Cavities based on grazing-incidence optics for VUV radiation and soft X-rays

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The operating principles of several cavities for VUV radiation and soft X-rays (1000 Å ≥ λ ≥ 1 Å) are described. The operation of the cavities is based on grazing-incidence X-ray optics. Some types of ring cavities and a cavity with pear-shaped mirrors are considered, as well as a semi-ring cavity, in which structural X-ray optics is used in addition to the grazing-incidence optics. The performance of the cavities and their efficiency are discussed.

Designing cavities for lasers, whose operating range covers a part of the VUV region (1000 Å ≥ λ ≥ 400 Å), soft X-ray region, and a part of the hard X-ray region (400 Å ≥ λ ≥ 1 Å) (referred to, in what follows, as the X-ray region), is a more difficult and, in some aspect, more important problem than for optical lasers.

The main difficulty is connected with the fact that X-ray optics has the efficiency much lower than that of optics for visible radiation because of the high absorption of X-ray radiation, low reflection coefficients, and other factors. Therefore, it is quite problematic to create high-efficiency X-ray cavities.

In spite of the intense development of X-ray optics during the last three decades, no efficient X-ray cavities have been designed, but many ideas have been put forward. Cavities operating based on different principles of X-ray optics (crystal optics and multilayer X-ray optics, grazing-incidence optics and mixed X-ray optics) have been addressed.

Rivlin was one of the first to consider X-ray cavities and proposed a ring cavity, in which polished crystal surfaces were used as mirrors.[1] This cavity employed the Bragg reflection, and therefore it has the property of radiation monochromatization.

Bragg reflection cavities based on multilayer mirrors were proposed in Refs. 2 and 3. In particular, a tunable cavity with such mirrors was described in Ref. 4. A cavity based on one multilayer mirror with the reflection coefficient about 15% has been applied, largely, for adjustment in an X-ray laser on the Nova laser facility (Lawrence Livermore National Laboratory).[5,6]

Cavities based on grazing-incidence X-ray optics are simpler in fabrication, and, unlike the cavities with multilayer mirrors, they can operate in a rather wide wavelength range, though with a varying efficiency.

Consider X-ray cavities, whose operation is based on grazing-incidence optics.

The optical properties of a material reflecting X-ray radiation are determined by the refractive index n [Ref. 7]:

$$n = 1 - \delta - i\beta \approx n' - i\beta,\ n' = \mathrm{Re}\, n,$$

where n' is the real part of the refractive index; δ is the refractive index decrement of the reflecting material; β is the parameter characterizing absorption of X-rays by the medium ($\beta \ll \delta$). It can be seen from this equation that δ and β are below zero, and this makes the refractive index of materials less than unity in the X-ray region. For different materials, δ and β can be determined from the atomic scattering factors f_1 and f_2 tabulated in Ref. 8:

$$\delta = (2\pi)^{-1} N_a r_e \lambda^2 f_1,\ \beta = (2\pi)^{-1} N_a r_e \lambda^2 f_2,$$

where N_a is the density of atoms; $r_e = e^2 m^{-1} c^{-2}$ is the classic electron radius; λ is the wavelength of X-rays.

Actual substances have, at different X-ray wavelengths, the refractive index decrement $\delta = 10^{-2}–10^{-6}$. For the X-ray region, the real part of the refractive index differs only slightly from unity: $n' = 1 - \delta \approx 1$. This is indicative of the low refractivity for X-rays (several orders lower than in the visible region).

The grazing-incidence X-ray optics is based on the effect of the total reflection of X-rays from a smooth surface at the grazing angles θ not exceeding the critical angle θ_c, $\theta \le \theta_c = \sqrt{2\delta}$.

At multiple reflection of X-rays with $\theta \le \theta_c$, the rays can be turned by rather large angles at a relatively low loss.

The transmission coefficient of X-rays propagating along the surface $K(\psi)$ at the angle ψ in the case of multiple reflection for small grazing angles $\theta \to 0$ can be expressed[9,10] as

$$K = \exp(-\psi\beta\delta^{-3/2}). \tag{1}$$

The X-ray waves propagating along smooth surfaces, by analogy with the phenomenon of acoustic wave propagation along concave surfaces, are called *whispering gallery modes.*

The real values of the critical grazing angle depend on the X-ray wavelength and are described by the approximate equations for different wavelengths[11]:

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$$\theta_c \approx \begin{cases} (0.1-0.2)\lambda, \ 15\,\text{Å} \le \lambda \le 200\,\text{Å}, \\ (0.1-0.3)\lambda, \ \lambda \le 15\,\text{Å}, \end{cases}$$

where angles are in degrees, the wavelength λ is in Angströms, and the constants of proportionality increase with the increasing mean charge of nuclei of the medium substance.

A characteristic property of the grazing-incidence X-ray optics is a wide spectral pass band, which depends on the material and quality of the surface. Therefore, cavities based on the grazing-incidence X-ray optics can operate in a rather wide region, which can be changed by changing the material.

A particular case of the grazing-incidence X-ray optics is *capillary X-ray optics*,[12] in which internal surfaces of thin and, usually, cylindrical tubes (capillaries) serve as waveguides.

Simple ring cavities

Ring cavities for the X-rays are made as one or several convex surfaces (usually, the surfaces of a circular cylinder; Fig. 1), and X-rays propagate along the inner side of these surfaces due to multiple reflections at small grazing angles with the total turning angle equal to 360°. For the first time, such cavities were considered in Refs. 13–15.

The wave of the resonant radiation forms a traveling wave. No standing waves are generated in such cavities, and this makes the laser with a ring cavity somewhat more efficient when operated in the X-ray region. The transmission coefficient for X-rays in the cavity is practically independent of the cavity radius, because, the number of reflections needed for a radiation quantum to turn by a certain angle depends only on the number of reflections at propagation along a surface.

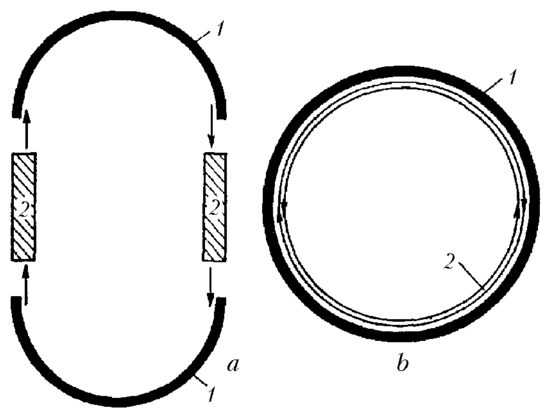

Fig. 1. Cavities with the circular-cylinder surfaces (*1*) and working medium of a laser (*2*): cavity with two mirror and working-medium sections (*a*) and single-section cavity with the counterpropagating pump beam (*b*) [Ref. 15].

The cavity radius $R \gg \lambda$ determines the period, for which the radiation quanta travel round the cavity $T = R/c$ (c is the speed of light). This period may prove important for the following reasons:

1. The shorter the period, the greater is the number of cycles, the quanta travel round the cavity per unit time, and the more efficient is the feedback in the cavity. In this case, the wave of the induced radiation increases more quickly and the cavity efficiency becomes higher. The ratio of the periods T_1 and T_2 for two cavities with the radii R_1 and R_2 is $T_1/T_2 = R_1/R_2$.

2. If short-lived plasma is used as a working medium, then the time of pumping of the working medium is short, and the lifetime of the population inversion is even shorter.

The above-said suggests that ring cavities with a shorter period will be more efficient in X-ray lasers.

A cavity with a capillary used to turn the radiation[16] (Fig. 2) also falls in the category of simple ring cavities.

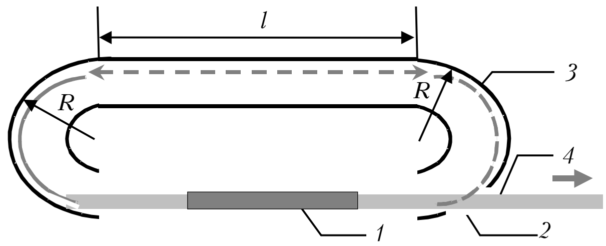

Fig. 2. Basic diagram of a simple ring cavity[16]: working medium *1*; outlet *2*; inner wall *3*, along which a part of the laser beam is turned; laser beam *4*.

Consider the operation of this cavity in a more detail. The X-ray radiation from the laser working body *1* exits at the face end and forms the laser beam *4*. A part of the beam falls on the inner wall *3* of the cavity, turns round by 360° due to multiple reflections, and thus enters the working medium from the rear end. The radiation exiting at the rear end of the working medium is directed back by a ring, leaves it, and enters the working medium from the left. The transmission coefficient along the cavity surface K for different substances, different wavelengths, and different cavities ranges from 0.1 to 0.4, which can be easily obtained from Eq. (1).

The ray propagating in the counterclockwise direction is a working one, but, in addition to it, a counterpropagating beam of the induced radiation is generated in the cavity and this beam does not take part in the formation of the laser beam. The counterpropagating beam induces some active centers of the working medium to transit from the upper working state, thus decreasing the population inversion, and, as a consequence, the laser efficiency. This disadvantage is removed in a ring cavity with a return ring.[17]

Ring cavity with a return ring

This cavity is a more complicating version of the simple ring cavity, but with the problem of the counterpropagating wave resolved[17] (Fig. 3).

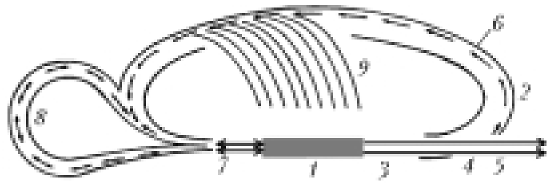

Fig. 3. Basic diagram of the ring cavity for X-ray laser[17]: laser working medium *1*; ring cavity *2*; beam from the face end of the working medium *3*; outlet of the cavity *4*; laser beam *5*; beam turning in the cavity *6*; part of the beam upon turning in the cavity and return ring *7*; return ring *8*; system of director surfaces *9*.

This cavity operates as follows. The flux of quanta exiting at the face end of the working medium *1* enters the cavity, where, due to the outlet *4*, the beam *3* is divided into the laser beam *5* and the beam *6*, which is transported by the cavity *2* through multiple reflections from its surface at small grazing angles. The beam *6* provides for the positive feedback in the laser. After turning by 360°, the beam exits at the rear end of the cavity (beam *7*) and comes into the working plasma, where it is amplified due to stimulated emission in the presence of population inversion. As the amplified beam exits at the front end of the plasma, the cycle is repeated.

The intensity of the X-ray beam after accomplishing a single full cycle in the cavity for the far-from-saturation intensities can be presented in the form

$$I = I_0\, kK\,(1 - \mu_1 - \mu_2)\exp(\Delta N\sigma l) = I_0\, K\,\xi\exp(\Delta N\sigma l),\ \xi \equiv k\,(1 - \mu_1 - \mu_2). \quad (2)$$

Here I_0 is the initial beam intensity; $\Delta N = N_2 - (g_2/g_1)\,N_1 > 0$ is the population inversion of laser levels (N_1 and N_2 are the concentrations of the lasing centers at the upper and lower laser levels; g_1 and g_2 are the statistical weights of the levels); l is the length of the laser working medium; $\sigma = (8\pi)^{-1}\nu^{-2}c^2A_{21}\times\varphi(\nu - \nu_0)$ is the cross section of the induced radiation, where A_{21} is the probability of spontaneous transition between the lasing levels; $\varphi(\nu - \nu_0)$ is the line profile, ν and ν_0 are the radiation frequency and the resonant frequency of the laser radiation; μ_1 and μ_2 are the losses due to scattering and absorption in the medium; k is the part of the X-ray beam captured by the cavity; $1 \ge k \ge 0$. The condition for lasing is the critical population inversion ΔN_c, which can be easily found from Eqs. (1) and (2):

$$\Delta N_c = -\frac{\chi}{\sigma l} = \frac{8\pi\nu^2}{c^2 l\varphi(\nu_0)}(360°\beta\delta^{-3/2} - \ln\xi),$$

where it is taken into account that $\psi = 360°$, the logarithmic losses in the cavity $\chi = 360°\,\beta\delta^{-3/2} - \ln\xi$ are introduced, and the line profile is taken at $\nu = \nu_0$.

The normal operation of the cavity is possible if the lifetime of the population inversion in plasma τ is longer than or comparable with the time needed for a single cycle $\tau_c = c^{-1}L \approx 3.3\cdot 10^{-8}$, where $L \approx 10$ cm is the mean path of unabsorbed photons in the cavity ring for one cycle. The lifetime of the laser plasma or high-current discharge plasma ranges within 10^{-9}–10^{-8} s, so the condition $\tau > \tau_c$ can be believed quite realizable at a proper duration of pumping, which may be photopumping, recombination, or combined pumping.

Let us note some possibilities of improving the operation of a laser with the ring cavity. To reduce the losses due to counterpropagating wave, it is proposed to direct the photons exiting from the rear end of the plasma back to the rear end using a return loop *8*. In this case, the intensity I_0 coming for amplification will be supplemented with the intensity I_0' coming from the return loop:

$$I_0' \approx I_P \exp(\psi°\,\beta\delta^{-3/2}),$$

where I_P is the flux entering the loop; ψ is the total turning angle in the loop with the allowance made for the particular geometry, which is roughly equal to 260–280° depending on the shape of the return ring.

It is also possible to collect a part of the resonant radiation (mostly spontaneous) using the system of director surfaces *9*, which direct the radiation quanta into the rear end for amplification. In this case, the intensity coming for amplification increases by

$$I_0'' \approx I_s\,(4\pi)^{-1}\sum_i \Omega_i \exp(\psi_i\beta\delta^{-3/2}),$$

where I_s is the intensity of radiation from the working medium propagating to sides at the resonant frequency; Ω_i is the solid angle, from which the X-ray radiation is collected by the ith surface; ψ_i is the turning angle needed for the ray propagating along the ith surface.

Finally, for calculation of the laser radiation intensity, the sum $I_0 + I_0' + I_0''$ should be used in place of I_0 in Eq. (2). The values of I_0, I_P, I_s should be calculated from the particular conditions of existence and properties of plasma and the used theoretical models.

A common disadvantage of the considered ring cavities is the asymmetric profile of the beam exiting after passage through the ring, which becomes closer to the circular surface as the turning angle increases. In the case that a capillary serves as a wave guide of the cavity, the beam propagating in it takes the characteristic "banana-shaped" profile.

It is possible to create X-ray cavities with two return rings along the optical axis as reflectors. But such a cavity is no longer a ring cavity, and its efficiency must be somewhat higher than that of the ring cavity because of the higher transmission coefficient. The disadvantage of this cavity, as well as of the cavities considered above, is the axially asymmetric laser beam. The planes of location of the return rings may either coincide or be arranged at a nonzero angle. In the latter case, the beam intensity will be somewhat smoothed and its axial symmetry will be improved. The right angle seems to be optimal in this case.

The disadvantage of the considered cavities is the absence of axial symmetry, which results in the asymmetric feedback distribution and, as a result, to the axial asymmetry of the laser beam. Cavities with cylindrical symmetry will result in generation of more symmetric laser beams. Consider some axially symmetric X-ray cavities.

Axisymmetric ring cavities

Ring cavity based on grazing-incidence mirrors for X-ray laser

The proposed cavity consists of two mirrors arranged coaxially, that is, so that their optical axes coincide (Figs. 4 and 5) [Refs. 18, 19].

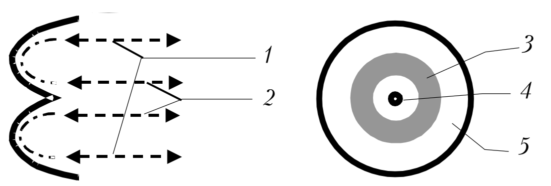

Fig. 4. Diagram of the grazing-incidence mirror of the X-ray cavity. Side (*a*) and end (*b*) views[19]: external *1* and internal *2* parts of the beam; absorbing ring *3*; inner *4* and outer *5* working areas.

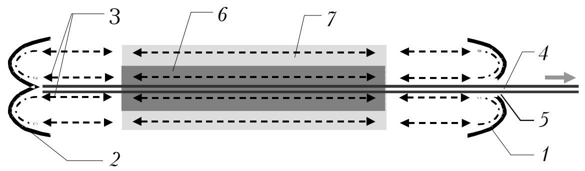

Fig. 5. Block-diagram of the ring cavity operating in the double-pass mode[19]: front mirror *1*; back mirror *2*; closed ray trajectories *3*; laser beam *4*; outlet *5*; working medium *6* and *7* as the cavity operates in the single-pass and double-pass modes, respectively.

The X-rays in such mirrors also propagate along the surface due to multiple reflections at small grazing angles θ less than the critical one θ_c at the total reflection $\theta \le \theta_c$. The mirrors in the cavity considered (see Fig. 4) are the surfaces of revolution of some generating curve around the axis coinciding with the optical axis of the mirror. The resulting mirror surface is axisymmetric. The generating curve may be a part of a circle or other convex curve oriented in some way about the axis of revolution. The possibility of varying the surface geometry in order to change the properties of the X-rays indicates some flexibility of such a cavity.

The gray color in Fig. 4*b* indicates the ring area of the mirror *3* with low reflectivity, if the X-ray quanta fall in parallel to the optical axis. This area does not work in the mirror, or its work is inefficient because of the high absorption, so it will be referred to as the *absorbing ring.* The white ring area on the inner and outer sides of the absorbing ring are, respectively, the *outer 5* and *inner 4 working areas.* The boundaries between the working and non-working areas are not sharp, but somewhat blurred.

This cavity is an axisymmetric open cavity. The closed trajectory of X-ray quanta inside the cavity indicates that it is a sort of a ring cavity.

The induced radiation in this cavity is generated in two opposing running waves and forms the *principal* and *backward beams,* as in simple ring cavities. No standing waves are generated in this cavity.

In the right-hand mirror, as shown in Fig. 5, a part of the X-ray radiation from the internal part of the principal beam is directed to the central aperture, while the other part is directed by the ring trajectory through the laser working medium, where it is amplified.

Imperfect treatment of the mirror surface and the geometry of the reflection surface suggest that the intensity distribution of the laser beam has a radially inhomogeneous structure with a minimum in the center and a maximum closer to the outer boundary of the beam. Note that the laser beam may have a tubular structure, which is not always convenient. It is possible to make the output laser beam more homogeneous by correcting the mirror surfaces and the laser working medium. But this, in turn, may result in a higher divergence of the laser beam.

Let us describe the operation of the cavity. The flux of the X-ray quanta exiting from the face end of the working medium (see Fig. 5) is incident on the right-hand mirror, where it is separated into the output beam passing through the outlet and the rest part, which radially propagates over the mirror surface and turns by 180°. Quanta pass along the mirror surface due to multiple reflections from it at small angles. After turning, the X-ray quanta in the form of a beam are directed to the left-hand mirror. This part of the beam, providing for the feedback in the laser, will be referred to as the *outer* part, while the part formed along the optical axis of the mirror and making the laser beam will be called the *inner* part. Then in the second mirror, the beam also turns by 180° and thus comes to the central part of the mirror and then toward the right-hand mirror. Then the cycle is repeated.

If a laser with this cavity is employed, it is possible to organize operation so that only the inner part of the principal beam (*single-pass mode*) or both inner and outer parts of the beam (*double-pass mode*) pass through the working medium and experience amplification (Fig. 5). The stimulated emission of quanta takes place as the radiation passes through the working medium, and the longer the distance passed by a quantum in the working medium, the more efficient the amplification. Therefore, the operation of the X-ray laser in the double-pass mode is much more efficient than that in the single-pass mode.

For the laser beam intensity per single passage of the X-rays round the cavity in the region far from saturation, the following equation is valid:

$$I = I_0 K\xi \exp(qbl\sigma\Delta N). \quad (3)$$

The coefficient q takes the values $q = 1$ for the single-pass mode and $q = 2$ for the double-pass mode. In this case $b = 1$, if only the principal beam is used for generation in the cavity, and $b = 2$, if both the principal and the backward beams are used.

The condition for lasing is achievement of the critical population inversion ΔN_c, which can be easily determined from Eqs. (1) and (3):

$$\Delta N_c = -\frac{\chi}{qbl\sigma} = \frac{8\pi\nu^2}{c^2 qblA_{21}\varphi(\nu_0)}(360°\beta\delta^{-3/2} - \ln\xi),$$

where it is taken into account that the turning angle is $\psi = 360°$, that is, the logarithmic loss in the cavity $\chi = 360°\beta\delta^{-3/2} - \ln\xi$ for one cycle is introduced and the line profile is taken at $\nu = \nu_0$.

At the double-pass mode of cavity operation and the homogeneous population inversion density, the following peculiarity is observed in the working medium. As the radiation transits from the inner part of the beam into the outer one, the radiation density decreases, because the cross section area of the inner beam s is smaller than the annular cross section area of the outer beam S. The cross section areas of the outer and inner parts of the beam can be taken equal to the areas of projections of, respectively, the outer and inner parts of the working areas of the mirrors onto the plane normal to the optical axis of the cavity. To the contrary, as the radiation transits from the outer part of the beam into the inner one, the beam density increases. If we neglect the losses at transportation of quanta along the mirror surface and the losses for lasing, then the following equation:

$$I_0 = sj_0 = Sj_1$$

is valid for the densities of the inner j_0 and the outer j_1 beams in the cavity.

Because of the high density of radiation in the inner part of the beam, saturation is possible at its significant amplification. In this case, the population inversion of the energy levels is zero ($\Delta N = 0$), amplification and absorption processes in the inner part of the beam are balanced, and the medium "*clears up.*" As this takes place, amplification is still possible in the outer part of the beam because the population inversion there is nonzero $\Delta N > 0$ due to the lower density. Such cavities, in which different population density is formed in different parts of the working medium due to the different densities of the induced radiation, will be referred to as *cavities with distributed inversion.*

A cavity version is possible, when a part of the radiation from the backward wave is also outputted through a similar outlet in the left mirror (symmetric version of the cavity).

To improve the efficiency of the cavity, it is desirable to direct the backward beam of the induced radiation into the principal beam.

The backward beam can be turned in the *return loop,* as in the cavity shown in Fig. 3. The return loop is attached to the hole on the side opposite to the outlet for the laser beam. The return loop returns the quanta back, so that they take part in the formation of the laser beam. The disadvantage of the cavity with the return loop is distortion of the axial symmetry of the laser beam (after passage in the return loop), which takes the banana-shaped profile.

If the right-hand mirror is replaced by a system of two mirrors, as shown in Fig. 6, then, possibly, the problem of the return wave will be solved in a different way. To believe this, it is sufficient to look at the directions of X-ray quanta along their trajectories.

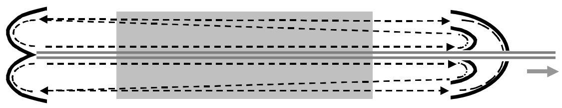

Fig. 6. The diagram of the ring cavity with combination of the principal and backward beams.[19]

In this cavity, larger volume of the working medium is involved than in the cavity shown in Fig. 5, which additionally improves its efficiency. Other versions of the cavity, in which the backward beam is either not generated or turned and directed into the principal beam, are also possible.

This cavity, unlike those considered above, is more compact and technological, and the laser beam generated in it will be axisymmetric.

So we have considered the simplest cases of cavities with return mirrors. The principle proposed allows us to design many versions of cavities with different mirror parameters. For example, the outer and inner beams in the cavity may be not only parallel to the optical axis as in Fig. 4, but also convergent or divergent. In this case, the convergent beams will converge at the focal points, and divergent ones will have imaginary focuses. Real and imaginary focuses, in their turn, may have different mutual orientations on the optical axis of the cavity. The combination of all possible versions of such mirrors gives a wide variety of cavities (more than hundred), which is a subject of independent consideration. All these cavities will have different stability and efficiency of lasing.

X-ray cavities with pear-shaped reflectors

The proposed cavity[20] (Fig. 7) consists of two *pear-shaped* reflectors *1* and *2*, whose surfaces serve the grazing-incidence X-ray mirrors. The reflectors are oriented face to face with the inlets *3* lying on the common optical axis. One of the reflectors *2* has an outlet *4* for the beam from the working medium of the X-ray laser *5* lying between the cavity mirrors.

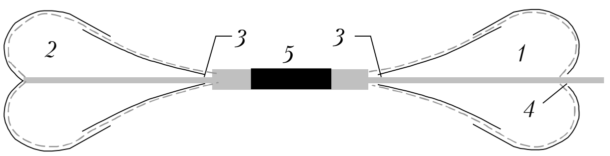

Fig. 7. Diagram of a cavity with pear-shaped reflectors.[20] Dashed curves show the trajectories of X-rays along the reflector surface: front and rear reflectors and their inlets *1*, *2*; inlets and outlet *3*, *4*; working medium *5*.

Each reflector consists of two sections, which are the surfaces of revolution of two *generating curves* around the cavity's optical axis; the geometry of these curves may be different (Fig. 8).

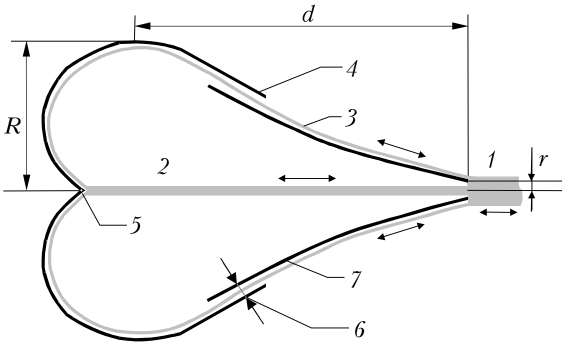

Fig. 8. Diagram of the pear-shaped reflector[20]: X-ray beam incident on the reflector inlet *1*, inner *2* and outer *3* beams; large section of the reflector *4*; cavity cone *5*; cavity slit *6*; external transportation mirror *7*; R is the reflector radius; r is the radius of the reflector inlet; d is the part of the distance from inlet to the plane passing through the reflector cross section with the maximum area.

The X-ray beam *1* incident on the reflector inlet is divided into two beams: inner *2* and outer *3* ones. The inner beam propagates inside a large section of the reflector *4* and, upon incidence on the cone of the cavity *5*, diverges radially over the internal surface. The X-ray quanta are transported along this surface to the cavity slit *6*, where they skip to the external transportation mirror *7*. Then, propagating along the mirror surface, they return into the working medium from the same side, but in the opposite direction. It is obvious that, for the X-ray quanta to propagate along the surface of the external transportation mirror and on the "cone" of the return mirror, the surfaces should be concave along the optical axis of the cavity, but it will be convex normally to this direction. Therefore, the propagation of the beam there is unstable. Because of this instability, some quanta can go away and be lost, which will reduce the efficiency of the cavity. The higher is the quality of the cavity surfaces, the smaller is the loss of X-ray quanta due to this instability.

The outer beam after separation of the incident beam obviously passes the same path as the inner beam, but in the opposite direction. The difference between the two reflectors is that one of them has an aperture for exiting a part of radiation. This aperture is made by cutting the top of the cone (see Fig. 8).

The pear-shaped reflector is, essentially, realization of the idea of a plane return loop of the ring X-ray cavity proposed in Ref. 3 for a volume axisymmetric cavity.

Estimate the efficiency of the proposed cavity.

The total turning angle of X-rays ψ in the reflectors depends on the particular geometry of the mirrors is $360° > \psi > 180°$:

$$\psi = 180° + 2\alpha,\ \alpha = \arctan\frac{R-r}{d},$$

where α is the *angle of deflection* in the cavity.

Assuming that $R \gg r$, we obtain a simplified equation for the angle of deflection $\alpha \approx \arctan(R/d)$.

The ratio $(R - r)/d \approx R/d$ will be called the elongation factor of the reflector.

Really, it is possible to create cavities with reflectors having the total turning angle $\psi = 210$–$270°$, which is less that $360°$, characteristic of ring cavities.[15–19] Such cavities must have lower loss as compared to ring cavities and, unlike ring cavities, generate a radially symmetric X-ray beam.

For different substances and different wavelengths, the coefficient K usually ranges from 0.1 to 0.4. The transmission coefficients for some substances, wavelengths, and different elongation factors were estimated based on the experimental data[11] and are tabulated below.[20]

Transmission coefficient

Material	λ, Å	K at turn by 270°, (R/d = 1)	K at turn by 270°, ($R/d \approx 0.27$)
Ru	120	0.51	0.6
Au	100–125	0.31	0.41
In	76	0.16	0.24

It can be seen from the Table that the highest value of the transmission coefficient K is 0.6, which is somewhat higher than in ring cavities.

Stronger elongated reflectors have the higher transmission coefficient and generate a beam with lower divergence. But it takes longer time for an X-ray quantum to pass in them, and this may prove significant for X-ray lasers, which employs short-lived plasma as a working medium.

The increase in the intensity of the laser beam I for passage of the resonant radiation in M reflections along the mirrors until saturation is described by the following equation:

$$I = I_0\,[(K\,\gamma_1^1\,\gamma_1^2\,K_2\,\gamma_2^1\,\gamma_2^2\,\xi]^M \times$$
$$\times \exp(MlG),\ G = \Delta N\sigma, \qquad (4)$$

where I_0 is the initial intensity of the beam; K_1 and K_2 are the reflection coefficients of the first and second reflectors; G is the amplification coefficient at the line center for a small signal without saturation.

Besides, we have introduced the coefficients γ characterizing the degree of stability of the beams with respect to quanta "sliding" from the cone (superscript 1) and from the external transportation mirror (superscript 2); subscripts correspond to the first and second reflectors.

The condition for operation of a laser with this cavity is achievement of the critical population inversion ΔN_c, which can easily be found from Eqs. (1) and (4):

$$\Delta N_c = -\frac{\chi}{Ml\sigma} = \frac{8\pi\nu^2}{c^2 l A_{21}\varphi(\nu_0)}\{\psi\beta\delta^{-3/2} - \ln(\gamma_1^1\gamma_2^1\gamma_1^2\gamma_1^2\xi)\},$$

where χ are the logarithmic losses in this cavity, which are equal to the expression in braces, the line profile is taken at $\nu = \nu_0$. Note that the obtained equation for ΔN_c is independent of M.

For the cavity with the reflectors having the same size and all other characteristics identical (symmetric case), the following equalities are valid:

$$K_1 = K_2 \equiv K;\ \gamma_1^2 = \gamma_2^2 \equiv \gamma^2.$$

Semi-ring cavity based on mixed X-ray optics

Such a cavity consists of two reflecting surfaces arranged coaxially. The first surface is a multilayer (interference) X-ray mirror.[21] The second surface is a return mirror, as in Fig. 4. This mirror operates in the grazing-incidence mode. Its cone is directed toward the multilayer mirror.[22] This cavity can be obtained, if we replace the rear or front return mirror of the axisymmetric ring cavity (see Fig. 5) by a multilayer mirror. This cavity is also axisymmetric.

The trajectory of resonant X-ray quanta in the cavity is shaped as a semi-ring, so the cavity can be called a semi-ring one. The laser radiation can leave the cavity through an aperture in one of the sides. This cavity operates at the resonant frequency, which is determined by the properties of the multilayer mirror.

Two beams are generated in the cavity: the inner one in the form of a continuous round beam and the outer one in the form of a circular cylinder; the density of the beams may depend on the radius. This cavity can operate in both single- and double-pass modes, as the axisymmetric ring cavity.

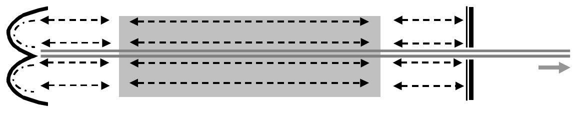

Fig. 9. Diagram of semi-ring X-ray cavity.

Let us list some properties of the considered X-ray cavities.

1. Due to the low reflection and transmission coefficients of the X-ray mirrors, the Q-factor of X-ray cavities is no more than a few units.

2. The angle of diffraction-limited divergence of the X-ray cavity with a characteristic size a can be determined as $\varphi = \lambda/a$ and is negligibly small.

3. The diffraction losses A_{dif} in the considered cavities are low: $A_{dif} = (4l\lambda)/a^2 = 4/N_F$, where the Fresnel number is $N_F = a^2/l\lambda$.

In conclusion, it can be noted that the cavities considered can prove to be useful to provide for the positive feedback in X-ray lasers at the critical values of population inversion in the working medium.

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