DOMAIN OF EXISTENCE OF THE INVERSE PROBLEM SOLUTION IN SLANT SENSING OF THE TROPOSPHERE IN THE MICROWAVE RANGE

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Based on the analysis of the frequency shift vs the viewing angle when the microwaves and the optical waves refract in a stratified spherical troposphere, a near-horizon angular interval in which the inverse problem of retrieval of the elevation profile of the refractive index may be solved is identified. An example of processing very noisy experimental data is presented.

Most of the refractive techniques that have been developed in investigating the Earth's atmosphere are based on solving the inverse problems in which the refraction angle and the atmospheric Doppler frequency shift in both microwave and optical ranges are measured. The problem of retrieval of the elevation profile of the refractive index from the results of observations of various aerospace sources of radiation is of great practical importance. In contrast to the techniques of atmospheric sensing in the microwave range with the help of two satellites,[1] the analytical solution of the inverse problem of sensing both in the microwave and optical range when radiation has passed once through the propagation path "satellite—Earth" is extremely unstable.[2–4] In the present article we treat theoretically the dependence of the stability of solution of this inverse problem on the sensing angle and suggest a simple technique for retrieval of the profile of the normalized refractive index of the troposphere which would not entail the direct solution of the ill-posed inverse problem.

1. When the radiation source orbited, a Doppler frequency shift appears in the satellite signal[5]

$$\Delta F(E) = f\, a \cos E \frac{dE}{dt}\ \upsilon(E)/c, \tag{1}$$

where f is the signal frequency, E is the source elevation angle, a is the Earth's radius, and c is the speed of light. This frequency shift is related to the elevation profile of the refractive index of the troposphere via the so–called refraction integral $\upsilon(E)$, which enters into the expression for the Doppler frequency shift and, according to Ref. 5, has the form

$$\nu(E) = \int_0^\infty \frac{N(u)\, u\, du}{[u^2 + s^2]^{3/2}} . \tag{2}$$

Here $s = n_a\, a \sin E$, $u^2 = r^2\, n^2\,(r) - a^2 n_0^2$, $r = a + h$, $n = 1 + N$ is the atmospheric refractive index, and n_0 is its surface value. Using the first order approximation in Eq. (2), i.e., taking into account only the terms linear in N, the refraction integral may be easily written in the form

$$\nu(E) = a \sin E \int_0^\infty \frac{N(h)\, r\, dr}{[r^2 - a^2 \cos^2 E]^{3/2}} , \tag{3}$$

where h is the height above the Earth's surface.

Let the normalized refractive index profile be described by a polyexponential model

$$N(h) = \sum_i N_i(h) = \sum_i N_{0i} \exp(-h/H_i) . \tag{4}$$

The parameters N_{0i} and H_i that, in a one-dimensional case have the meaning of the surface refractive index and the effective height of the troposphere, are used as the parameters of this model. With the aim of further solution of this problem, we assume this profile in somewhat different form

$$N_i(h) \approx N_{0i} \exp(-h/H_i) \exp(-h^2/2aH_1). \tag{5}$$

Since the running height is $h \ll a$, the exponential addition is small, and expression (5) deviates insignificantly from the exact profile (4). The advantage of such a choice of the function $N_1(h)$ consists in the possibility of integrating Eq. (3) and obtaining a closed expression for the function $\upsilon(E)$:

$$\nu(E) = \sum_i N_{0i}\, \mu\,(a \sin^2 E/2H_i),$$
$$\mu(y) = y^{1/2} \exp(y)\, \Gamma(-\tfrac{1}{2}, y)/2, \tag{6}$$

where the function $\mu(y)$ is expressed in terms of the gamma function.

Thus the problem of retrieval of our profile is reduced to approximation of the experimental data by expressions (1) and (6). The parameters of this dependence determine the profile sought. These parameters (N_{0i}, H_i) may be found employing the least-squares technique and simultaneously linearizing the resultant system of equations.

2. Direct processing of published experimental data has demonstrated that the problem is extremely unstable, so it appears impossible to retrieve the two assumed parameters using a simple exponential model. Therefore, it seems to be necessary to study the problem stability.

Starting from the asymptotic of the gamma function for large and small arguments and taking into account the fact that $H_i \ll a$, we find that the function $\upsilon(E)$ is as follows:

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$$\nu(E) \approx \sum_i N_{0i}, E - 0,$$

$$a \sin^2 E\, \nu(E) \approx \sum_i N_{0i} H_i, E - \frac{\pi}{2},$$

for glancing and vertical viewing angles. It may then be stated that for near–vertical viewing angles ($E - \pi/2$), the total normalized refractive index is decisive

$$N_\Sigma = \int_0^\infty N(h)\, dh = \sum_i N_{0i} H_i,$$

while for glancing angles ($E - 0$) the surface value of the refractive index is important

$$N_0 = N(h=0) = \sum_i N_{0i}.$$

These are the two parameters (N_0 and N_Σ) that determine the Doppler frequency shift in the corresponding angular intervals. One can hardly speak about retrieving a greater number of parameters in this case. An attempt of retrieval of the overparameterized profile results in a sharp increase of the conditional number for the corresponding system of equations of the least–squares technique and in a loss of the solution stability. In other words, both intervals of glancing and vertical angles of atmospheric sensing are found to be inapplicable for retrieval of the vertical profile of the refractive index. The solution is most stable at intermediate viewing angles. As for the vertical viewing angles, our conclusion agrees fairly well with the available data,[3,4] while it seems to be new and unexpected result for the interval of glancing angles.

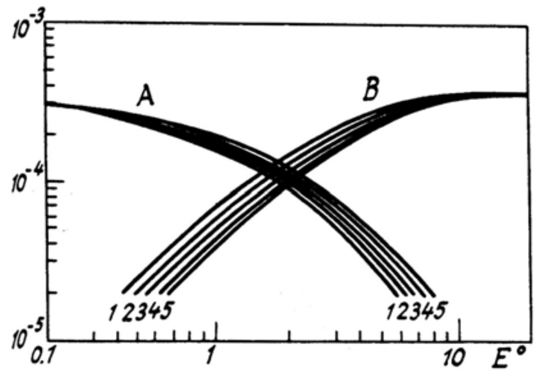

FIG. 1. Angular dependences of the functions $\upsilon(E)$ *(family of curves A) and* $\upsilon(E)\sin^2E$ *(family of curves B) at altitudes H = 6–10* km *(curves 1–5).*

To estimate the stability region, let us consider a simple exponential profile. Figure 1 shows the character of the angular dependence of the values $\upsilon(E)$ (family of curves A) and $\upsilon(E)\sin^2 E$ (family of curves B) for various effective heights of the troposphere. Curves 1–5 correspond to the values H = 6, 7, 8, 9, and 10 km. The dependence of $\upsilon(E)$ and $\upsilon(E)\sin^2E$ on the parameter H is manifested only in the interval of angles $E \in [1, 10°)$. This interval seems to be a transition from one decisive parameter – the total refractive index, to the other – the surface value of $N(h)$, so that it required the two–parametric description. We may choose any pair from the parameters N_0, N_Σ, and H for the profile parameters. In one of these possible combinations (N_o, N_o/H) the second parameter N_o/H has the meaning of the surface refractive index gradient. The above–indicated interval embraces the stability region of the inverse problem of tropospheric sensing in the microwave range in which one can to retrieve the fine structure of the elevation profile of the refractive index.

Note that the atmosphere Is assumed to be spherically symmetric. As we have pointed out above, if this condition were strictly satisfied, the single parameter to be determined would be the surface refractive index N_0 in the troposphere. However, certain deviations from this trend may be found in reality resulting from the possible effect of horizontal gradients In the refractive index.

3. Let us consider an example of solving the Inverse problem of retrieval the profile from the experimental data given in Ref. 6 using an exponential approximation of $N(h)$. The experimental data are represented in the form of the Doppler frequency shifts obtained in sensing of the atmosphere at frequency f = 108 MHz. The angle E varied within the limits 1–14° during the sensing session. Comparing these data with those in Fig. 1, one can see that these values lie in the angular interval which enables one to retrieve both parameters of the profile. Nevertheless, these two parameters cannot be retrieved independently, because of strong noise in the data. Therefore, in solving the problem we applied the normalization condition which permits to relate the profile parameters with the help of the typical values of the profile itself. In such a way the number of the parameters sought is reduced to 1. The following normalizing values were used: $N(h) = 93 \cdot 10^{-8}$ at an altitude of h = 10 km.

Figure 2 shows the graphic interpretation of the solution. The circles show the set of experimental data and solid lines give the approximating curves corresponding to the different prescribed values of the parameter H within the 5–10 km altitude range. The parameters of the approximating curve that correspond to experimental data and are retrieved following the above–given technique, are as follows: the effective height of the troposphere H is equal to 8.6 km, the total refractive index N_Σ is equal to 2.5 m, and the surface refractive index N_0 is $298.7 \cdot 10^{-6}$. The obtained values are in good agreement with the results of meteorological measurements carried out at different times.[7]

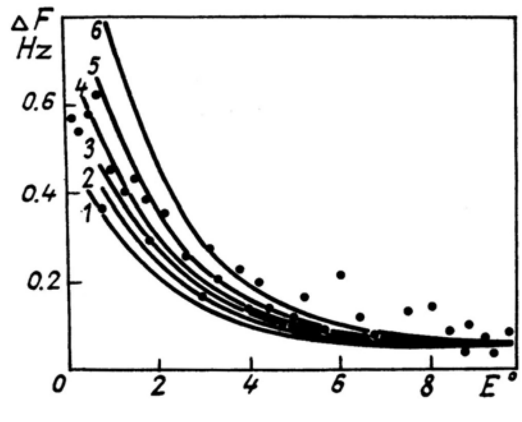

FIG. 2. Single–parameter approximation of the observed Doppler frequency shift in the troposphere at H = 5–10 km *(curves 1–6).*

In the case of a single–parameter linear problem the solution accuracy coincides with that of the initial data on microwave refraction. In other words, it is determined by their spread around the mean value. In our case, this value is close to 10%. Apparently, if more accurate data were available that could result in accuracy of the solution, so that a solution for the case of polyexponential structure could be obtained. The above analysis of the solution stability for the inverse problem of sensing of the atmosphere showed that in the case, in which one of the correspondents was at the Earth's surface, the retrieval of the vertical structure of the normalized refractive index in the troposphere was possible only with the use of the data obtained in the interval of the viewing angles from 1 to 10°. An example presented here illustrates the possibilities of retrieval of the vertical profile of the refractive index from the very noisy experimental data.

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