Algorithms for estimating the phase-matching angular width

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I present concretized algorithms for estimating the phase-matching angular width characteristic of different frequency conversion processes. These algorithms exclude necessity of using different estimation algorithms for the cases of noncritical 90°-phase matching and out of it. Data are presented in the paper on the angular width for CO_2-laser second-harmonic generation in $ZnGeP_2$ at 20 and 200°C, estimated by use of proposed and conventional algorithms.

Introduction

Phase-matching angular width for the case of three-frequency conversion processes in nonlinear crystals can be expressed, by use of quadratic approximation, through the first and second derivatives of the wave detuning Δk with respect to the phase-matching angle θ as follows

$$\Delta k(\delta\theta) = \Delta k(0) + \frac{\partial(\Delta k)}{\partial(\delta\theta)}\Delta\theta + \frac{1}{2}\frac{\partial^2(\Delta k)}{\partial(\delta\theta)^2}\Delta\theta^2. \qquad (1)$$

According to the common procedure,[1] the second derivative in Eq. (1) is neglected at the angles, which are far from noncritical 90°-phase matching and the angular bandwidth is determined through the first derivative only. The width value is taken to be equal to such a range of angles where the wave detuning $\Delta k = 0.886\pi/L$ (L is the crystal length) and the power of frequency-converted radiation decreases down to half of its maximum value which occurs at the exact phase matching, $\Delta k(0) = 0$.

The first derivative vanishes under conditions of noncritical 90°-phase matching and close to it and the angular bandwidth is estimated only through the second derivative. In so doing the wavelengths where one should pass from one estimation algorithm to another remain uncertain and the problem arises on sewing data obtained by the algorithms. Estimation algorithms for phase-matching angular width, proposed in this paper, allow the above disadvantages to be removed.

Algorithms for estimating phase-matching angular width of the second-harmonic generation

Let us denote the angular deviation from the exact angle of phase matching by $\delta\theta = \theta - \theta_{pm}$, where θ is the current angle and θ_{pm} is the phase-matching angle. Then transform Eq. (1) to the quadratic form:

$$\frac{1}{2}\frac{\partial^2(\Delta k)}{\partial(\delta\theta)^2}\Delta\theta^2 + \frac{\partial(\Delta k)}{\partial(\delta\theta)}\Delta\theta - 0.886\frac{\pi}{L} = 0. \qquad (2)$$

Define the phase-matching angular width as

$$\Delta\theta = \frac{\sqrt{1.772\pi\alpha + L\beta^2}}{\sqrt{L}\alpha} - \frac{\beta}{\alpha}, \qquad (3)$$

where $\alpha = \dfrac{\partial^2(\Delta k)}{\partial(\delta\theta)^2}$ and $\beta = \dfrac{\partial(\Delta k)}{\partial(\delta\theta)}$.

For second-harmonic generation (SHG) due to interaction of the first type *ee-o* in a positive crystal, we obtain

$$\Delta k = 4\pi[n_o(\lambda_2) - n_e(\theta,\lambda_1)]/\lambda_1; \qquad (4)$$

$$\beta(\lambda_1,\theta) = \frac{2\pi\sin(2\theta)\left|n_o^2(\lambda_1) - n_e^2(\lambda_1)\right|n_e^3(\lambda_1,\theta)}{\lambda n_o^2(\lambda_1)n_e^2(\lambda_1)}; \qquad (5)$$

$$\alpha(\lambda_1,\theta) = \big\{\pi n_e^5(\lambda_1,\theta)[n_e^2(\lambda_1) - n_o^2(\lambda_1)]\times \times\{[n_e^2(\lambda_1) - n_o^2(\lambda_1)][\cos(4\theta) - 5] - 4\cos(2\theta)[n_e^2(\lambda_1) + n_o^2(\lambda_1)]\}\big\}/2\lambda n_e^4(\lambda_1)n_o^4(\lambda_1). \qquad (6)$$

For SHG due to interaction of the second type *oe-o* in a positive crystal, we obtain

$$\Delta k = 2\pi[n_o(\lambda_1) + n_e(\theta,\lambda_1) - 2n_o(\lambda_2)]/\lambda_1; \qquad (7)$$

$$\beta(\lambda_1,\theta) = \frac{\pi\sin(2\theta)[n_e^2(\lambda_1) - n_o^2(\lambda_1)]n_e^3(\lambda_1,\theta)}{\lambda n_o^2(\lambda_1)n_e^2(\lambda_1)}; \qquad (8)$$

$$\alpha(\lambda_1,\theta) = \big\{\pi n_e^5(\lambda_1,\theta)[n_e^2(\lambda_1) - n_o^2(\lambda_1)]\times \times\{4\cos(2\theta)[n_e^2(\lambda_1) + n_o^2(\lambda_1)] - [n_e^2(\lambda_1) - n_o^2(\lambda_1)][\cos(4\theta) - 5]\}\big\}/4\lambda n_e^4(\lambda_1)n_o^4(\lambda_1). \qquad (9)$$

For SHG due to interactions of the first and second types in a negative crystal, we obtain

$$\Delta k = 2\pi[n_o(\lambda_1) - n_e(\theta,\lambda_2)]/\lambda_2; \qquad (10)$$

$$\beta(\lambda_2,\theta) = \frac{\pi\sin(2\theta)[n_o^2(\lambda_2) - n_e^2(\lambda_2)]n_e^3(\lambda_2,\theta)}{\lambda n_o^2(\lambda_2)n_e^2(\lambda_2)}; \qquad (11)$$

$$\alpha(\lambda_2,\theta) = \pi n_e^5(\lambda_2,\theta)[n_e^2(\lambda_2) - n_o^2(\lambda_2)]\times \times\{[n_e^2(\lambda_2) - n_o^2(\lambda_2)][\cos(4\theta) - 5] - 4\cos(2\theta)[n_e^2(\lambda_2) + n_o^2(\lambda_2)]\}\big\}/4\lambda_2 n_e^4(\lambda_2)n_o^4(\lambda_2); \qquad (12)$$

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$$\Delta k = \pi[n_e(\theta,\lambda_1) + n_o(\lambda_1) - 2n_e(\theta,\lambda_2)]/\lambda_2; \quad (13)$$

$$\beta(\lambda_1,\lambda_2,\theta) = \pi\sin(2\theta)\times$$
$$\times\left(\frac{n_e^3(\lambda_1,\theta)\left|n_e^2(\lambda_1) - n_o^2(\lambda_1)\right|}{\lambda_1 n_o^2(\lambda_1) n_e^2(\lambda_1)} - \frac{n_e^3(\lambda_2,\theta)\left|n_e^2(\lambda_2) - n_o^2(\lambda_2)\right|}{\lambda_2 n_o^2(\lambda_2) n_e^2(\lambda_2)}\right); \quad (14)$$

$$\alpha(\lambda_1,\lambda_2,\theta) = \frac{\pi}{4}\Big\{\{n_e^5(\lambda_1,\theta)\{4\cos(2\theta)[n_e^4(\lambda_1) - n_o^4(\lambda_1)] -$$
$$- [n_e^2(\lambda_1) - n_o^2(\lambda_1)]^2[\cos(4\theta) - 5]\}\}/\lambda_1 n_e^4(\lambda_1) n_o^4(\lambda_1)\Big\} -$$
$$- \frac{\pi}{4}\Big\{\{n_e^5(\lambda_2,\theta)[4\cos(2\theta)[n_e^4(\lambda_2) - n_o^4(\lambda_2)] -$$
$$- [n_e^2(\lambda_2) - n_o^2(\lambda_2)]^2[\cos(4\theta) - 5]]\}/\lambda_2 n_e^4(\lambda_2) n_o^4(\lambda_2)\Big\}. \quad (15)$$

The results estimated

The estimations obtained for the SHG in $ZnGeP_2$ with the account for temperature dependences of the refractive indices,[2] using the algorithms proposed, are shown, as an example, in Fig. 1.

As is seen from the figure, calculation of the phase-matching angular width using conventional estimation algorithms with regard to the second derivative alone provides for a fundamental error in spectral ranges, which are far from noncritical phase-matching. Conventional algorithms with regard to the first derivative give rise to a significant overestimation under phase-matching conditions close to noncritical 90° ones near the long-wave boundary of the phase-matching. The algorithms presented here provide for self-restricted estimations, which better agree (to ~0.3°) with the calculated ones and the experimental results. In particular, $\theta_{pm} = 82.97°$ when second-harmonic generation of a CO_2 laser radiation at $\lambda = 10.25$ μm is being done at the crystal temperature of 20°C. In this case, conventional estimation algorithms give the phase-matching angular width $\Delta\theta_{con} = 26.78°$, while the presented one – $\Delta\theta_{new} = 16.88°$. The difference between estimations of 9.9° can significantly change parameters of the optical instruments that may be developed using the second-harmonic generators considered above.

Note, that in calculating by use of conventional algorithms, formulae are chosen depending on the results of comparison of angular widths obtained. The discrepancy between the results does not vary significantly for the SHG at other CO_2 laser lines in $ZnGeP_2$ and other crystal temperatures. Thus, $\theta_{pm} = 83.67°$ for SHG at $\lambda = 10.58$ μm at 200°C. Phase-matching angular widths, calculated using conventional algorithms and those proposed here, are $\Delta\theta_{con} = 26.85°$ and $\Delta\theta_{new} = 17.22°$, respectively, while the divergence of estimations remains virtually invariant and amounts to 9.63°. Estimation algorithms for phase-matching angular width for other processes of parametric frequency conversion can be concretized in a similar way.

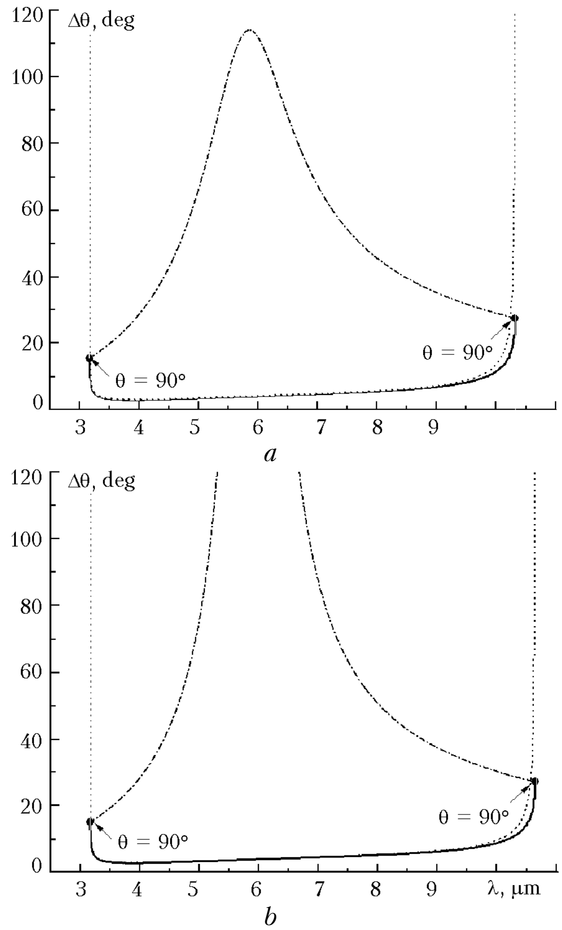

Fig. 1. Total angular width of the phase-matching (FWHM) for the SHG due to interaction of the first type (*ee-o*) in a $ZnGeP_2$ crystal of 1-mm length at 20 (*a*) and 200°C (*b*) estimated using conventional algorithms (dotted line corresponds to the first-derivative algorithms and dash-dotted line – to the second-derivative ones) and the algorithms proposed (solid line).

Conclusion

A simple procedure has been proposed to concretize estimation algorithms for phase-matching angular width for different parametric frequency-conversion processes. It allows the estimation precision to be improved to 10° for SHG in positive nonlinear $ZnGeP_2$ crystals.

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