Possibility of creating a nonstationary waveguide channel based on nanoparticles

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Received December 4, 2003

For microwave radiation in the wavelength range λ = 1–10 cm, the possibility of creating a non-stationary waveguide channel from conducting nanoparticles is shown at the concentration of nanoparticles in air equal to 1 g/m^3. The estimates show that 1–10 kg of nanoparticles is enough for creation of a channel with the length of 100 m. The power range of the transmitted radiation depends on how ellipsoidal are the particles. For strongly prolate nanoparticles, ignition of the wave channel under certain conditions is possible.

Waveguides with a light transmitting core and a transparent cladding have gained wide technological application in recent years.[1] Both the core and the cladding are dielectrics, that is, such waveguides have no conducting parts. The radiation is localized in a core because the dielectric constant of the core is higher than that of the cladding. For example, in the case of a waveguide for the radiation in the visible region, the relative difference between the dielectric constant of the core and the cladding is $(2-3)\cdot 10^{-3}$. Such a small difference between the refractive indices of the core and the cladding means that a small amount of substance with higher constant shaped as long cylinder in a homogeneous medium of other kind forms a waveguide channel. Let air be a homogeneous medium, and dielectric nanoparticles be another one substance. To achieve the effect, we estimate the optimal concentration of particles in a waveguide channel. It should be noted that a stable volume formation of nanoparticles is needed for creation of a stationary or quasistationary waveguide channel. The quasistationary waveguide channel can be created by a vortex. For this case, it is needed to estimate the vortex lifetime.

Consider the problem formulated. Assume that nanoparticles with the radius R and the dielectric constant ε_2 are sprayed in a long air cylinder L of radius ρ_0. The dielectric constant of the gas medium is ε_1 (for air $\varepsilon_1 = 1$), and the concentration of nanoparticles is much less than that of air molecules. In this case, the dielectric constant of the mixture ε_m is [Ref. 2, p. 69]:

$$\varepsilon_m = \varepsilon_1 + nv\frac{3(\varepsilon_2-\varepsilon_1)}{\varepsilon_2+2\varepsilon_1}\varepsilon_1, \qquad (1)$$

where $v = (4\pi/3)R^3$ is the nanoparticle volume; n is the concentration of nanoparticles. If nanoparticles are uncharged conductors, we should assume $\varepsilon_2 \to \infty$ in Eq. (1). Then

$$\varepsilon_m = \varepsilon_1 + 3nv\varepsilon_1. \qquad (2)$$

Consider the case of $nv = 10^{-3}$ (this corresponds to the averaged nanoparticle density of 1.7 kg/m^3). Suppose that the concentration of nanoparticles in the cylinder is independent of coordinates. Then the refractive index of the medium $\tilde{n} = \sqrt{\varepsilon}$ is a stepwise function:

$$\tilde{n}^2 = \begin{cases} n_{co}^2\varepsilon_1 = (1+3nv), & 0 \le r \le \rho_0 \\ n_{cl}^2 = \varepsilon_1, & r \ge \rho_0 \end{cases}, \qquad (3)$$

where $\varepsilon_2 \to \infty$ for nanoparticles (here n_{co} and n_{cl} are the refractive indices of, respectively, the core and the cladding of the waveguide).

Depending on ρ_0 and the wavelength λ, the waveguide (3) can transmit one or more modes. The initiated modes can propagate to long distances virtually without loss.

Determine now ρ_0 for the unimodal conditions and estimate the fraction of the radiation power accounted for by the launched mode, as well as assess the waveguide length L for $c = nv = 10^{-3}$. Introduce the optical volume

$$V = n_{co}k\rho_0\sqrt{2\Delta} \equiv k\rho_0\sqrt{n_{co}^2 - n_{cl}^2}, \qquad (4)$$

where $k = 2\pi/\lambda$, λ is the wavelength of electromagnetic radiation; Δ = const. The unimodal conditions of the waveguide are determined from the condition $0 < V \le 2.405$ [Ref. 1, p. 272]. At $V \to 0$ the launched mode accounts for a small fraction of power, since in this case the modal spot tends to infinity, and at $V \to 2.405$ another launched mode appears. Therefore, the optimal condition is $V \approx 1$ (see Appendix). In this case, from Eq. (4) we obtain

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$$\rho_0 = \lambda/(2\pi\sqrt{n_{co}^2 - n_{cl}^2}). \quad (5)$$

Assume that graphite of weight $m = 1.7$ kg is sprayed (graphite density $\rho_g = 1.7 \cdot 10^3$ kg/m^3). Then the length of the waveguide channel L is

$$L = \Lambda/(\pi\rho_0^2), \quad (6)$$

where $\Lambda = m/(nv\rho_g) = 10^6$ cm^3; ρ_0 is measured in centimeters.

Let $\lambda = 1$ to 10 cm. Then, according to Eq. (5), at $nv = 10^{-3}$ we obtain the core radius ρ_0 (radius of the waveguide channel) equal to 2.9 to 29 cm, and the length of the waveguide channel L ranges within 379 to 3.79 m.

One of the ways to make the waveguide channel stable is to induce vortical rotation of the waveguide channel with nanoparticles. In this case, the waveguide properties of such a channel almost do not change, but the system becomes much more stable (Ref. 3, p. 143). Estimate the characteristic lifetime of the vortical channel τ. Assume the linear velocity of points in the rotating waveguide channel to have only the φ-component; the gas is incompressible. Then from the Navier–Stokes equation we have for the φ-component of the velocity $\mathbf{v}$:

$$\rho_g \frac{\partial v_\varphi}{\partial t} = \eta \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_\varphi}{\partial r}\right). \quad (7)$$

Suppose that v_φ varies by the exponential law:

$$v_\varphi \sim \exp(-t/\tau). \quad (8)$$

From Eqs. (7) and (8) we obtain v_φ, which satisfies the equation

$$\frac{\partial^2 v_\varphi}{\partial r^2} + \frac{1}{r}\frac{\partial v_\varphi}{\partial r} + \frac{\rho_g}{\eta\tau} v_\varphi = 0. \quad (9)$$

In the vicinity of $r = 0$, the solution of Eq. (9) is the Bessel function

$$v_\varphi \quad J_0\left(\sqrt{\frac{\rho_g}{\tau\eta}}\, r\right), \quad (10)$$

where $J_0(x)$ is the Bessel function of the variable x.

Let the radius of the waveguide channel is ρ_0. Assuming that $v_\varphi \approx 0.5$ at $r = \rho_0$, where $v_\varphi(0) = 1$, from Eq. (10) we obtain $\sqrt{\rho_g/(\tau\eta)}\rho_0 \approx 1.5$. Finally $\tau \approx \rho_g\rho_0^2/1.5^2\eta$. For air $\rho_g = 1.3$ kg/m^3, $\eta = 1.8 \cdot 10^{-5}$ kg/(m · s). Let $\rho_0 = 6$ cm (this corresponds to the case $\lambda = 2.1$ cm, $L \approx 88$ m), then $\tau \approx 116$ s. Similar value follows from the results of mathematical simulation. Equation (7) was solved numerically with the following initial and boundary conditions:

$$\frac{\partial}{\partial r} v_\varphi(r = 0) = 0, \; v_\varphi(r = a_r) = 0, \; a_r = 5\rho_0.$$

Figure 1 depicts the dependence of v_φ on r at $t = 0$, 120, and 240 s. It can be seen from Fig. 1 that the maximum value of v_φ halves at $t \approx 120$ s.

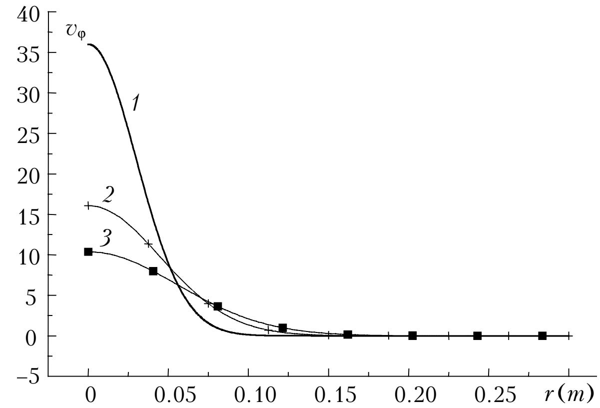

Fig. 1. Dependence of the φ-component of the linear velocity of medium points on r in the vortical channel at $t = 0$ (curve *1*), 120 (*2*), and 240 s (*3*).

As ρ_0^2 increases depending on t, the concentration of nanoparticles decreases, and $\rho_0^2 n = \text{const}$ at $L = \text{const}$. It can be seen from Eqs. (3) and (4) that the value of V^2 determining the unimodal conditions of radiation propagation is proportional to $\rho_0^2 n$. This, in turn, means that the unimodal conditions of radiation propagation do not change with time.

Thus, these estimates demonstrate the possibility of creating a waveguide channel using graphite-type nanoparticles. It should be noted that the waveguide channel, we often see, is a contrail behind aircraft at high altitude. This contrail can be used as, for example, a waveguide channel for radiation of meter wavelength. Depending on the weather conditions, such channel may be as long as 100 km.

Appendix

Consider a waveguide channel with the stepwise profile of the refractive index (PRI) at $\lambda = 1$ cm and $\rho_0 = 5$ cm:

$$\tilde{n}^2 = \begin{cases} n_{co}^2 = 1.01, & 0 \le r \le \rho_0, \\ n_{cl}^2 = 1, & \rho_0 \le r \le 10\rho_0. \end{cases} \quad (A.1)$$

In this case, the optical volume is $V = 3.14$. The waveguide contains two modes: $m = 0$, $N = 1$ and $m = 1$, $N = 1$. The propagation constants are $\beta_{m=0,\,N=1} = 6.3043$ cm^{-1} and $\beta_{m=1,\,N=1} = 6.29$ cm^{-1}. Figure 2 depicts the dependences of the mode intensities on the radius; the functions were normalized to meet the condition $\max \Psi_{m,N} = 1$.

Consider a waveguide channel with the following stepwise PRI at $\lambda = 1$ cm, $\rho_0 = 5$ cm:

$$\tilde{n}^2 = \begin{cases} n_{co}^2 = 1.001, & 0 \le r \le \rho_0, \\ n_{cl}^2 = 1, & \rho_0 \le r \le 10\rho_0. \end{cases} \quad (A.2)$$

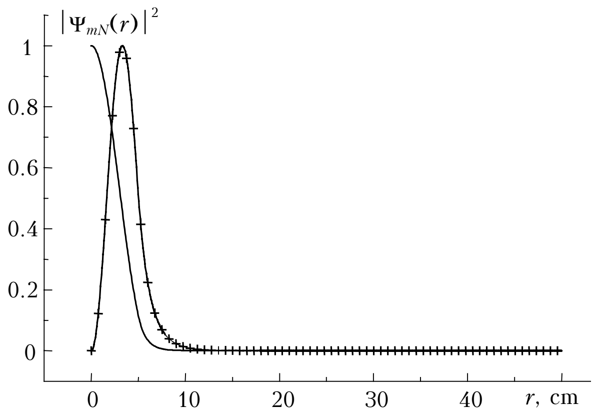

Fig. 2. Dependences of the normalized mode intensities $|\Psi_{m=1,\,N=1}|^2$ on the radius with $m = 0$, $N = 1$ (solid curve) and $m = 1$, $N = 1$ (curve marked by +).

In this case the optical volume is $V = 0.993$. The waveguide contains one mode $m = 0$, $N = 1$. The propagation constant is $\beta_{m=0,\,N=1} = 6.2833$ cm^{-1}. The dependence of the intensity of this mode on the radius is shown in Fig. 3.

Consider the waveguide channel with the following PRI:

$$\tilde{n}^2 = \begin{cases} n_{\rm co}^2 = 1.0003, & 0 \le r \le \rho_0, \\ n_{\rm cl}^2 = 1, & \rho_0 \le r \le 10\rho_0. \end{cases} \qquad \text{(A.3)}$$

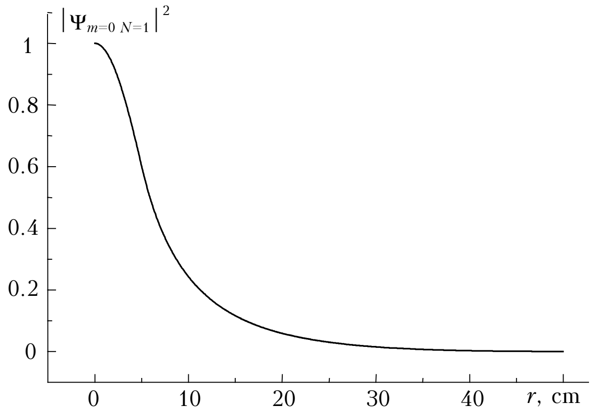

Fig. 3. Dependence of the intensity of the principal mode ($m = 0$, $N = 1$) on r.

The waveguide contains one mode $m = 0$, $N = 1$. The propagation constant of the mode is $\beta_{m=0,\,N=1} = 6.2831$ cm^{-1}. In this case V amounts to 0.77.

References

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