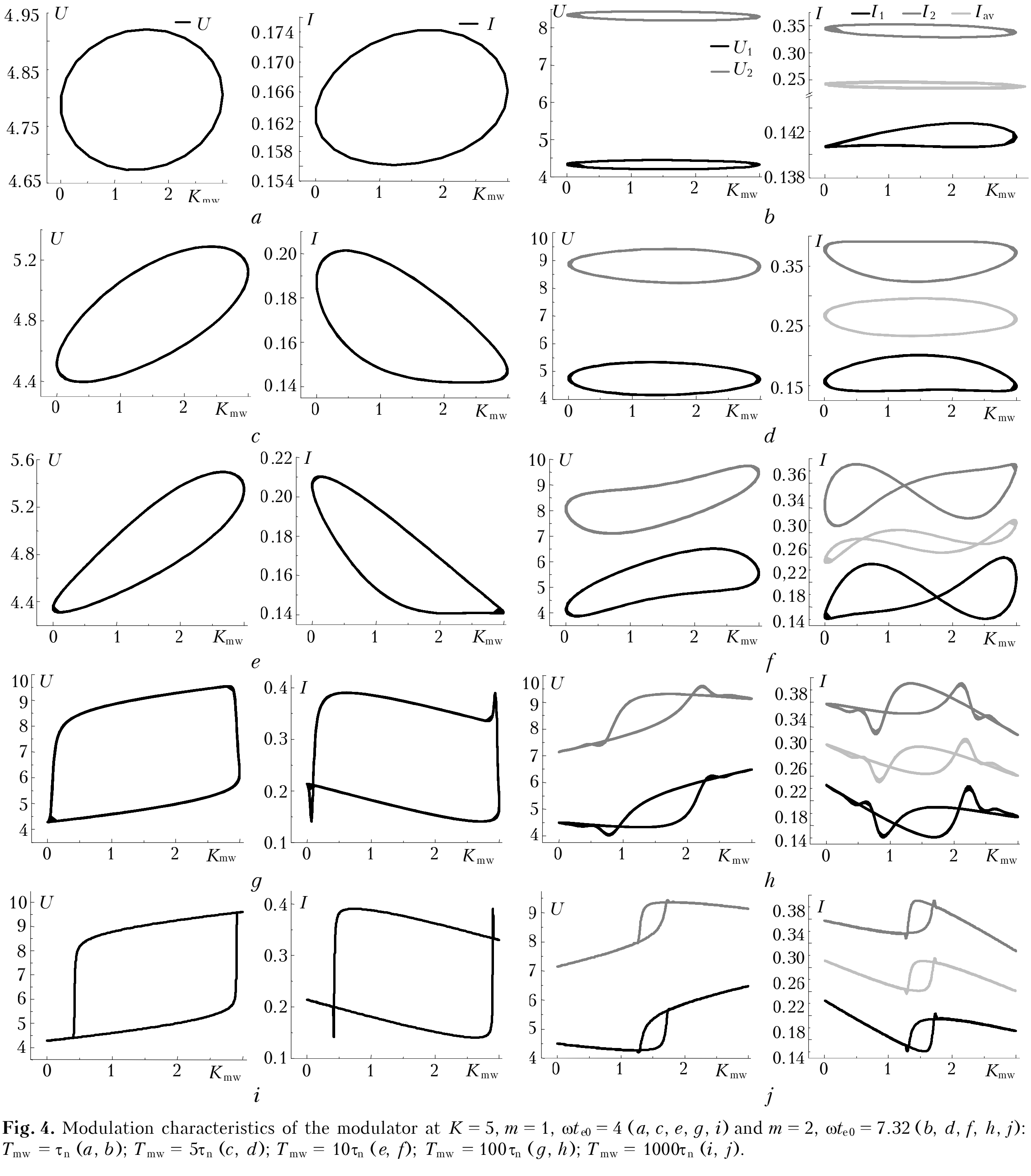

Fig. 4. Modulation characteristics of the modulator at $K = 5$, $m = 1$, $\omega t_{e0} = 4$ (a, c, e, g, i) and $m = 2$, $\omega t_{e0} = 7.32$ (b, d, f, h, j): $T_{mw} = \tau_n$ (a, b); $T_{mw} = 5\tau_n$ (c, d); $T_{mw} = 10\tau_n$ (e, f); $T_{mw} = 100\tau_n$ (g, h); $T_{mw} = 1000\tau_n$ (i, j).

phases, position of the plane of polarization of the input fields, the field delay time in NRI, slow relaxation of the nonlinear part of the refractive index of the nonlinear medium, as well as single passage of the field through the NRI.

It is shown that the presence of the external modulating action at the given polarization of the microwave leads to the periodic alternation of the structure of BD branches. As this takes place, the optical and the microwave fields act additively on the phase shift $U(\mathbf{r}, t)$. The relation of equivalence is established for the parameter $K_{mw}(\mathbf{r}, t)$, describing the effect of the microwave field on NM, and the linear part ωt_e of the phase change for the passage through the NRI. As $K_{mw}(\mathbf{r}, t)$ changes, the alternation of dynamic regimes and the complication of the dynamics in the modulator are also possible.

Selecting the working point in terms of the entire set of NRI parameters, the modulation amplitude $K_{mw0}/2$, and the modulation period T_{mw}, it is possible to control the parameters of the laser beam at the NRI exit. It is proposed to interpret BDs as static transfer functions. The data simulated allow us to expect that the optimization of the device described will open a way for creating practical modulators of laser radiation.

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